# Wavelet Toolbox™

Reference

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R2015a

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*Wavelet Toolbox™ Reference*

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# addlift

Add lifting steps to lifting scheme

## Syntax

```
LSN = addlift(LS,ELS)
LSN = addlift(LS,ELS,'begin')
LSN = addlift(LS,ELS,'end')
addfilt(LS,ELS)
```

## Description

`LSN = addlift(LS,ELS)` returns the new lifting scheme *LSN* obtained by appending the elementary lifting step *ELS* to the lifting scheme *LS*.

`LSN = addlift(LS,ELS,'begin')` prepends the specified elementary lifting step.

*ELS* is either a cell array (see `lsinfo`)

```
{TYPEVAL, COEFS, MAX_DEG}
```

or a structure (see `liftfilt`)

```
struct('type',TYPEVAL,'value',LPVAL)
```

with

```
LPVAL = laurpoly(COEFS, MAX_DEG)
```

`LSN = addlift(LS,ELS,'end')` is equivalent to `addfilt(LS,ELS)`.

If *ELS* is a sequence of elementary lifting steps, stored in a cell array or an array of structures, then each of the elementary lifting steps is added to *LS*.

For more information about lifting schemes, see `lsinfo`.

## Add Primal Lifting Step

This example shows how to start with the Haar lifting scheme and add a primal lifting step.

```
LSbegin = liftwave('haar');
```

Display the lifting scheme.

```
displs(LSbegin);
```

```
LSbegin = {...
'd'               [ -1.00000000]  [0]
'p'               [  0.50000000]  [0]
[  1.41421356]  [  0.70710678]  []
};
```

Create a primal lifting step.

```
pstep = { 'p', [-1 2 -1]/4 , 1 };
```

Add the primal lifting step.

```
LSend = addlift(LSbegin,pstep);
```

Display the final lifting scheme.

```
displs(LSend);
```

```
LSend = {...
'd'               [ -1.00000000]                              [0]
'p'               [  0.50000000]                              [0]
'p'               [ -0.25000000  0.50000000 -0.25000000]  [1]
[  1.41421356]  [  0.70710678]                              []
};
```

### See Also

`liftfilt`

## allnodes

Tree nodes

### Syntax

```
N = allnodes(T)
N = allnodes(T,'deppos')
```

### Description

`allnodes` is a tree management utility that returns one of two node descriptions: either indices, or depths and positions.

The nodes are numbered from left to right and from top to bottom. The root index is 0.

`N = allnodes(T)` returns the indices of all the nodes of the tree *T* in column vector *N*.

`N = allnodes(T,'deppos')` returns the depths and positions of all the nodes in matrix *N*.

`N(i,1)` is the depth and `N(i,2)` the position of the node `i`.

### Return Nodes of Wavelet Packet Tree

This example shows how to obtain the depth-position and linear indices of a wavelet packet tree.

Load the noisy Doppler signal and obtain the wavelet packet decomposition down to the level 4 using the `'db2'` wavelet.

```
load noisdopp;
T = wpdec(noisdopp,4,'db2');
```

Obtain the depth-position indices.

```
DepthPosition = allnodes(T,'deppos');
```

Obtain the corresponding linear indices.

```
LinearIndices = allnodes(T);
```

Display the correspondance in a table.

```
table(DepthPosition,LinearIndices)
```

```
ans = 

    DepthPosition    LinearIndices
    _____________    _____________

    0     0                0      
    1     0                1      
    1     1                2      
    2     0                3      
    2     1                4      
    2     2                5      
    2     3                6      
    3     0                7      
    3     1                8      
    3     2                9      
    3     3               10      
    3     4               11      
    3     5               12      
    3     6               13      
    3     7               14      
    4     0               15      
    4     1               16      
    4     2               17      
    4     3               18      
    4     4               19      
    4     5               20      
    4     6               21      
    4     7               22      
    4     8               23      
    4     9               24      
    4    10               25      
    4    11               26      
    4    12               27      
    4    13               28      
    4    14               29      
    4    15               30      
```

# appcoef

1-D approximation coefficients

## Syntax

```
A = appcoef(C,L,'wname',N)
A = appcoef(C,L,'wname')
A = appcoef(C,L,Lo_R,Hi_R)
A = appcoef(C,L,Lo_R,Hi_R,N)
```

## Description

`appcoef` is a one-dimensional wavelet analysis function.

`appcoef` computes the approximation coefficients of a one-dimensional signal.

`A = appcoef(C,L,'wname',N)` computes the approximation coefficients at level *N* using the wavelet decomposition structure `[C,L]` (see `wavedec` for more information).

'*wname*' is a string containing the wavelet name. Level *N* must be an integer such that `0 ≤ N ≤ length(L)-2`.

`A = appcoef(C,L,'wname')` extracts the approximation coefficients at the last level: length`(`*L*`)-2`.

Instead of giving the wavelet name, you can give the filters.

For `A = appcoef(C,L,Lo_R,Hi_R)` or `A = appcoef(C,L,Lo_R,Hi_R,N)`, *Lo_R* is the reconstruction low-pass filter and *Hi_R* is the reconstruction high-pass filter (see `wfilters` for more information).

## Level 3 Approximation Coefficients

This example shows how to extract the level 3 approximation coefficients.

Load the signal consisting of electricity usage data.

```
load leleccum;
```

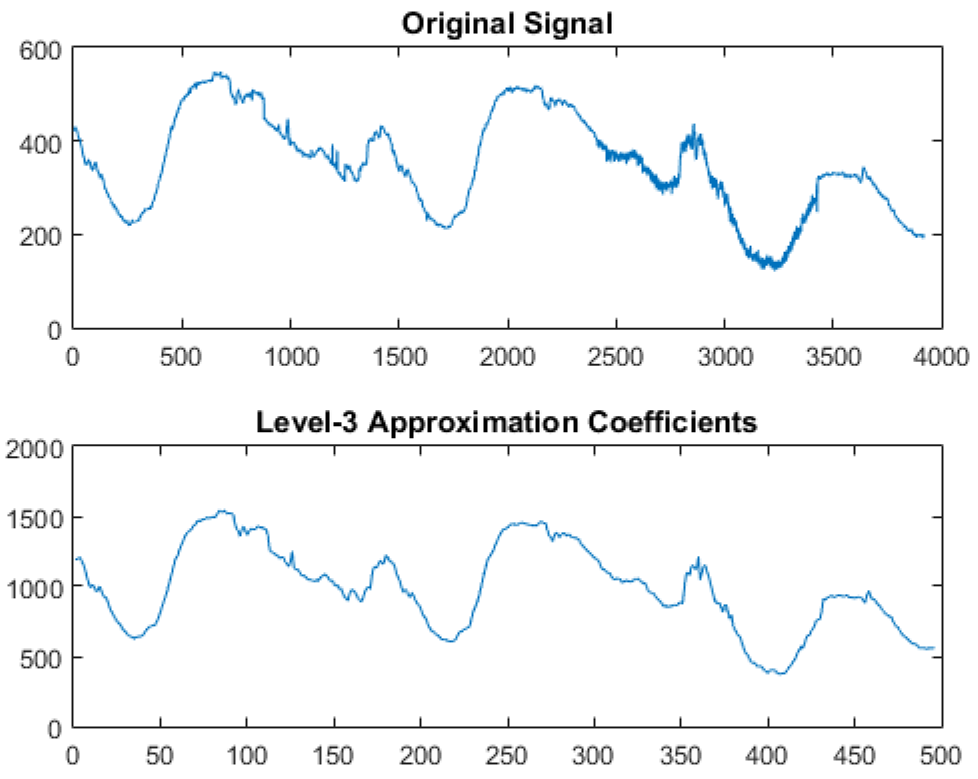
```
sig = leleccum(1:3920);
```

Obtain the DWT down to level 5 with the `'sym4'` wavelet.

```
[C,L] = wavedec(sig,5,'sym4');
```

Extract the level-3 approximation coefficients. Plot the original signal and the approximation coefficients.

```
Lev = 3;
a3 = appcoef(C,L,'sym4',Lev);
subplot(2,1,1)
plot(sig); title('Original Signal');
subplot(2,1,2)
plot(a3); title('Level-3 Approximation Coefficients');
```



You can substitute any value from 1 to 5 for `Lev` to obtain the approximation coefficients for the corresponding level.

## More About

### Algorithms

The input vectors *C* and *L* contain all the information about the signal decomposition.

Let `NMAX = length(L)-2`; then `C = [A(NMAX) D(NMAX) ... D(1)]` where *A* and the *D* are vectors.

If `N = NMAX`, then a simple extraction is done; otherwise, `appcoef` computes iteratively the approximation coefficients using the inverse wavelet transform.

## See Also

`detcoef` | `wavedec`

## appcoef2

2-D approximation coefficients

### Syntax

```
A = appcoef2(C,S,'wname',N)
A = appcoef2(C,S,'wname')
A = appcoef2(C,S,Lo_R,Hi_R)
A = appcoef2(C,S,Lo_R,Hi_R,N)
```

### Description

`appcoef2` is a two-dimensional wavelet analysis function. It computes the approximation coefficients of a two-dimensional signal. The syntaxes allow you to give the wavelet name or the filters as inputs.

`A = appcoef2(C,S,'wname',N)` computes the approximation coefficients at level *N* using the wavelet decomposition structure [*C*,*S*] (see `wavedec2` for more information).

`'wname'` is a string containing the wavelet name. Level *N* must be an integer such that `0` ≤ `N` ≤ `size(S,1)-2`.

`A = appcoef2(C,S,'wname')` extracts the approximation coefficients at the last level: `size(S,1)-2`.

`A = appcoef2(C,S,Lo_R,Hi_R)` or `A = appcoef2(C,S,Lo_R,Hi_R,N)`, *Lo_R* is the reconstruction low-pass filter and *Hi_R* is the reconstruction high-pass filter (see `wfilters` for more information).

### Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load original image.
load woman;
```

```
% X contains the loaded image.

% Perform decomposition at level 2
% of X using db1.
[c,s] = wavedec2(X,2,'db1');
sizex = size(X)
sizex =
    256 256

sizec = size(c)

sizec =
     1     65536
    val_s = s

val_s =
    64     64
    64     64
   128    128
   256    256

% Extract approximation coefficients
% at level 2.
ca2 = appcoef2(c,s,'db1',2);
sizeca2 = size(ca2)

sizeca2 =
    64     64

% Compute approximation coefficients
% at level 1.
ca1 = appcoef2(c,s,'db1',1);
sizeca1 = size(ca1)

sizeca1 =
   128    128
```

## More About

### Tips

If C and S are obtained from an indexed image analysis or a truecolor image analysis, A is an m-by-n matrix or an m-by-n-by-3 array, respectively.

For more information on image formats, see the `image` and `imfinfo` reference pages.

### Algorithms

The algorithm is built on the same principle as `appcoef`.

## See Also

`detcoef2` | `wavedec2`

# bestlevt

Best level tree wavelet packet analysis

## Syntax

```
T = bestlevt(T)
[T,E] = bestlevt(T)
```

## Description

`bestlevt` is a one- or two-dimensional wavelet packet analysis function.

`bestlevt` computes the optimal complete subtree of an initial tree with respect to an entropy type criterion. The resulting complete tree may be of smaller depth than the initial one.

`T = bestlevt(T)` computes the modified wavelet packet tree *T* corresponding to the best level tree decomposition.

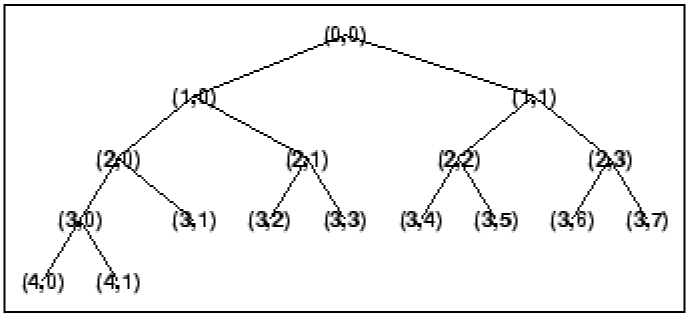
`[T,E] = bestlevt(T)` computes the best level tree *T*, and in addition, the best entropy value *E*.

The optimal entropy of the node, whose index is `j-1`, is `E(j)`.

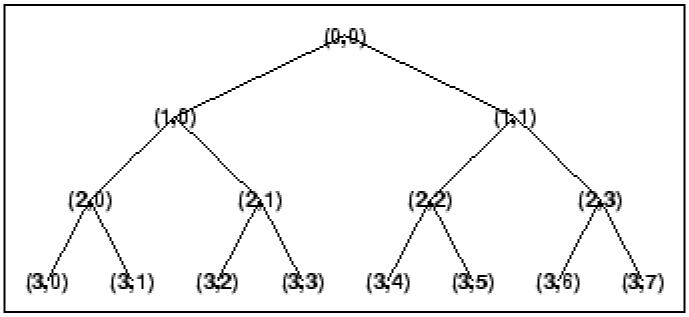
## Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load signal.
load noisdopp;
x = noisdopp;

% Decompose x at depth 3 with db1 wavelet, using default
% entropy (shannon).
wpt = wpdec(x,3,'db1');
```

```
% Decompose the packet [3 0].
wpt = wpsplt(wpt,[3 0]);

% Plot wavelet packet tree wpt.
plot(wpt)
```



```
% Compute best level tree.
blt = bestlevt(wpt);

% Plot best level tree blt.
plot(blt)
```



## More About

### Algorithms

See `besttree` algorithm section. The only difference is that the optimal tree is searched among the complete subtrees of the initial tree, instead of among all the binary subtrees.

## See Also

`besttree` | `wpdec` | `wenergy` | `wpdec2`

## besttree

Best tree wavelet packet analysis

### Syntax

```
T = besttree(T)
[T,E] = besttree(T)
[T,E,N] = besttree(T)
```

### Description

`besttree` is a one- or two-dimensional wavelet packet analysis function that computes the optimal subtree of an initial tree with respect to an entropy type criterion. The resulting tree may be much smaller than the initial one.

Following the organization of the wavelet packets library, it is natural to count the decompositions issued from a given orthogonal wavelet.

A signal of length $N = 2^L$ can be expanded in α different ways, where α is the number of binary subtrees of a complete binary tree of depth $L$.

As a result, we can conclude that $\alpha \geq 2^{N/2}$ (for more information, see the Mallat's book given in References at page 323).

This number may be very large, and since explicit enumeration is generally intractable, it is interesting to find an optimal decomposition with respect to a convenient criterion, computable by an efficient algorithm. We are looking for a minimum of the criterion.

`T = besttree(T)` computes the best tree *T* corresponding to the best entropy value.

`[T,E] = besttree(T)` computes the best tree *T* and, in addition, the best entropy value *E*.

The optimal entropy of the node, whose index is `j-1`, is `E(j)`.

`[T,E,N] = besttree(T)` computes the best tree *T*, the best entropy value *E* and, in addition, the vector *N* containing the indices of the merged nodes.

## Best Wavelet Packet Tree

This example shows to obtain the optimal wavelet packet tree based on an entropy criterion.

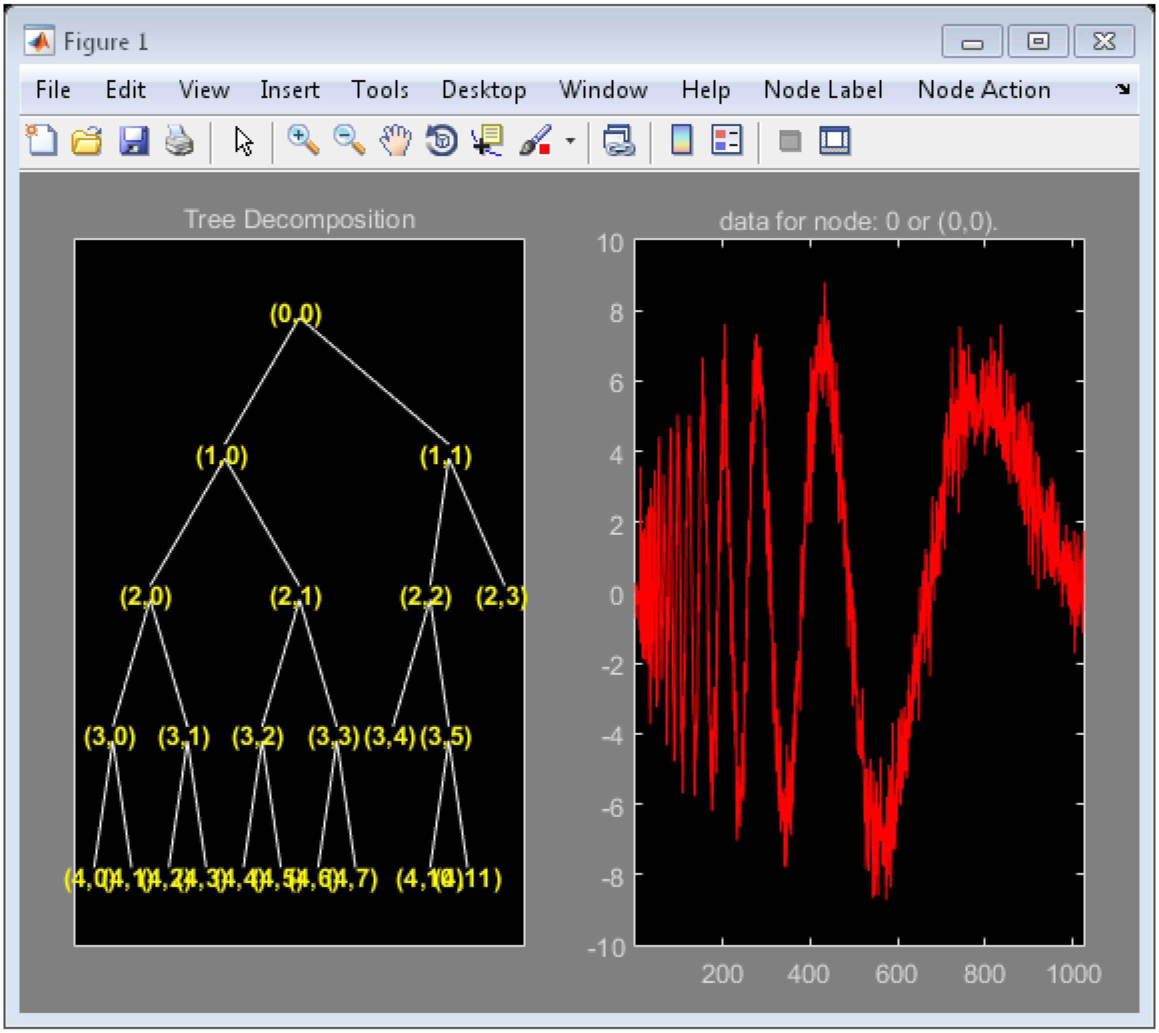
Load the noisy Doppler signal. Obtain the wavelet packet tree down to level 4 with the `'sym4'` wavelet. Use the periodic extension mode.

```
dwtmode('per');
load noisdopp;
T = wpdec(noisdopp,4,'sym4');
```

```
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!  WARNING: Change DWT Extension Mode  !
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

****************************************
**  DWT Extension Mode: Periodization  **
****************************************
```

Obtain the best wavelet packet tree and plot the result.

```
BstTree = besttree(T);
plot(BstTree)
```



Return the DWT extension mode to the default value.

```
dwtmode('sym');
```

```
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!  WARNING: Change DWT Extension Mode  !
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
```

```
**************************************************
**  DWT Extension Mode: Symmetrization (half-point)  **
**************************************************
```

## More About

### Algorithms

Consider the one-dimensional case. Starting with the root node, the best tree is calculated using the following scheme. A node N is split into two nodes N1 and N2 if and only if the sum of the entropy of N1 and N2 is lower than the entropy of N. This is a local criterion based only on the information available at the node N.

Several entropy type criteria can be used (see `wenergy` for more information). If the entropy function is an additive function along the wavelet packet coefficients, this algorithm leads to the best tree.

Starting from an initial tree T and using the merging side of this algorithm, we obtain the best tree among all the binary subtrees of T.

- "Reconstructing a Signal Approximation from a Node"

## References

Coifman, R.R.; M.V. Wickerhauser (1992), "Entropy-based algorithms for best basis selection," *IEEE Trans. on Inf. Theory*, vol. 38, 2, pp. 713–718.

Mallat, S. (1998), *A wavelet tour of signal processing*, Academic Press.

## See Also

`bestlevt` | `wenergy` | `wpcoef` | `wpdec` | `wpdec2` | `wprcoef`

# biorfilt

Biorthogonal wavelet filter set

## Syntax

```
[Lo_D,Hi_D,Lo_R,Hi_R] = biorfilt(DF,RF)
[Lo_D1,Hi_D1,Lo_R1,Hi_R1,Lo_D2,Hi_D2,Lo_R2,Hi_R2] =
biorfilt(DF,RF,'8')
```

## Description

The `biorfilt` command returns either four or eight filters associated with biorthogonal wavelets.

`[Lo_D,Hi_D,Lo_R,Hi_R] = biorfilt(DF,RF)` computes four filters associated with the biorthogonal wavelet specified by decomposition filter *DF* and reconstruction filter *RF*. These filters are

| | |
|---|---|
| `Lo_D` | Decomposition low-pass filter |
| `Hi_D` | Decomposition high-pass filter |
| `Lo_R` | Reconstruction low-pass filter |
| `Hi_R` | Reconstruction high-pass filter |

`[Lo_D1,Hi_D1,Lo_R1,Hi_R1,Lo_D2,Hi_D2,Lo_R2,Hi_R2] = biorfilt(DF,RF,'8')` returns eight filters, the first four associated with the decomposition wavelet, and the last four associated with the reconstruction wavelet.

It is well known in the subband filtering community that if the same FIR filters are used for reconstruction and decomposition, then symmetry and exact reconstruction are incompatible (except with the Haar wavelet). Therefore, with biorthogonal filters, two wavelets are introduced instead of just one:

One wavelet, $\tilde{\psi}$ , is used in the analysis, and the coefficients of a signal $s$ are

$$\tilde{c}_{j,k} = \int s(x)\tilde{\psi}_{j,k}(x)dx$$

The other wavelet, ψ, is used in the synthesis:

$$s = \sum_{j,k} \tilde{c}_{j,k}\psi_{j,k}$$

Furthermore, the two wavelets are related by duality in the following sense:
$\int \tilde{\psi}_{j,k}(x)\psi_{j',k'}(x)dx = 0$ as soon as $j \neq j'$ or $k \neq k'$ and

$\int \tilde{\phi}_{0,k}(x)\phi_{0,k'}(x)dx = 0$ as soon as $k \neq k'$.

It becomes apparent, as A. Cohen pointed out in his thesis (p. 110), that “the useful properties for analysis (e.g., oscillations, null moments) can be concentrated in the $\tilde{\psi}$ function; whereas, the interesting properties for synthesis (regularity) are assigned to the ψ function. The separation of these two tasks proves very useful.”

$\tilde{\psi}$ and ψ can have very different regularity properties, ψ being more regular than $\tilde{\psi}$ .

The $\tilde{\psi}$ , ψ, $\tilde{\phi}$ and ϕ functions are zero outside a segment.

## Biorthogonal Filters and Transfer Functions

This example shows how to obtain the decompositition (analysis) and reconstruction (synthesis) filters for the `'bior3.5'` wavelet.

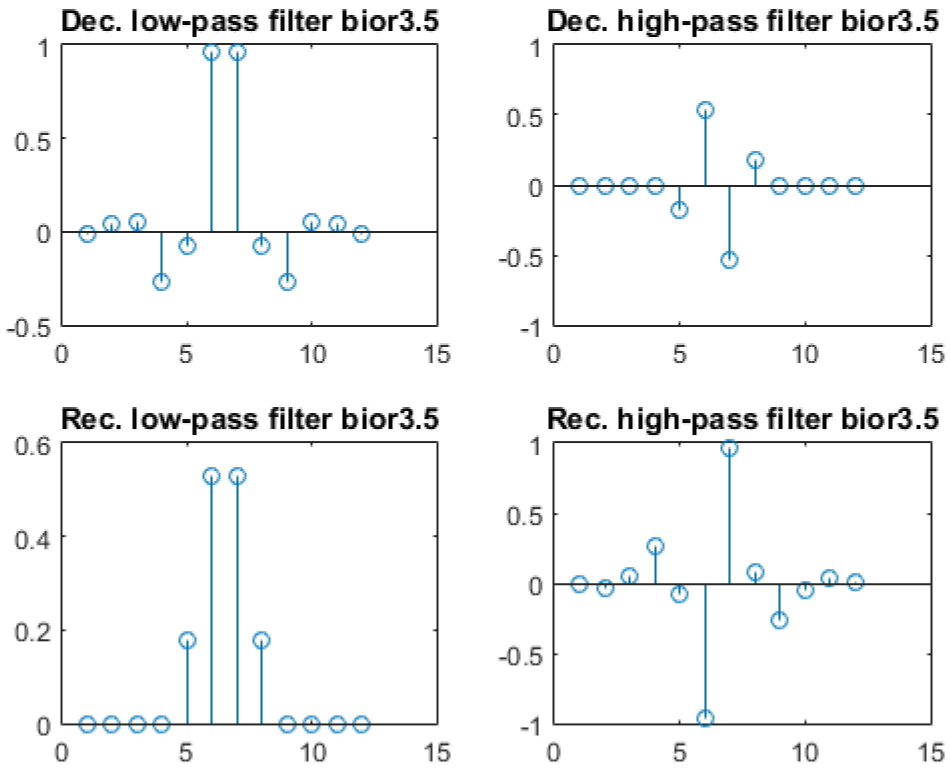
Determine the two scaling and wavelet filters associated with the `'bior3.5'` wavelet.

```
[Rf,Df] = biorwavf('bior3.5');
[LoD,HiD,LoR,HiR] = biorfilt(Df,Rf);
```

Plot the filter impulse responses.

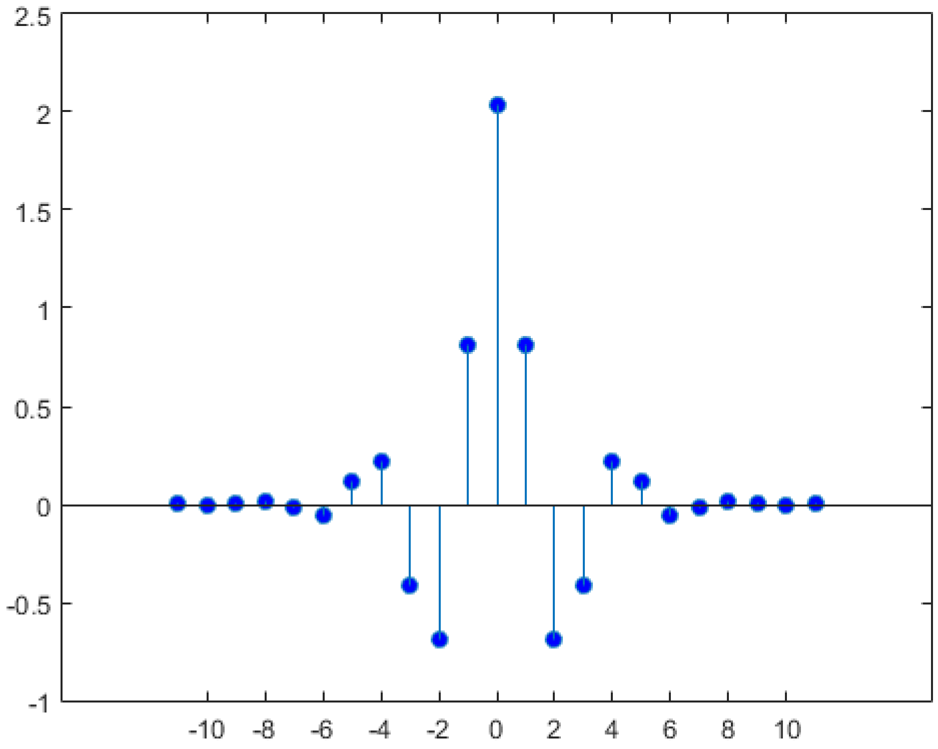
```
subplot(221); stem(LoD);
title('Dec. low-pass filter bior3.5');
subplot(222); stem(HiD);
title('Dec. high-pass filter bior3.5');
```

```
subplot(223); stem(LoR);
title('Rec. low-pass filter bior3.5');
subplot(224); stem(HiR);
title('Rec. high-pass filter bior3.5');
```



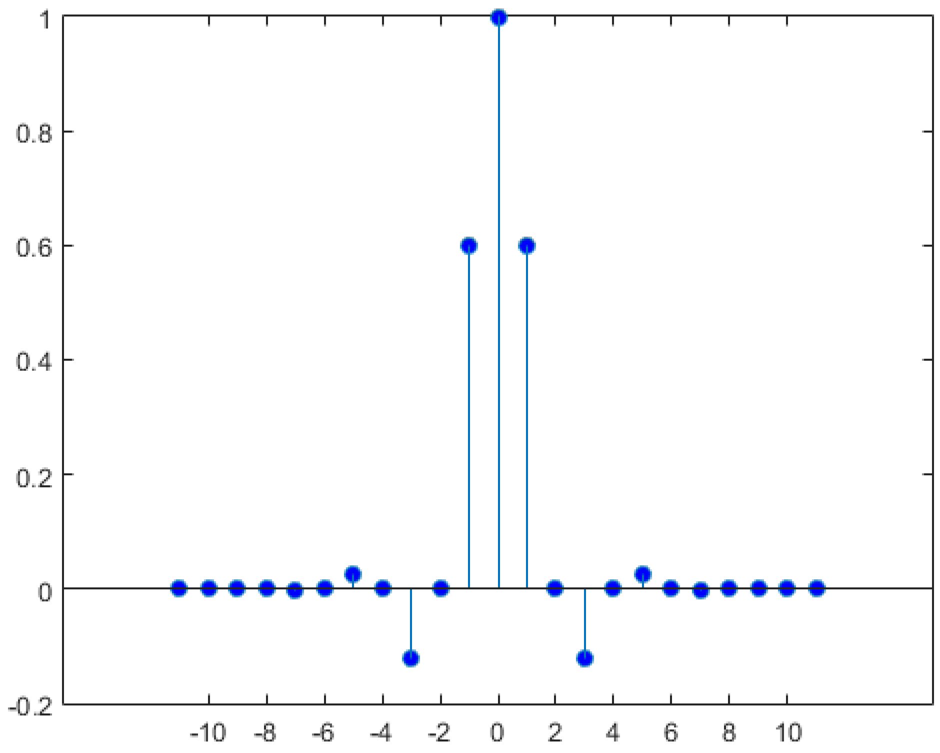
Demonstrate that autocorrelations at even lags are only zero for dual pairs of filters. Examine the autocorrelation sequence for the lowpass decomposition filter.

```
npad = 2*length(LoD)-1;
LoDxcr = fftshift(ifft(abs(fft(LoD,npad)).^2));
lags = -floor(npad/2):floor(npad/2);
figure;
stem(lags,LoDxcr,'markerfacecolor',[0 0 1])
set(gca,'xtick',-10:2:10)
```



Examine the cross correlation sequence for the lowpass decomposition and synthesis filters. Compare the result with the preceding figure.

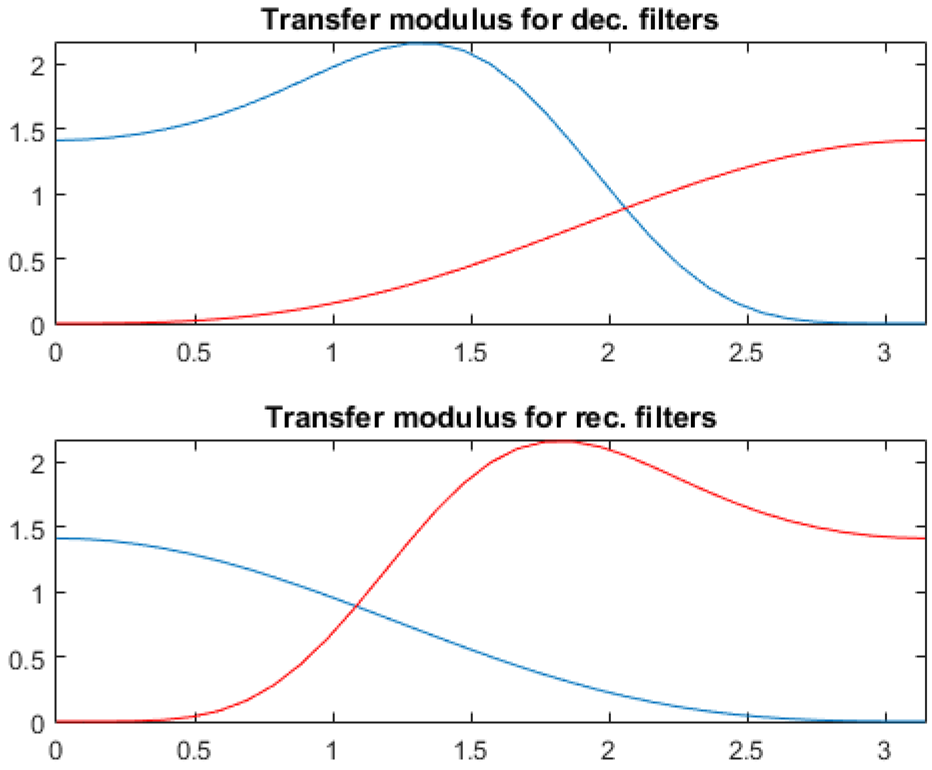
```
npad = 2*length(LoD)-1;
xcr = fftshift(ifft(fft(LoD,npad).*conj(fft(LoR,npad))));
lags = -floor(npad/2):floor(npad/2);
stem(lags,xcr,'markerfacecolor',[0 0 1])
set(gca,'xtick',-10:2:10)
```



Compare the transfer functions of the analysis and synthesis scaling and wavelet filters

```
dftLoD = fft(LoD,64);
dftLoD = dftLoD(1:length(dftLoD)/2+1);
dftHiD= fft(HiD,64);
dftHiD = dftHiD(1:length(dftHiD)/2+1);
dftLoR = fft(LoR,64);
dftLoR = dftLoR(1:length(dftLoR)/2+1);
dftHiR = fft(HiR,64);
dftHiR = dftHiR(1:length(dftHiR)/2+1);
df = (2*pi)/64;
freqvec = 0:df:pi;

subplot(211); plot(freqvec,abs(dftLoD),freqvec,abs(dftHiD),'r');
```

```
axis tight;
title('Transfer modulus for dec. filters')
subplot(212); plot(freqvec,abs(dftLoR),freqvec,abs(dftHiR),'r');
axis tight;
title('Transfer modulus for rec. filters')
```



## References

Cohen, A. (1992), “Ondelettes, analyses multirésolution et traitement numérique du signal,” *Ph. D. Thesis*, University of Paris IX, DAUPHINE.

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics. SIAM Ed.

## See Also

`biorwavf` | `orthfilt`

# biorwavf

Biorthogonal spline wavelet filter

## Syntax

```
[RF,DF] = biorwavf(W)
```

## Description

`[RF,DF] = biorwavf(W)` returns the reconstruction (synthesis) and decomposition (analysis) filters associated with the biorthogonal wavelet specified by the string `W`.

`W = 'biorNr.Nd'` where possible values for *`Nr`* and *`Nd`* are

| | |
|---|---|
| `Nr = 1` | `Nd = 1 , 3 or 5` |
| `Nr = 2` | `Nd = 2 , 4 , 6 or 8` |
| `Nr = 3` | `Nd = 1 , 3 , 5 , 7 or 9` |
| `Nr = 4` | `Nd = 4` |
| `Nr = 5` | `Nd = 5` |
| `Nr = 6` | `Nd = 8` |

The output arguments are filters.

- `RF` is the reconstruction filter.
- `DF` is the decomposition filter.

## Examples

Return the biorthogonal spline wavelet scaling filters with 2 vanishing moments.

```
wname = 'bior2.2';
[RF,DF] = biorwavf(wname);
```

## See Also

`biorfilt` | `waveinfo`

# bswfun

Biorthogonal scaling and wavelet functions

## Syntax

```
[PHIS,PSIS,PHIA,PSIA,XVAL] = bswfun(LoD,HiD,LoR,HiR)
bswfun(LoD,HiD,LoR,HiR,ITER)
bswfun(LoD,HiD,LoR,HiR,'plot')
bswfun(LoD,HiD,LoR,HiR,ITER,'plot')
bswfun(LoD,HiD,LoR,HiR,'plot',ITER)
```

## Description

`[PHIS,PSIS,PHIA,PSIA,XVAL] = bswfun(LoD,HiD,LoR,HiR)` returns approximations on the grid `XVAL` of the two pairs of biorthogonal scaling and wavelet functions. `PHIS` and `PSIS` are the scaling and wavelet functions constructed from the decomposition filters, `LoD` and `HiD`. `PHIA` and `PSIA` are the scaling and wavelet functions constructed from the reconstruction filters, `LoR` and `HiR`.

`bswfun(LoD,HiD,LoR,HiR,ITER)` computes the two pairs of scaling and wavelet functions using ITER iterations.

`bswfun(LoD,HiD,LoR,HiR,'plot')` or `bswfun(LoD,HiD,LoR,HiR,ITER,'plot')` or `bswfun(LoD,HiD,LoR,HiR,'plot',ITER)` computes and plots the functions.

## Examples

### Biorthogonal Scaling and Wavelet from Lifting Scheme

This example shows how to obtain the biorthogonal scaling and wavelet functions corresponding to a lifting scheme. Obtain the lifting scheme for the CDF 3/1 wavelet.

```
lscdf = liftwave('cdf3.1');
```

Display the lifting scheme, which consists of two primal and one dual step.

```
Sc = displs(lscdf);
Sc
```

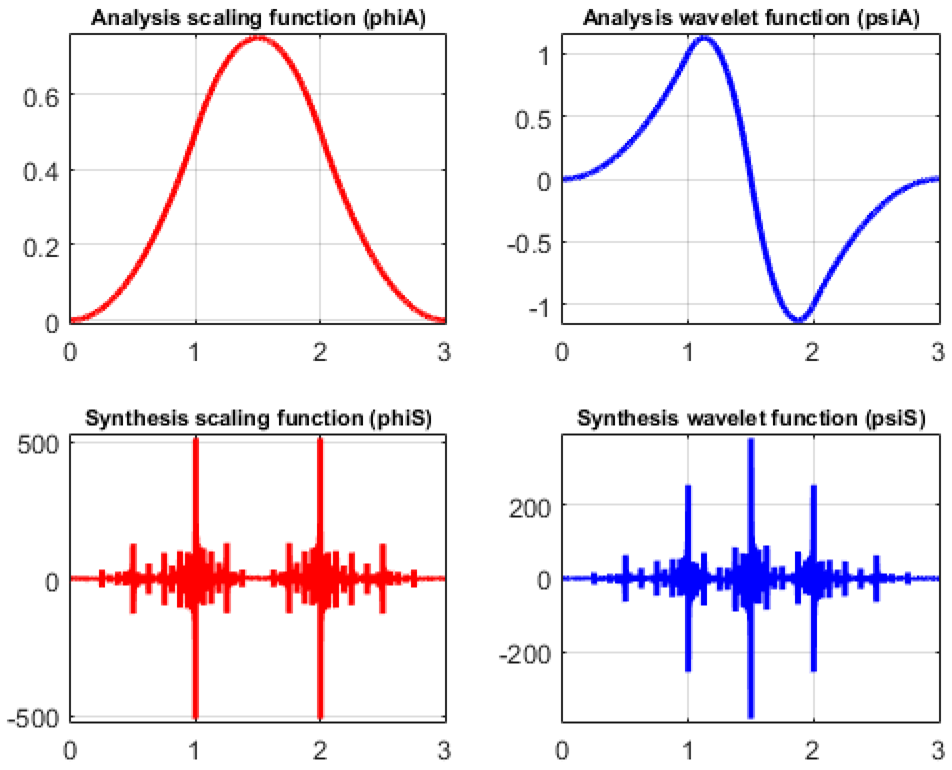
```
Sc =

lscdf = {...
'p'             [ -0.33333333]               [-1]
'd'             [ -0.37500000 -1.12500000]  [1]
'p'             [  0.44444444]               [0]
[  2.12132034]  [  0.47140452]               []
};
```

Obtain the decomposition and reconstruction filters from the lifting scheme.

```
[LoD,HiD,LoR,HiR] = ls2filt(lscdf);
```

Visualize the scaling and wavelet function and their duals.

```
bswfun(LoD,HiD,LoR,HiR,'plot');
```



## More About

### Algorithms

This function uses the cascade algorithm.

## See Also

`wavefun`

## centfrq

Wavelet center frequency

### Syntax

```
FREQ = centfrq('wname')
FREQ = centfrq('wname',ITER)
[FREQ,XVAL,RECFREQ] = centfrq('wname',ITER,'plot')
```

### Description

`FREQ = centfrq('wname')` returns the center frequency in herz of the wavelet function, `'wname'` (see `wavefun` for more information).

For `FREQ = centfrq('wname',ITER)`, `ITER` is the number of iterations performed by the function `wavefun`, which is used to compute the wavelet.

`[FREQ,XVAL,RECFREQ] = centfrq('wname',ITER,'plot')` returns, in addition, the associated center frequency based approximation `RECFREQ` on the $2^{\mathrm{ITER}}$ points grid `XVAL` and plots the wavelet function and `RECFREQ`.

### Examples

#### Determine Center Frequency

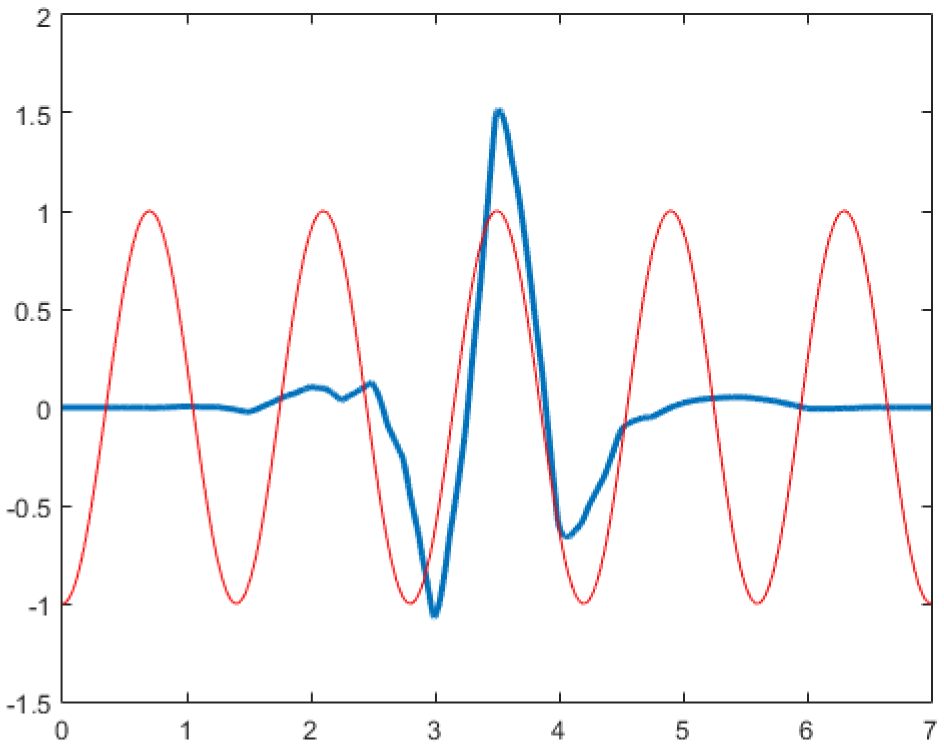
This example shows how to determine the center frequency in hertz for Daubechies' least-asymmetric wavelet with 4 vanishing moments.

```
cfreq = centfrq('sym4');
```

Obtain the wavelet and create a sine wave with a frequency equal to the center frequency, `cfreq`, of the wavelet. Use a starting phase of $-\pi$ for the sine wave to visualize how the oscillation in the sine wave matches the oscillation in the wavelet.

```
[~,psi,xval] = wavefun('sym4');
y = cos(2*pi*cfreq*xval-pi);
```

```
plot(xval,psi,'linewidth',2);
hold on;
plot(xval,y,'r');
```



### Convert Scales to Frequencies

This example shows to convert scales to frequencies for the Morlet wavelet. There is an approximate inverse relationship between scale and frequency. Specifically, scale is inversely proportional to frequency with the constant of proportionality being the center frequency of the wavelet.

Construct a vector of scales with 32 voices per octave over 5 octaves for data sampled at 1 kHz.

```
Fs = 1000;
numvoices = 32;
a0 = 2^(1/numvoices);
numoctaves = 5;
scales = a0.^(numvoices:1/numvoices:numvoices*numoctaves).*1/Fs;
```

Convert the scales to approximate frequencies in hertz for the Morlet wavelet.

```
Frq = centfrq('morl')./scales;
```

You can also use `scal2frq` to convert scales to approximate frequencies in hertz.

## See Also

`scal2frq` | `wavefun`

# cfs2wpt

Wavelet packet tree construction from coefficients

## Syntax

## Description

`CFS2WPT` builds a wavelet packet tree (T) and the related analyzed signal or image (X) using the following input information:

*`WNAME`*: name of the wavelet used for the analysis

*`SIZE_OF_DATA`*: size of the analyzed signal or image

*`TN_OF_TREE`*: vector containing the terminal node indices of the tree

*`ORDER`*: 2 for a signal or 4 for an image

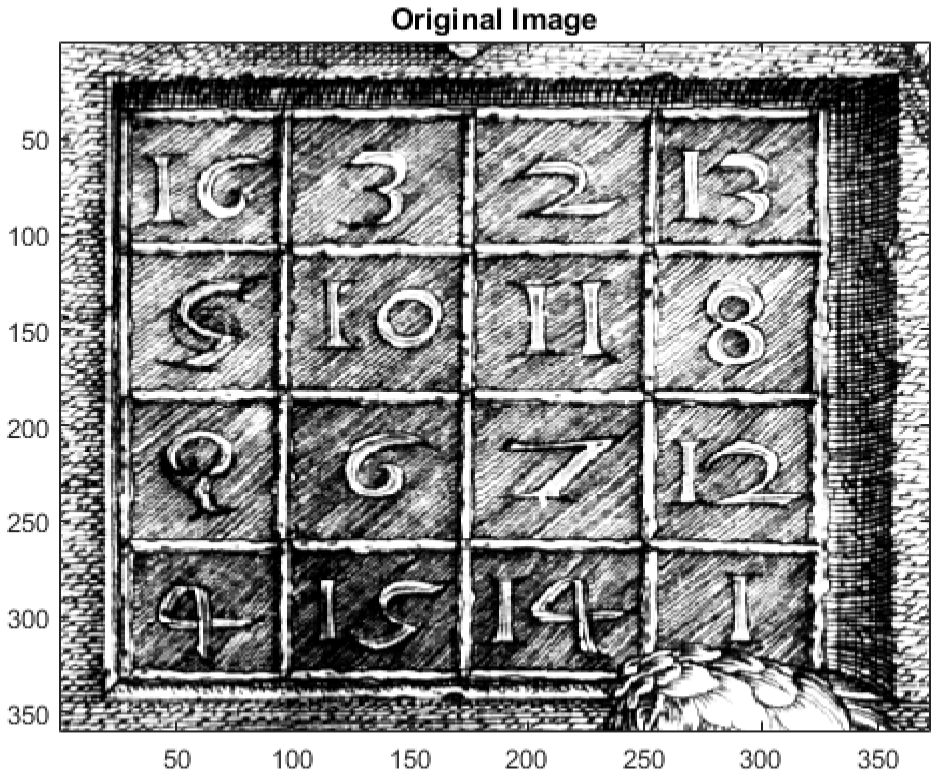
*`CFS`*: coefficients used to reconstruct the original signal or image. *`CFS`* is optional. When `CFS2WPT` is used without the *`CFS`* input parameter, the wavelet packet tree structure (T) is generated, but all the tree coefficients are null (including X).

## Build Wavelet Packet Tree

This example shows how to build a wavelet packet tree in two ways: 1.) By filling the wavelet packet tree with coefficients, and 2.) By creating the wavelet packet tree and using `write`

Load an image and obtain the wavelet packet decomposition down to level 2 with the `'sym4'` wavelet.

```
load detail;
imagesc(X); colormap gray; title('Original Image');
Tr = wpdec2(X,2,'sym4');
```



Read the coefficients from the wavelet packet tree. Add $N(0,40^2)$ noise to the coefficients and plot the new wavelet packet tree.

```
cfs = read(Tr,'allcfs');
noisyCfs = cfs + 40*rand(size(cfs));
noisyT = cfs2wpt('sym4',size(X),tnodes(Tr),4,noisyCfs);
plot(noisyT)
```

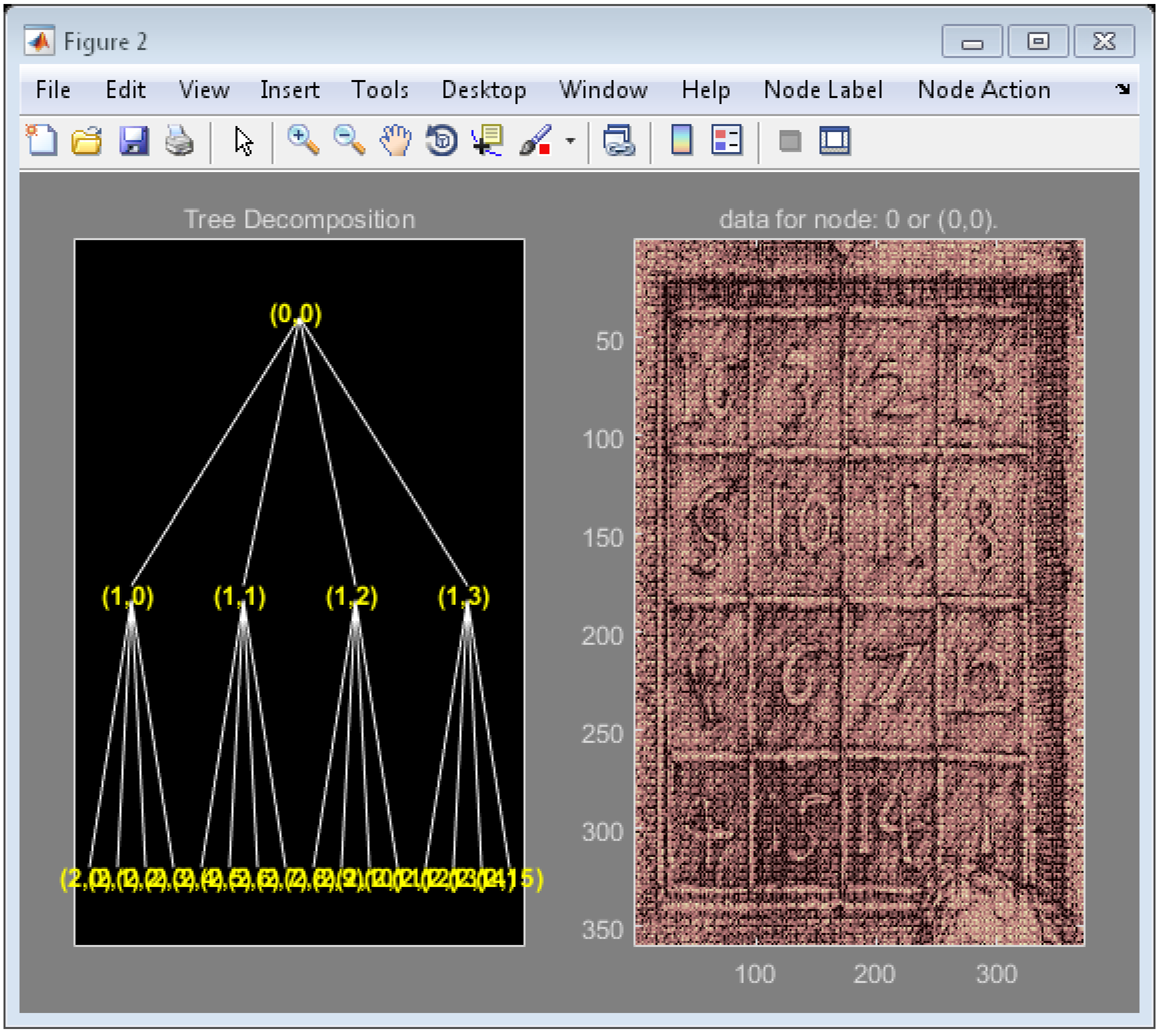


Figure 2
File Edit View Insert Tools Desktop Window Help Node Label Node Action
Tree Decomposition
(0,0)
(1,0) (1,1) (1,2) (1,3)
data for node: 0 or (0,0).
50
100
150
200
250
300
350
100
200
300

To illustrate building a wavelet packet tree using `write`, construct an admissible binary wavelet packet tree with terminal nodes `[2 3 9 10]`. The analyzing wavelet is `'sym4'` and the signal length is 1024.

```
tr = cfs2wpt('sym4',[1 1024],[2 3 9 10]',2);
```

Fill terminal nodes `[3 9]` with $N(0,1)$ coefficients.

```
sN = read(tr,'sizes',[3,9]);
sN3 = sN(1,:); sN9 = sN(2,:);
cfsN3 = randn(sN3);
cfsN9 = randn(sN9);
tr = write(tr,'cfs',3,cfsN3,'cfs',9,cfsN9);
```

Plot the resulting wavelet packet tree and synthesized signal.

```
plot(tr)
```

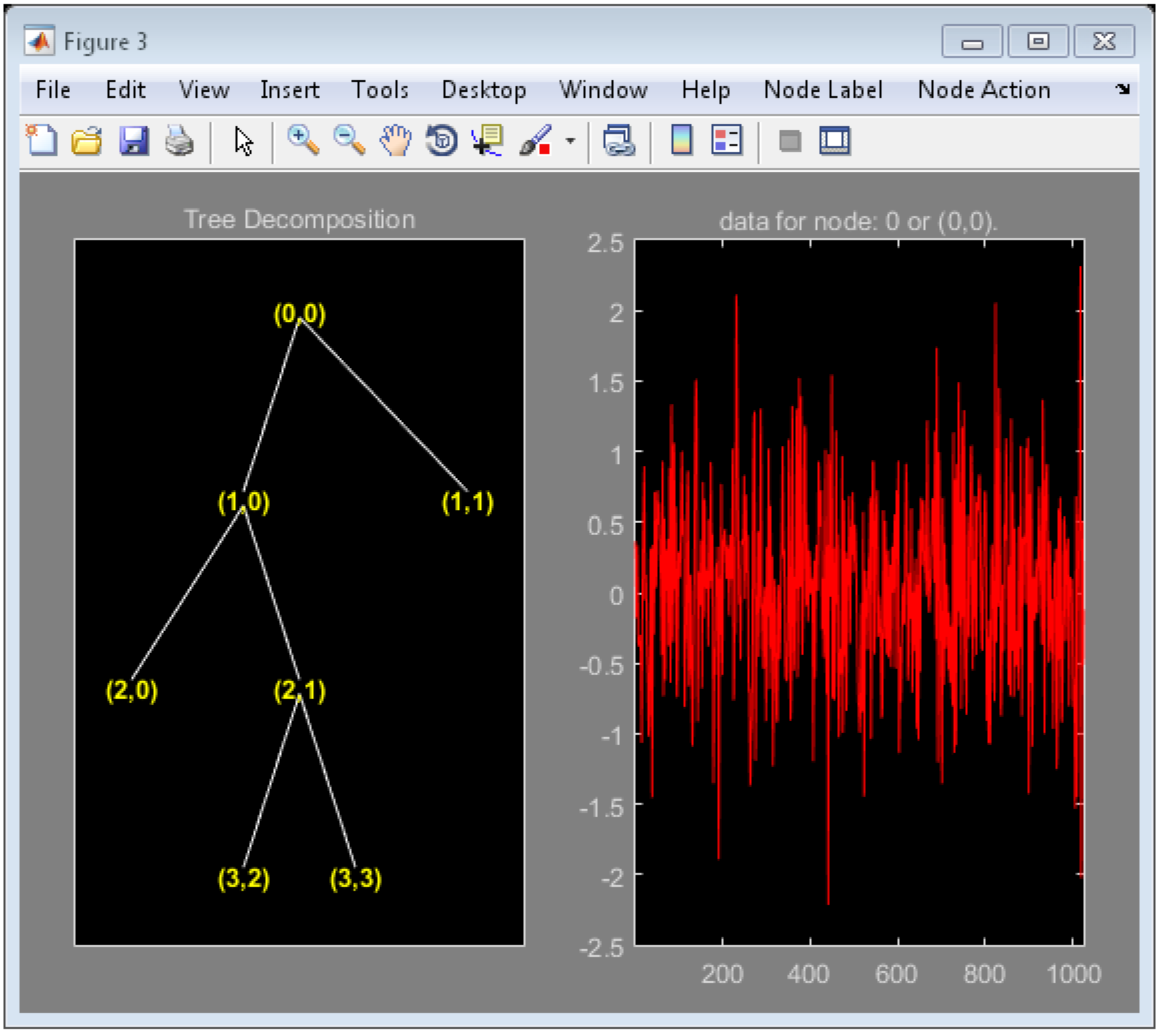


Figure 3
File
Edit
View
Insert
Tools
Desktop
Window
Help
Node Label
Node Action
Tree Decomposition
(0,0)
(1,0)
(1,1)
(2,0)
(2,1)
(3,2)
(3,3)
data for node: 0 or (0,0).
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
200
400
600
800
1000

# cgauwavf

Complex Gaussian wavelet

## Syntax

```
[PSI,X] = cgauwavf(LB,UB,N,P)
[PSI,X] = cgauwavf(LB,UB,N)
[PSI,X] = cgauwavf(LB,UB,N,1)
```

## Description

`[PSI,X] = cgauwavf(LB,UB,N,P)` returns values of the P-th derivative of the complex Gaussian function on an N point regular grid for the interval [LB,UB].

For P > 8, Symbolic Math Toolbox™ software is required.

Output arguments are the wavelet function PSI computed on the grid X.

`[PSI,X] = cgauwavf(LB,UB,N)` is equivalent to
`[PSI,X] = cgauwavf(LB,UB,N,1)`.

These wavelets have an effective support of [-5 5].

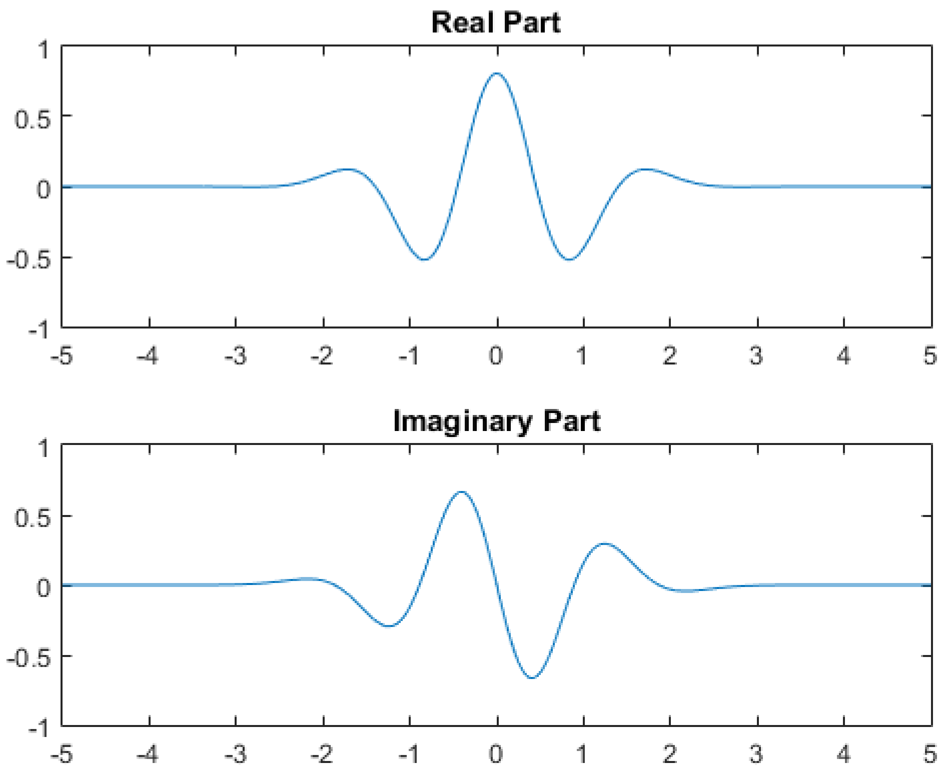
## Examples

### Create Complex Gaussian Wavelet

This example shows how to create a complex-valued Gaussian wavelet of order 4. The wavelet has an effective support of [-5,5] and is constructed using 1,000 samples.

```
lb = -5;
ub = 5;
n = 1000;
order = 4;
[psi,x] = cgauwavf(lb,ub,n,order);
subplot(211)
```

```
plot(x,real(psi))
title('Real Part');
subplot(212)
plot(x,imag(psi))
title('Imaginary Part');
```



## See Also

`waveinfo`

## chgwdeccfs

Change multisignal 1-D decomposition coefficients

### Syntax

```
DEC = chgwdeccfs(DEC,'ca',COEFS)
DEC = chgwdeccfs(DEC,'cd',COEFS,LEV)
DEC = chgwdeccfs(DEC,'all',CA,CD)
DEC = chgwdeccfs(DEC,'all',V)
DEC = chgwdeccfs(...,IDXSIG)
```

### Description

`DEC = chgwdeccfs(DEC,'ca',COEFS)` replaces the approximation coefficients at level `DEC.level` with those contained in the matrix `COEFS`. If `COEFS` is a single value *V*, all coefficients are replaced by *V*.

`DEC = chgwdeccfs(DEC,'cd',COEFS,LEV)` replaces the detail coefficients at level `LEV` with those contained in the matrix `COEFS`. If `COEFS` is a single value *V*, then `LEV` can be a vector of levels and all the coefficients that belong to these levels are replaced by *V*. `LEV` must be such that `1 ≤ LEV ≤ DEC.level`

`DEC = chgwdeccfs(DEC,'all',CA,CD)` replaces all the approximation and detail coefficients. *CA* must be a matrix and *CD* must be a cell array of length `DEC.level`.

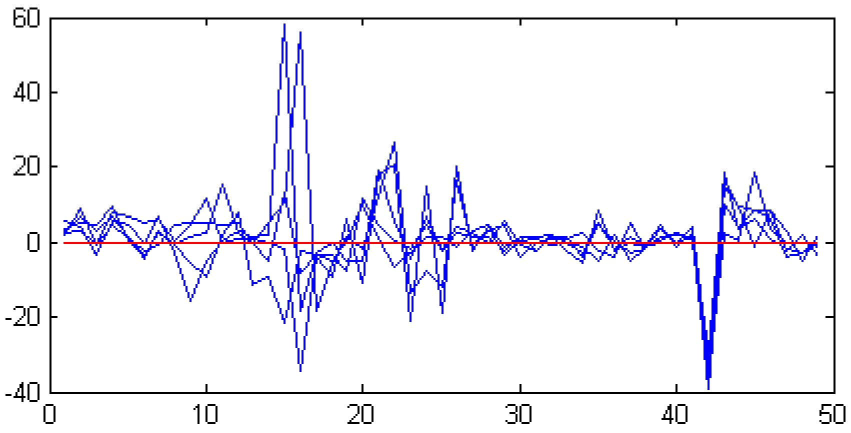
If `COEFS` (or *CA* or *CD*) is a single number, then it replaces all the related coefficients. Otherwise, `COEFS` (or *CA*, or *CD*) must be a matrix of appropriate size.

For a real value *V*, `DEC = chgwdeccfs(DEC,'all',V)` replaces all the coefficients by *V*.

`DEC = chgwdeccfs(...,IDXSIG)` replaces the coefficients for the signals whose indices are given by the vector `IDXSIG`. If the initial data are stored row-wise or column-wise in a matrix `X`, then `IDXSIG` contains the row or column indices, respectively, of the data.

## Examples

```
% Load original 1D-multisignal
load thinker

% Perform a decomposition at level 2 using wavelet db2
dec = mdwtdec('r',X,2,'db2');

% Change the coefficients of details at level 1.
% Replace all values by 0.
decBIS = chgwdeccfs(dec,'cd',0,1);

% Change the coefficients of details at level 1 and
% level 2 for signals 31 to 35. Replace all values by 0.
decTER = chgwdeccfs(dec,'cd',0,1:2,31:35);

% Compare original and new coefficients for details
% at level 1 for signals 31 to 35.
plot(dec.cd{1}(31:35,:)','b'); hold on;
plot(decTER.cd{1}(31:35,:)','r')
```



## See Also

mdwtdec | mdwtrec

## cmddenoise

Interval-dependent denoising

### Syntax

```
sigden = cmddenoise(sig,wname,level)
sigden = cmddenoise(sig,wname,level,sorh)
sigden = cmddenoise(sig,wname,level,sorh,nb_inter)
sigden = cmddenoise(sig,wname,level,sorh,nb_inter,thrParamsIn)

[sigden,coefs] = cmddenoise( ___ )
[sigden,coefs,thrParamsOut] = cmddenoise( ___ )

[sigden,coefs,thrParamsOut,int_DepThr_Cell] = cmddenoise(sig,wname,
level,sorh,nb_inter)
[sigden,coefs,thrParamsOut,int_DepThr_Cell,BestNbofInt] =
cmddenoise(sig,wname,level,sorh,nb_inter)
```

### Description

`sigden = cmddenoise(sig,wname,level)` returns the denoised signal, sigden, obtained from an interval-dependent denoising of the signal, sig, using the orthogonal or biorthogonal wavelet and scaling filters, wname. `cmddenoise` thresholds the wavelet (detail) coefficients down to level, level, and reconstructs a signal approximation using the modified detail coefficients. `cmddenoise` partitions the signal into intervals based on variance change points in the first level detail coefficients and thresholds each interval separately. The location and number of variance change points are automatically selected using a penalized contrast function [2]. The minimum delay between change points is 10 samples. Thresholds are obtained using a minimax threshold rule and soft thresholding is used to modify the wavelet coefficients [1] .

`sigden = cmddenoise(sig,wname,level,sorh)` returns the denoised signal, sigden, using the thresholding method, sorh, to modify the wavelet coefficients. Valid choices for sorh are `'s'` for soft thresholding or `'h'` for hard thresholding.

`sigden = cmddenoise(sig,wname,level,sorh,nb_inter)` returns the denoised signal, sigden, with the number of denoising intervals as a positive integer between 1 and 6: 1≤ nb_inter ≤6. For nb_inter ≥ 2, `cmddenoise` estimates the location of the change points with a contrast function [2].

`sigden = cmddenoise(sig,wname,level,sorh,nb_inter,thrParamsIn)` returns the denoised signal, sigden, with the denoising intervals and corresponding thresholds specified as a cell array of matrices with length equal to level. Each element of the cell array contains the interval and threshold information for the corresponding level of the wavelet transform. The elements of thrParamsIn are N-by-3 matrices with N equal to the number of intervals. The 1st and 2nd columns contain the beginning and ending indices of the intervals and the 3rd column contains the corresponding threshold value. If you specify thrParamsIn, `cmddenoise` ignores the value of nb_inter.

`[sigden,coefs] = cmddenoise(___)` returns the approximation (scaling) and detail (wavelet) coefficients, coefs. The organization of coefs is identical to the structure returned by `wavedec`. This syntax can include any of the input arguments used in previous syntaxes.

`[sigden,coefs,thrParamsOut] = cmddenoise(___)` returns a cell array, thrParamsOut, with length equal to level. Each element of thrParamsOut is an N-by-3 matrix. The row dimension of the matrix elements is the number of intervals and is determined by the value of the input arguments. Each row of the matrix contains the beginning and end points (indices) of the thresholded interval and the corresponding threshold value.

`[sigden,coefs,thrParamsOut,int_DepThr_Cell] = cmddenoise(sig,wname,level,sorh,nb_inter)` returns a cell array, int_DepThr_Cell, with length equal to 6. int_DepThr_Cell contains interval and threshold information assuming the number of change points ranges from 0 to 5. The N-th element of int_DepThr_Cell is a N-by-3 matrix containing the interval information assuming N-1 change points. Each row of the matrix contains the beginning and end points (indices) of the thresholded interval and the corresponding threshold value. Attempting to output int_DepThr_Cell if you use the input argument, thrParamsIn, results in an error.

`[sigden,coefs,thrParamsOut,int_DepThr_Cell,BestNbofInt] = cmddenoise(sig,wname,level,sorh,nb_inter)` returns the optimal number of signal intervals based on the estimated variance change points in the level-1 detail coefficients. To estimate the number of change points, `cmddenoise` assumes the total number is less than or equal to 6 and uses a penalized contrast [2]. Attempting to output BestNbofInt if you use the input argument, thrParamsIn, results in an error.

## Examples

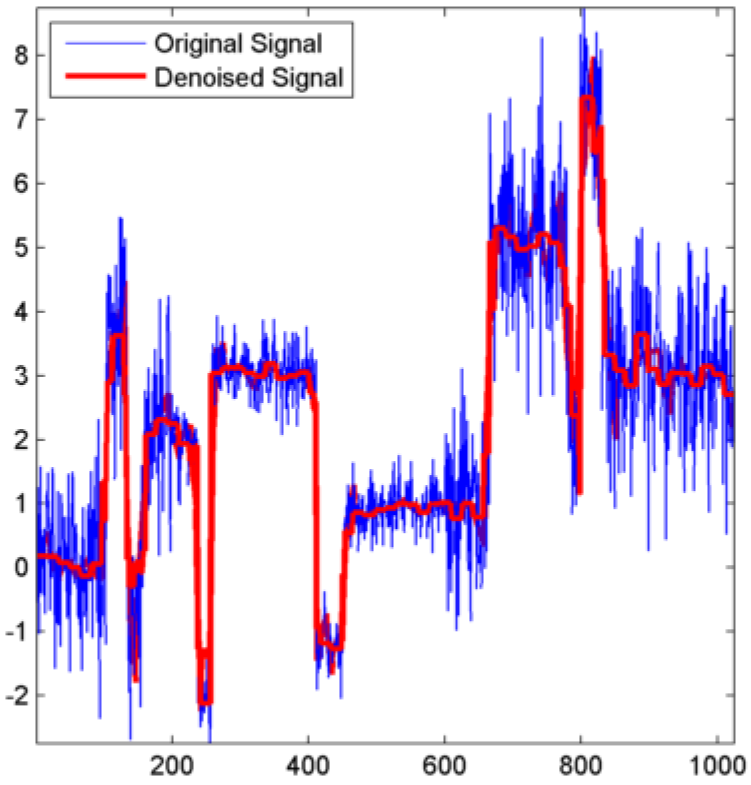
### Denoising Blocks Signal with Haar Wavelet

Load the noisy blocks signal, `nblocr1.mat`. The signal consists of a piecewise constant signal in addtive white Gaussian noise. The variance of the additive noise differs in three disjoint intervals.

```
load nblocr1;
```

Apply interval-dependent denoising down to level 4 using the Haar wavelet. `cmddenoise` automatically determines the optimal number and locations of the variance change points. Plot the denoised and original signal for comparison.

```
sigden = cmddenoise(nblocr1,'db1',4);
plot(nblocr1);
hold on;
plot(sigden,'r','linewidth',2);
axis tight;
legend('Original Signal','Denoised Signal','Location','NorthWest');
```



### Denoising Blocks Signal with Hard Thresholding

Load the noisy blocks signal, `nblocr1.mat`. The signal consists of a piecewise constant signal in additive white Gaussian noise. The variance of the additive noise differs in three disjoint intervals.

```
load nblocr1;
```

Apply interval-dependent denoising down to level 4 using the Haar wavelet and a hard thresholding rule. `cmddenoise` automatically determines the optimal number and locations of the intervals. Plot the original and denoised signals.

```
sorh = 'h';
sigden = cmddenoise(nblocr1,'db1',4,sorh);
plot(nblocr1);
hold on;
```

```
plot(sigden,'r','linewidth',2);
axis tight;
legend('Original Signal','Denoised Signal','Location','NorthWest');
```

### Specify the Number of Intervals

Create a signal sampled at 1 kHz. The signal consists of a series of bumps of various widths.

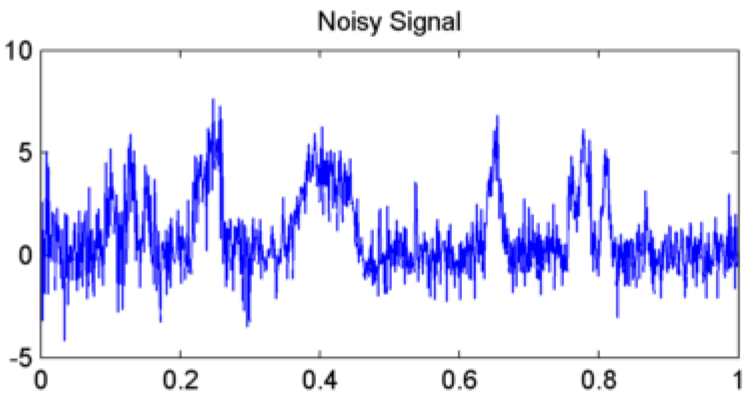
```
t = [0.1  0.13  0.15  0.23  0.25  0.40  0.44  0.65  0.76  0.78  0.81];
h = [4  -5 3 -4 5  -4.2   2.1   4.3  -3.1   5.1  -4.2];
h  = abs(h);
len = 1000;
w  = 0.01*[0.5 0.5 0.6 1 1 3 1 1 0.5 0.8 0.5];
tt = linspace(0,1,len);  x = zeros(1,len);
for j=1:11
  x = x + ( h(j) ./ (1+ ((tt-t(j))/w(j)).^4));
end
plot(tt,x); title('Original Signal');
```

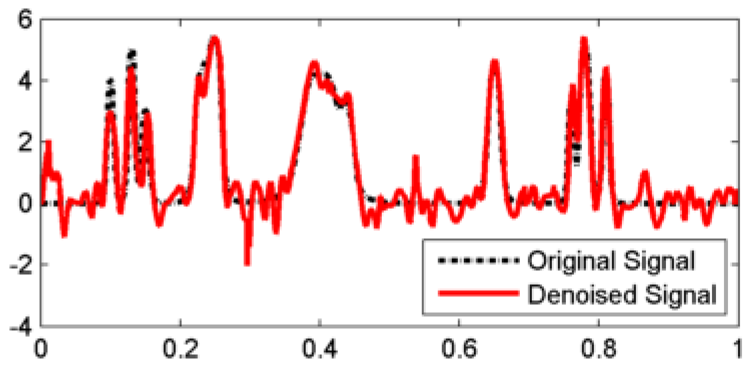
Add white Gaussian noise with different variances to two disjoint segments of the signal. Add zero-mean white Gaussian noise with variance equal to 2 to the signal segment from 0 to 0.3 seconds. Add zero-mean white Gaussian noise with unit variance to the signal segment from 0.3 seconds to 1 second. Set the random number generator to the default settings for reproducible results.

```
rng default;
nv1 = sqrt(2).*randn(size(tt)).*(tt<=0.3);
nv2 = randn(size(tt)).*(tt>0.3);
xx = x+nv1+nv2;
```

Apply interval-dependent denoising using the Daubechies' least-asymmetric wavelet with 5 vanishing moments down to level 3. Set the number of intervals to 2. Plot the noisy signal, original signal, and denoised signal for comparison.

```
sigden = cmddenoise(xx,'sym5',3,'s',2);
subplot(211)
plot(tt,xx); title('Noisy Signal');
subplot(212)
plot(tt,x,'k-.','linewidth',2);
hold on;
plot(tt,sigden,'r','linewidth',2);
legend('Original Signal','Denoised Signal','Location','SouthEast');
```





### Specify Intervals and Thresholds

Load the example signal `nbumpr1.mat`. The variance of the additive noise differs in three disjoint intervals.

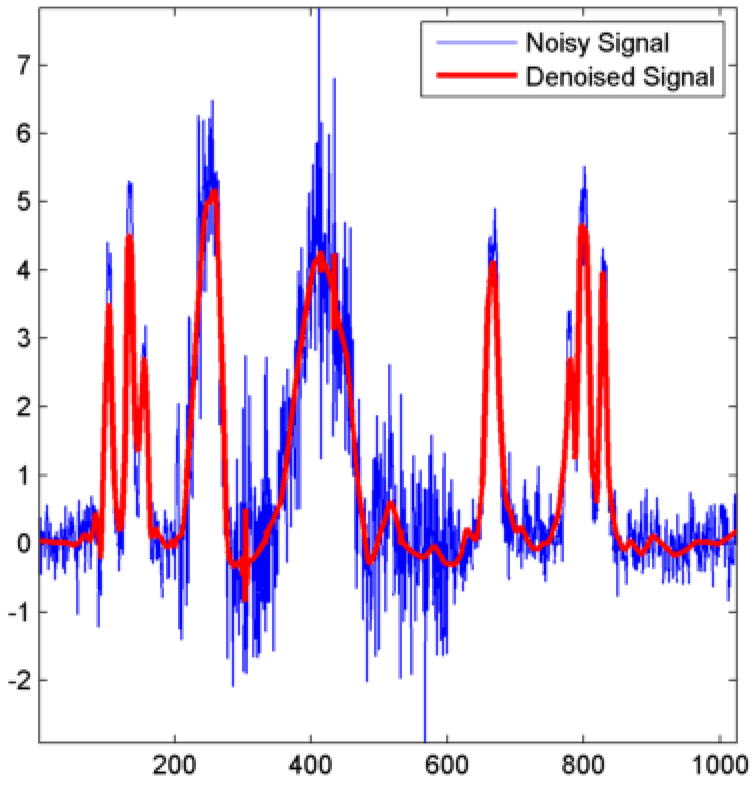
```
load nbumpr1.mat;
```

Use a level-5 multiresolution analysis. Create a cell array of length 5 consisting of 3-by-3 matrices. The first two elements of each row contain the beginning and ending indices of the interval and the last element of each row is the corresponding threshold.

```
thrParamsIn =  {...
    [...
    1     207      1.0482; ...
    207   613      2.5110; ...
    613   1024     1.0031; ...
    ]; ...
    [...
    1    207      1.04824; ...
    207  613      3.8718; ...
    613  1024     1.04824; ...
    ]; ...
    [...
    1    207      1.04824; ...
    207  613      1.99710; ...
    613  1024     1.65613; ...
    ]; ...
    [...
    1    207      1.04824; ...
    207  613      2.09117; ...
    613  1024     1.04824; ...
    ]; ...
    [...
    1    207      1.04824; ...
    207  613      1.78620; ...
    613  102      1.04824; ...
    ]; ...
    };
```

Denoise the signal using the threshold settings and the Daubechies' least-asymmetric wavelet with 4 vanishing moments. Use a soft thresholding rule. Plot the noisy and denoised signals for comparison.

```
wname = 'sym4';
level = 5;
sorh = 's';
sigden = cmddenoise(nbumpr1,wname,level,sorh,NaN,thrParamsIn);
plot(nbumpr1); hold on;
plot(sigden,'r','linewidth',2); axis tight;
legend('Noisy Signal','Denoised Signal','Location','NorthEast');
```



### Return Denoised Wavelet Coefficients

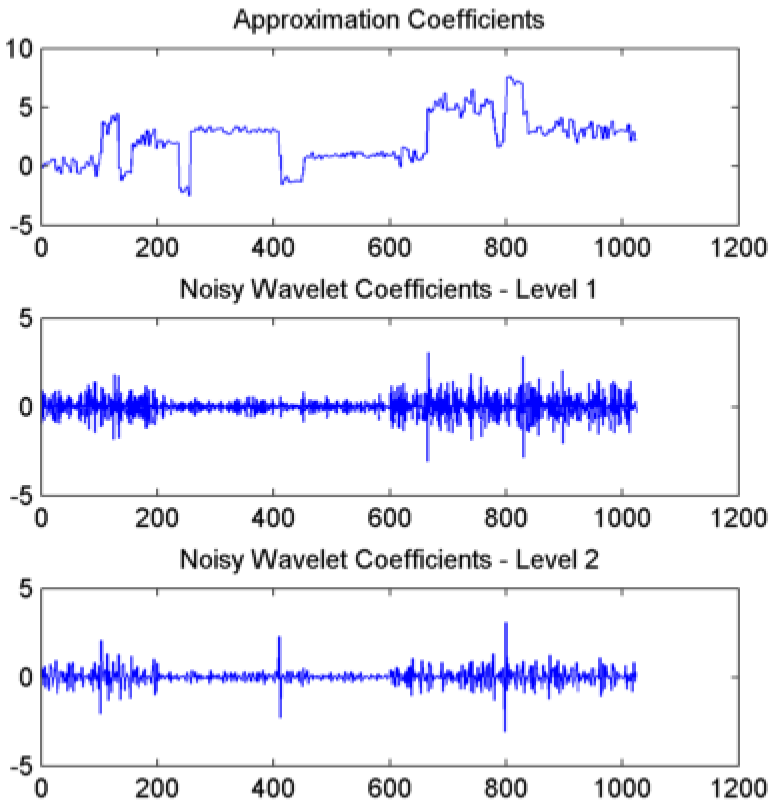
Load the example signal `nblocr1.mat`. Use the Haar wavelet and decompose the signal down to level 2. Obtain the discrete wavelet transform and denoise the signal. Return the wavelet coefficients of the noisy and denoised signals.

```
load nblocr1.mat;
[sigden,coefs] = cmddenoise(nblocr1,'db1',2);
[C,L] = wavedec(nblocr1,2,'db1');
```

Plot reconstructions based on the level-2 approximation and level-2 and level-1 detail coefficients for the noisy signal.

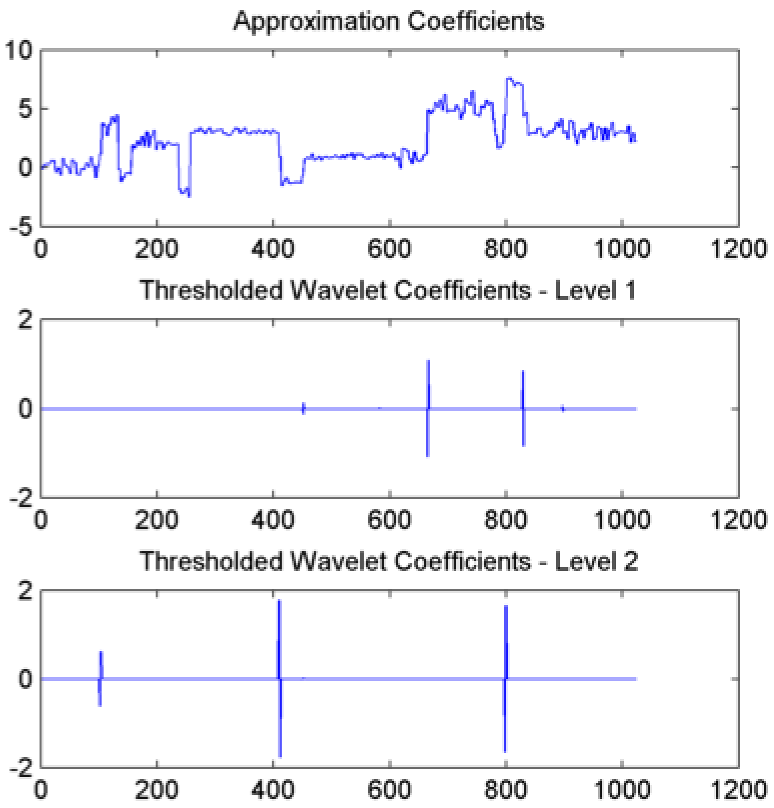
```
app = wrcoef('a',C,L,'db1',2);
subplot(3,1,1);
plot(app); title('Approximation Coefficients');
```

```
for nn = 1:2
    det = wrcoef('d',C,L,'db1',nn);
    subplot(3,1,nn+1)
    plot(det); title(['Noisy Wavelet Coefficients - Level ' num2str(nn)]);
end
```



Plot reconstructions based on the approximation and detail coefficients for the denoised signal at the same levels.

```
figure;
app = wrcoef('a',coefs,L,'db1',2);
subplot(3,1,1);
plot(app); title('Approximation Coefficients');
for nn = 1:2
    det = wrcoef('d',coefs,L,'db1',nn);
    subplot(3,1,nn+1)
    plot(det);
    title(['Thresholded Wavelet Coefficients - Level '  num2str(nn)]);
end
```



The approximation coefficients are identical in the noisy and denoised signal, but most of the detail coefficients in the denoised signal are close to zero.

### Output Intervals and Thresholds

Create a signal sampled at 1 kHz. The signal consists of a series of bumps of various widths.

```
t = [0.1  0.13  0.15  0.23  0.25  0.40  0.44  0.65  0.76  0.78  0.81];
h = [4  -5 3 -4 5  -4.2   2.1   4.3  -3.1   5.1  -4.2];
h  = abs(h);
```

```
len = 1000;
w  = 0.01*[0.5 0.5 0.6 1 1 3 1 1 0.5 0.8 0.5];
tt = linspace(0,1,len);  x = zeros(1,len);
for j=1:11
  x = x + ( h(j) ./ (1+ ((tt-t(j))/w(j)).^4));
end
plot(tt,x); title('Original Signal');
```

Add white Gaussian noise with different variances to two disjoint segments of the signal. Add zero-mean white Gaussian noise with variance equal to 2 to the signal segment from 0 to 0.3 seconds. Add zero-mean white Gaussian noise with unit variance to the signal segment from 0.3 seconds to 1 second. Set the random number generator to the default settings for reproducible results.

```
rng default;
nv1 = sqrt(2).*randn(size(tt)).*(tt<=0.3);
nv2 = randn(size(tt)).*(tt>0.3);
xx = x+nv1+nv2;
```

Apply interval-dependent denoising using the Daubechies' least-asymmetric wavelet with 4 vanishing moments down to level 5. Automatically choose the number of intervals and output the result.

```
[sigden,coefs,thrParamsOut] = cmddenoise(xx,'sym4',5);
thrParamsOut{1}
```

`cmddenoise` identifies one variance change point in the 1st level detail coefficients defining two intervals. The first interval contains samples 1 to 293. The second interval contains samples 293 to 1000. This is close to the true variance change point, which occurs at sample 299.

### Partition Signal into Increasing Numbers of Intervals with Thresholds

Load the example signal, `nbumpr1.mat`. Partition the signal into 1 to 6 intervals assuming 0 to 5 change points. Compute the thresholds for each interval. Using the Daubechies' least-asymmetric wavelet with 4 vanishing moments return the intervals and corresponding thresholds. Display the results in the command window.

```
load nbumpr1.mat;
[sigden,~,~,int_DepThr_Cell] = cmddenoise(nbumpr1,'sym4',1);
format bank;
disp('          Begin        End          Threshold ');
cellfun(@disp,int_DepThr_Cell,'UniformOutput',false);
```

```
format;
```

### Detect Number of Change Points

Load the example signal, `nbumpr1.mat`. The signal has two variance change points, which results in three intervals.

Use `cmddenoise` to detect the number of change points. Print the result.

```
load nbumpr1.mat;
[sigden,~,thrParamsOut,~,bestNbofInt] = cmddenoise(nbumpr1,'sym4',1);
fprintf('Found %d change points.\n',bestNbofInt-1);
```

## Input Arguments

### `sig` — Signal for interval-dependent denoising

1-D row or column vector

Input signal, specified as a 1-D row or column vector. sig is the real-valued input signal for interval-dependent denoising. The elements of sig are assumed to be equally spaced in time or space. If sig contains unequally-sampled data, `cmddenoise` is not appropriate. Use a lifting transform instead. See `lwt` for details.

Data Types: `double`

### `wname` — Wavelet name

string

Wavelet name, specified as a character array. wname is any valid orthogonal or biorthogonal wavelet. You can use the command: `wtype = wavemngr('fields',wname,'type','file');` to determine if the wavelet name is valid to use with `cmddenoise`. Valid wavelet names return a 1 or 2 for `wtype`.

Example: `'bior2.2'`, `'db4'`, `'sym4'`

Data Types: `char`

### `level` — Level of the decimated wavelet transform (multiresolution analysis)

positive integer

Wavelet transform (multiresolution analysis) level, specified as a positive integer. level gives the level of the multiresolution decomposition of the input signal using the decimated 1-D discrete wavelet transform, `wavedec`.

Data Types: `double`

**`sorh` — Threshold rule**
`'s'` (default) | `'h'`

Thresholding rule, specified as a character array. sorh is the threshold rule used in the modification of the detail coefficients. Valid choices for sorh are `'s'` (default) and `'h'` for soft and hard thresholding.

**`nb_inter` — Number of intervals**
positive integer in the set {1,2,3,4,5,6} | `NaN`

Number of intervals, specified as a positive integer less than 7. `cmddenoise` divides the input signal into nb_inter intervals. `cmddenoise` determines the location of the nb_inter change points using a contrast function [2]. If you enter `NaN` for nb_inter, `cmddenoise` ignores the input. If you use the input argument thrParamsIn, `cmddenoise` disregards any value you enter for nb_inter.

Data Types: `double`

**`thrParamsIn` — Intervals and thresholds by level**
cell array of matrices

Intervals and thresholds by level, specified as a cell array of matrices equal in length to level. Each element of thrParamsIn contains the interval and threshold information for the corresponding level of the multiresolution analysis. The elements of thrParamsIn are N-by-3 matrices with N equal to the number of intervals. The 1st and 2nd columns contain the beginning and ending indices of the intervals and the 3rd column contains the corresponding threshold value. If you specify thrParamsIn, you cannot specify the output arguments int_DepThr_Cell or BestNbofInt.

Data Types: `cell`

## Output Arguments

**`sigden` — Denoised signal**
1-D row or column vector

sigden is the denoised version of the input sig. sigden is a 1-D row vector equal in length to sig.

### `coefs` — Approximation coefficients and thresholded wavelet coefficients

1-D row vector of approximation coefficients and thresholded wavelet coefficients

coefs is a row vector of approximation (scaling) and thresholded detail (wavelet) coefficients. The ordering of the approximation and detail coefficients by level in coefs is the same as the output of `wavedec`. `cmddenoise` does not apply thresholding to the approximation coefficients.

Data Types: `double`

### `thrParamsOut` — Intervals and thresholds by level

cell array of matrices

thrParamsOut is a cell array of matrices equal in length to level. Each element of the cell array contains the interval and threshold information for the corresponding level of the multiresolution analysis. The elements of thrParamsOut are N-by-3 matrices with N equal to the number of intervals. N is determined by the value of the input arguments. The 1st and 2nd columns contain the beginning and ending indices of the intervals and the 3rd column contains the corresponding threshold value.

Data Types: `cell`

### `int_DepThr_Cell` — Intervals and thresholds assuming 0 to 5 change points

cell array of matrices

int_DepThr_Cell contains interval and threshold information assuming the number of change points ranges from 0 to 5. The N-th element of int_DepThr_Cell is a N-by-3 matrix containing the interval information assuming N-1 change points. Each row of the matrix contains the beginning and ending indices of the thresholded interval and the corresponding threshold value. Attempting to output int_DepThr_Cell if you input the number of intervals and thresholds, thrParamsIn, results in an error. `int_DepThr_Cell{BestNbofInt}` or `int_DepThr_Cell{nb_inter}` is equal to the matrix elements of thrParamsOut.

Data Types: `cell`

### `BestNbofInt` — Optimal number of intervals

positive integer ≤6

BestNbofInt is the optimal number of intervals based on estimated change points in the variance of the level-1 detail coefficients. The number and location of the change points are estimated using a penalized contrast method [2]. Attempting to output BestNbofInt if you input the number of intervals and thresholds, thrParamsIn, results in an error.

## References

[1] Donoho, D. and Johnstone, I. “Ideal spatial adaptation by wavelet shrinkage”, *Biometrika*, 1994, 81,3, 425–455.

[2] Lavielle, M. “Detection of multiple changes in a sequence of dependent variables”, *Stochastic Processes and their Applications*, 1999, 83, 79–102.

## See Also

`thselect` | `wavedec` | `wthresh` | `wvarchg`

# cmorwavf

Complex Morlet wavelet

## Syntax

```
[PSI,X] = cmorwavf(LB,UB,N,FB,FC)
```

## Description

`[PSI,X] = cmorwavf(LB,UB,N,FB,FC)` returns values of the complex Morlet wavelet defined by a positive bandwidth parameter *FB*, a wavelet center frequency *FC*, and the expression

```
PSI(X) = ((pi*FB)^(-0.5))*exp(2*i*pi*FC*X)*exp(-X^2/FB)
```

on an N point regular grid for the interval [*LB*,*UB*].

Output arguments are the wavelet function PSI computed on the grid X.

## Complex Morlet Wavelet

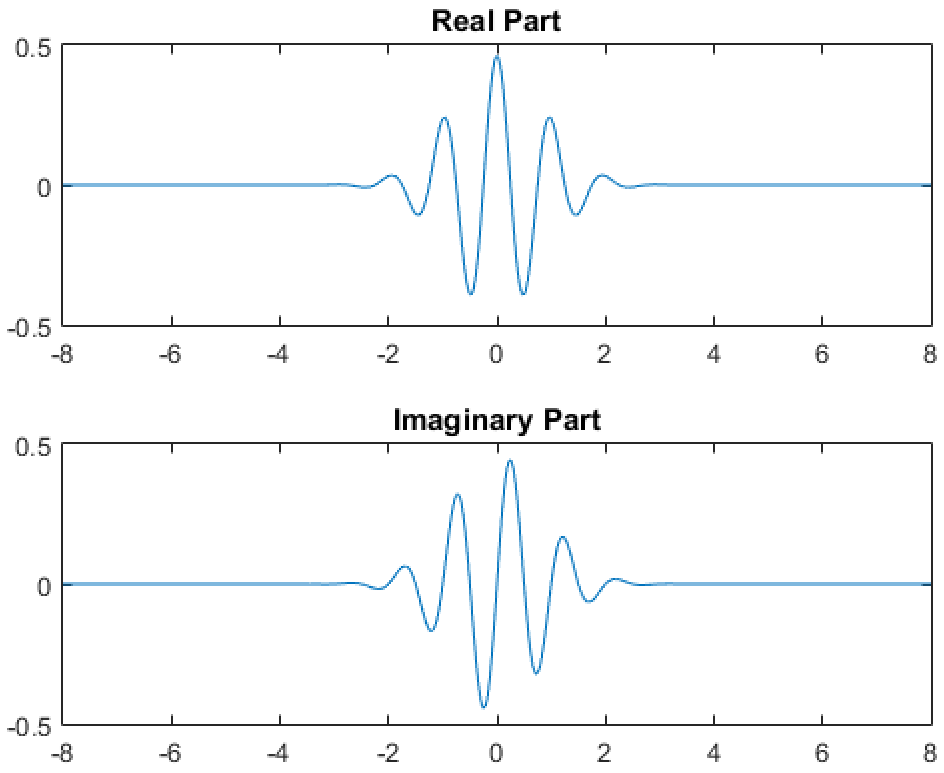
Construct a complex-valued Morlet wavelet with a bandwidth parameter of 1.5 and a center frequency of 1. Set the effective support to $[-8, 8]$ and the length of the wavelet to 1000.

```
N = 1000;
Lb = -8;
Ub = 8;
fb = 1.5;
fc = 1;
[psi,x] = cmorwavf(Lb,Ub,N,fb,fc);
```

Plot the real and imaginary parts of the wavelet.

```
subplot(2,1,1)
plot(x,real(psi)); title('Real Part');
```

```
subplot(2,1,2)
plot(x,imag(psi)); title('Imaginary Part');
```



## References

Teolis, A. (1998), *Computational signal processing with wavelets*, Birkhauser, p. 65.

## See Also

`waveinfo`

# coifwavf

Coiflet wavelet filter

## Syntax

```
F = coifwavf(W)
```

## Description

`F = coifwavf(W)` returns the scaling filter associated with the Coiflet wavelet specified by the string *W* where `W = 'coifN'`. Possible values for `N` are 1, 2, 3, 4, or 5.

## Examples

```
% Set coiflet wavelet name.
wname = 'coif2';

% Compute the corresponding scaling filter.
f = coifwavf(wname)

f =
Columns 1 through 7
0.0116  -0.0293  -0.0476 0.2730 0.5747  0.2949  -0.0541

Columns 8 through 12
-0.0420  0.0167  0.0040 -0.0013 -0.0005
```

## See Also

`waveinfo`

# conofinf

Cone of influence

## Syntax

```
cone = conofinf(wname,scales,LenSig,SigVal)
[cone,PL,PR] = conofinf(wname,scales,LenSig,SigVal)
[cone,PL,PR,PLmin,PRmax] = conofinf(wname,scales,LenSig,SigVal)
[PLmin,PRmax] = conofinf(wname,scales,LenSig)
[...] = conofinf(...,'plot')
```

## Description

`cone = conofinf(wname,scales,LenSig,SigVal)` returns the cone of influence (COI) for the wavelet wname at the scales in scales and positions in SigVal. LenSig is the length of the input signal. If SigVal is a scalar, cone is a matrix with row dimension `length(scales)` and column dimension LenSig. If  isa vector, cone is cell array of matrices.

`[cone,PL,PR] = conofinf(wname,scales,LenSig,SigVal)` returns the left and right boundaries of the cone of influence atscale1for the points in . PL and PR are `length(SigVal)`-by-2 matrices. The left boundaries are`(1-PL(:,2))./PL(:,1)` and therightboundariesare`(1-PR(:,2))./PR(:,1)`.

`[cone,PL,PR,PLmin,PRmax] = conofinf(wname,scales,LenSig,SigVal)` returns the equations of the lines that define the minimal left and maximal right boundaries of the cone of influence. PLmin and PRmax are 1-by-2 row vectors where `PLmin(1)` and `PRmax(1)` are the slopes of the lines. `PLmin(2)` and `PRmax(2)` are the points where the lines intercept the scale axis.

`[PLmin,PRmax] = conofinf(wname,scales,LenSig)` returns the slope and intercept terms for the first-degree polynomials defining the minimal left and maximal right vertices of the cone of influence.

`[...] = conofinf(...,'plot')` plots the cone of influence.

## Input Arguments

### `wname`

wname is a string corresponding to a valid wavelet. To verify that wname is a valid wavelet, `wavemngr('fields',wname)` must return a struct array with a `type` field of 1 or 2, or a nonempty `bound` field.

### `scales`

scales is a vector of scales over which to compute the cone of influence. Larger scales correspond to stretched versions of the wavelet and larger boundary values for the cone of influence.

### `LenSig`

LenSig is the signal length and must exceed the maximum of SigVal.

### `SigVal`

SigVal is a vector of signal values at which to compute the cone of influence. The largest value of SigVal must be less than the signal length, LenSig.If SigVal is empty, `conofinf` returns the slope and intercept terms for the minimal left and maximal right vertices of the cone of influence.

## Output Arguments

### `cone`

cone isthe cone of influence. If SigVal is a scalar, cone is a matrix. The row dimension is equal to the number of scales and column dimension equal to the signal length, LenSig. If SigVal is a vector, cone is a cell array of matrices. The elements of each row of the matrix are equal to 1 in the interval around SigVal corresponding to the cone of influence.

### `PL`

PL is the minimum value of the cone of influence on the position (time) axis.

### `PR`

PR is the maximum value of the cone of influence on the position (time) axis.

### `PLmin`

PLmin is a 1-by-2 row vector containing the slope and scale axis intercept of the line defining the minimal left vertex of the cone of influence. `PLmin(1)` is the slope and `PLmin(2)` is the point where the line intercepts the scale axis.

### `PRmax`

PRmax is a 1-by-2 row vector containing the slope and scale axis intercept of the line defining the maximal right vertex of the cone of influence. `PRmax(1)` is the slope and `PRmax(2)` is the point where the line intercepts the scale axis.

## Examples

Cone of influence for Mexican hat wavelet:

```
load cuspamax
signal = cuspamax;
wname  = 'mexh';
scales = 1:64;
lenSIG = length(signal);
x = 500;
figure;
cwt(signal,scales,wname,'plot');
hold on
[cone,PL,PR,Pmin,Pmax] = conofinf(wname,scales,lenSIG,x,'plot');
set(gca,'Xlim',[1 lenSIG])
```

Left minimal and right maximal vertices for the cone of influence (Morlet wavelet):

```
[PLmin,PRmax] = conofinf('morl',1:32,1024,[],'plot');
% PLmin = -0.1245*u+ 32.0000
% PRmax =  0.1250*u-96.0000
```

## More About

### Cone of Influence

Let ψ(t) be an admissible wavelet. Assume that the effective support of ψ(t) is [-B,B]. Letting *u* denote the translation parameter and *s* denote the scale parameter, the dilated and translated wavelet is:

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}}\psi(\tfrac{t-u}{s})$$

andhas effective support [u-sB,u+sB]. The cone of influence (COI) is the set of all $t$ included in the effective support of the wavelet at a given position and scale. This set is equivalent to:

$$|\, t-u \,| \leq sB$$

At each scale, the COI determines the set of wavelet coefficients influenced by the value of the signal at a specified position.

- “Continuous Wavelet Transform”

## References

Mallat, S. *A Wavelet Tour of Signal Processing*, London:Academic Press, 1999, p. 174.

### See Also

`cwt` | `wavsupport`

## cwt

Continuous 1-D wavelet transform

### Syntax

```
coefs = cwt(x,scales,'wname')
coefs = cwt(x,scales,'wname','plot')
coefs = cwt(x,scales,'wname','coloration')
[coefs,sgram] = cwt(x,scales,'wname','scal')
[coefs,sgram] = cwt(x,scales,'wname','scalCNT')
coefs = cwt(x,scales,'wname','coloration',xlim)
```

### Description

`coefs = cwt(x,scales,'wname')` computes the continuous wavelet transform (CWT) coefficients of the real-valued signal `x` at real, positive `scales`, using wavelet `'wname'` (see `waveinfo` for more information). The analyzing wavelet can be real or complex. `coefs` is an *la*-by-*lx* matrix, where *la* is the length of `scales` and *lx* is the length of the input `x`. `coefs` is a real or complex matrix, depending on the wavelet type.

`coefs = cwt(x,scales,'wname','plot')` plots the continuous wavelet transform coefficients, using default `coloration` `'absglb'`.

`coefs = cwt(x,scales,'wname','coloration')` uses the specified coloration.

`[coefs,sgram] = cwt(x,scales,'wname','scal')` displays a scaled image of the scalogram.

`[coefs,sgram] = cwt(x,scales,'wname','scalCNT')` displays a contour representation of the scalogram.

`coefs = cwt(x,scales,'wname','coloration',xlim)` colors the coefficients using coloration and `xlim`, where `xlim` is a vector, `[x1 x2]`, with 1 ≤ `x1` < `x2` ≤ `length(x)`.

## Examples

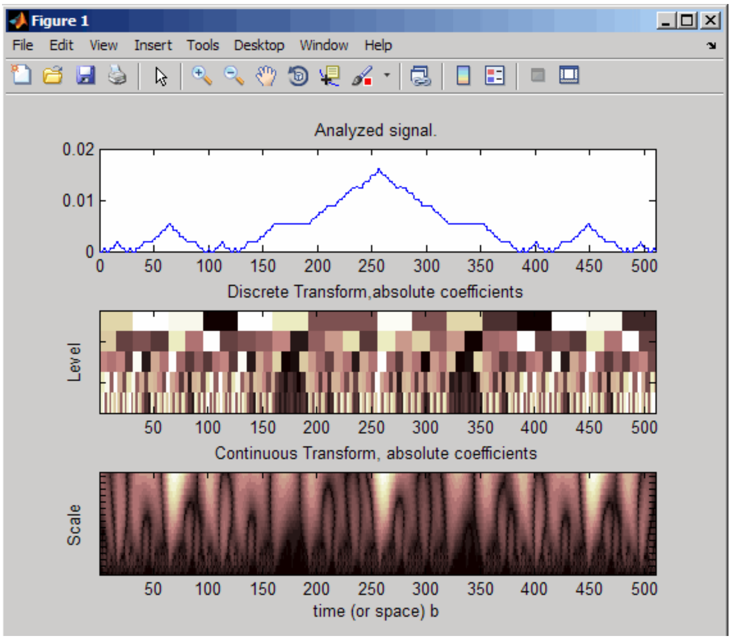
Plot the continuous wavelet transform and scalogram using sym2 wavelet at all integer scales from 1 to 32, using a fractal signal as input:

```
load vonkoch
vonkoch=vonkoch(1:510);
len = length(vonkoch);
cw1 = cwt(vonkoch,1:32,'sym2','plot');
title('Continuous Transform, absolute coefficients.')
ylabel('Scale')
[cw1,sc] = cwt(vonkoch,1:32,'sym2','scal');
title('Scalogram')
ylabel('Scale')
```

Compare discrete and continuous wavelet transforms, using a fractal signal as input:

```
load vonkoch
vonkoch=vonkoch(1:510);
len=length(vonkoch);
[c,l]=wavedec(vonkoch,5,'sym2');
% Compute and reshape DWT to compare with CWT.
cfd=zeros(5,len);
for k=1:5
    d=detcoef(c,l,k);
    d=d(ones(1,2^k),:);
    cfd(k,:)=wkeep(d(:)',len);
end
cfd=cfd(:);
I=find(abs(cfd) <sqrt(eps));
cfd(I)=zeros(size(I));
cfd=reshape(cfd,5,len);
% Plot DWT.
subplot(311); plot(vonkoch); title('Analyzed signal.');
set(gca,'xlim',[0 510]);
subplot(312);
image(flipud(wcodemat(cfd,255,'row')));
colormap(pink(255));
set(gca,'yticklabel',[]);
title('Discrete Transform,absolute coefficients');
ylabel('Level');
% Compute CWT and compare with DWT
subplot(313);
```

```
ccfs=cwt(vonkoch,1:32,'sym2','plot');
title('Continuous Transform, absolute coefficients');
set(gca,'yticklabel',[]);
ylabel('Scale');
```



## How To

"Continuous Wavelet Transform"

"1-D Continuous Wavelet Analysis"

"New Wavelet for CWT"

## More About

### Scale values

*Scale values* determine the degree to which the wavelet is compressed or stretched. Low scale values compress the wavelet and correlate better with high frequencies. The low scale CWT coefficients represent the fine-scale features in the input signal vector. High scale values stretch the wavelet and correlate better with the low frequency content of the signal. The high scale CWT coefficients represent the coarse-scale features in the input signal.

### Coloration

*Coloration* is the method used to scale the coefficient values for plotting. Each coefficient is divided by the resulting coloration value.

- `'lvl'` — uses maximum value in each scale
- `'glb'` — uses maximum value in all scales
- `'abslvl'` or `'lvlabs'` — uses maximum absolute value in each scale
- `'absglb'` or `'glbabs'` — uses maximum absolute value in all scales
- `'scal'` — produces a scaled image of the scalogram
- `'scalCNT'` — produces a contour representation of the scalogram

For 3-D plots (surfaces), use the `coloration` parameter preceded by `'3D'`, such as `coefs = cwt(...,'3Dplot')` or `coefs = cwt(...,'3Dlvl') ...`

### Scalogram

*Scalograms* are plots that represent the percentage energy for each coefficient.

## References

Daubechies, I. *Ten Lectures on Wavelets*, Philadelphia, PA: Society for Industrial and Applied Mathematics (SIAM), 1992.

Mallat, S. *A Wavelet Tour of Signal Processing*, San Diego, CA: Academic Press, 1998.

## See Also

`cwtext` | `dwt` | `wavedec` | `wavefun` | `waveinfo` | `wcodemat`

## cwtft

Continuous wavelet transform using FFT algorithm

### Syntax

```
cwtstruct = cwtft(sig)
cwtstruct = cwtft(sig,Name,Value)
cwtstruct = cwtft(...,'plot')
```

### Description

`cwtstruct = cwtft(sig)` returns the continuous wavelet transform (CWT) of the 1–D input signal sig. `cwtft` uses an FFT algorithm to compute the CWT. sig can be a vector, a structure array, or a cell array. If the sampling interval of your signal is not equal to `1`, you must input the sampling period with sig in a cell array or a structure array to obtain correct results. If `sig` is a cell array, `sig{1}` is equal to your signal and `sig{2}` is equal to the sampling interval. If `sig` is a structure array, the field `sig.val` contains your signal and `sig.period` contains the sampling interval.

By default, `cwtft` uses the analytic Morlet wavelet. See “Wavelet Definitions” on page 1-80 for a description of valid analyzing wavelets.

For additional default values, see scales in “Name-Value Pair Arguments” on page 1-75.

`cwtstruct = cwtft(sig,Name,Value)` returns the continuous wavelet transform (CWT) of the 1–D input signal sig with additional options specified by one or more Name,Value pair arguments. See “Name-Value Pair Arguments” on page 1-75 for a comprehensive list.

`cwtstruct = cwtft(...,'plot')` plots the continuous wavelet transform. If the analyzing wavelet is real-valued, the original signal along with the CWT coefficient magnitudes and signed CWT coefficients are plotted. If the analyzing wavelet is complex-valued, the original signal is plotted along with the moduli, real parts, imaginary parts, and angles of the CWT coefficients. You can select the radio button in the bottom left of the plot to superimpose the signal's reconstruction using `icwtft`.

## Input Arguments

### `sig`

The 1–D input signal. sig can be a vector, a structure array, or a cell array. If sig is a structure array, sig contains two fields: `val` and `period`. `sig.val` is the signal vector and `sig.period` is the sampling period. If sig is a cell array, `sig{1}` is the signal vector and `sig{2}` is the sampling period.

If sig is a vector, the sampling period defaults to `1`.

---

**Note:** If the sampling interval of your input signal is not `1`, you must input the sampling interval with sig in a cell array or structure array to obtain correct results. If `sig` is a cell array, `sig{1}` is the 1–D input signal and `sig{2}` is the sampling period. If `sig` is a structure array, the field `sig.val` is the 1–D input signal and `sig.period` is the sampling interval.

---

### Name-Value Pair Arguments

#### `'scales'`

Scales over which to compute the CWT. The value of scales can be a vector, a structure array, or a cell array. If scales is a structure array, it contains at most five fields. The first three fields are mandatory. The last two fields are optional.

1. `s0` — The smallest scale. The default `s0` depends on the wavelet. See “Wavelet Definitions” on page 1-80 for the wavelet-dependent default.
2. `ds` — Spacing between scales. The default `ds` depends on the wavelet. See “Wavelet Definitions” on page 1-80 for the wavelet-dependent default. You can construct a linear or logarithmic scale vector using `ds`. See type for a description of the type of spacing.
3. `nb` — Number of scales. The default `nb` depends on the wavelet. See “Wavelet Definitions” on page 1-80 for the wavelet-dependent default.
4. `type` — Type of spacing between scales. type can be one of `'pow'` or `'lin'`. The default is `'pow'`. If type is equal to `'pow'`, the CWT scales are `s0*pow.^((0:nb-1)*ds)`. This results in a constant spacing of `ds` if you take the logarithm to the base `power` of the scales vector. If type is equal to `'lin'`, the CWT scales are linearly spaced by `s0 + (0:nb-1)*ds`.

Use the default power of two spacing to ensure an accurate approximation to the original signal based only on select scales. See the second example in "Examples" on page 1-77 for a signal approximation based on select scales.

5. pow — The base for `'pow'` spacing. The default is 2. This input is valid only if the type argument is `'pow'`.

If scales is a cell array, the first three elements of the cell array are identical to the first three elements of the structure array described in the preceding list. The last two elements of the cell array are optional and match the two optional inputs in the structure array described in the preceding list.

**`'wavelet'`**

Analyzing wavelet. The supported analyzing wavelets are:

- `'dog'` — *m*-th order derivative of a Gaussian wavelet where *m* is a positive even integer. The default value of *m* is 2.
- `'morl'` — Morlet wavelet. Results in an analytic Morlet wavelet. The Fourier transform of an analytic wavelet is zero for negative frequencies.
- `'morlex'` — non-analytic Morlet wavelet
- `'morl0'` — non-analytic Morlet wavelet with zero mean
- `'mexh'` — Mexican hat wavelet. The Mexican hat wavelet is a special case of the *m*-th order derivative of a Gaussian wavelet with *m*=2.
- `'paul'` — Paul wavelet

See "Wavelet Definitions" on page 1-80 for formal definitions of the supported analyzing wavelets and associated defaults.

**Default:** `'morl'`

**`'padmode'`**

Signal extension mode. See `dwtmode` for supported extension modes. By default, `cwtft` does not extend the signal prior to computing the CWT. In a Fourier-transform-based CWT algorithm, extending a signal can mitigate wrap-around effects. The number of CWT coefficients in each row of the output matrix `cwtstruct.cfs` is truncated to match the length of the input signal.

## Output Arguments

### `cwtstruct`

A structure array with six fields. The fields of the structure array are:

- `dt` — The sampling interval of the 1–D input signal
- `cfs` — The CWT coefficient matrix. `cwtstruct.cfs` is an `nb`-by-N matrix where `nb` is the number of scales and N is the length of the input signal.
- `meanSIG` — Mean of the analyzed signal
- `omega` — Vector of angular frequencies
- `scales` — Vector of scales at which the CWT is computed. The length of `cwtstruct.scales` is equal to the row dimension of `cwtstruct.cfs`.
- `wav` — Analyzing wavelet

## Examples

Compute and display the CWT of sine waves with disjoint support. The sampling interval is 1/1023.

```
N = 1024;
% Sampling interval is 1/1023
t = linspace(0,1,N);
y = sin(2*pi*4*t).*(t<=0.5)+sin(2*pi*8*t).*(t>0.5);
% Because the sampling interval differs from the default
% you must input it along with the signal
% Using cell array input
sig = {y,1/1023};
cwtS1 = cwtft(sig,'plot');
```

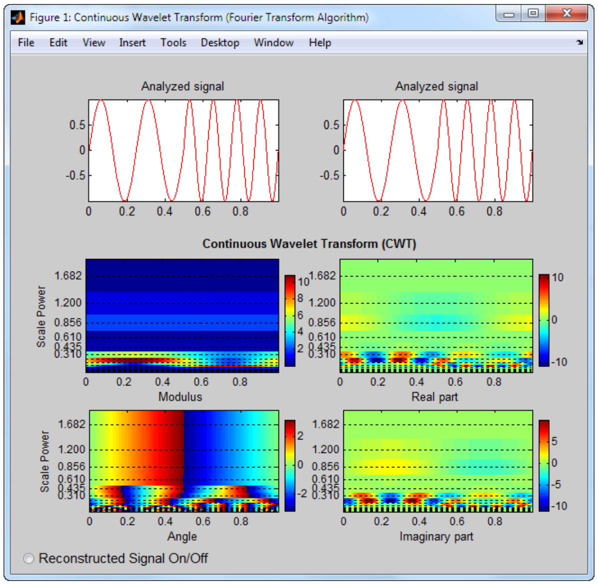


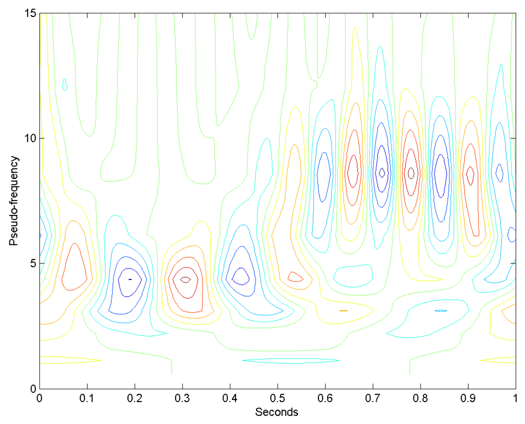
Figure 1: Continuous Wavelet Transform (Fourier Transform Algorithm)
File Edit View Insert Tools Desktop Window Help
Analyzed signal
Analyzed signal
Continuous Wavelet Transform (CWT)
Scale Power
Modulus
Real part
Angle
Imaginary part
Reconstructed Signal On/Off

You can display or hide the reconstructed signal using the radio button at the bottom left of the figure. When you select the radio button, the maximum and quadratic relative errors are computed and displayed along with the reconstructed signal.

Reconstruct an approximation to a sum of disjoint sine waves in noise using `cwtft` to decompose the signal and `icwtft` to reconstruct the approximation. Use the CWT coefficients to identify the scales isolating the sinusoidal components. Reconstruct an approximation to the signal based on those scales using the inverse CWT. To ensure an accurate approximation to the based on select scales, use the default power of two spacing in the CWT.

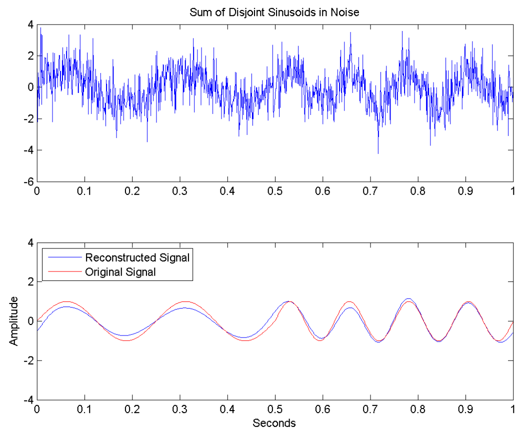
```
rng default % Reset random number generator for reproducible results
N = 1024;
% Sampling interval is 1/1023
t = linspace(0,1,N);
y = sin(2*pi*4*t).*(t<=0.5)+sin(2*pi*8*t).*(t>0.5);
ynoise = y+randn(size(t));
% Because the sampling interval differs from the default
% you must input it along with the signal
% Using structure array input
sig = struct('val',ynoise,'period',1/1023);
cwtS1 = cwtft(sig);
```

```
scales = cwtS1.scales;
MorletFourierFactor = 4*pi/(6+sqrt(2+6^2));
freq = 1./(scales.*MorletFourierFactor);
contour(t,freq,real(cwtS1.cfs));
xlabel('Seconds'); ylabel('Pseudo-frequency');
axis([0 t(end) 0 15]);
```



Extract the scales dominated by energy from the two sine waves and reconstruct a signal approximation using the inverse CWT.

```
cwtS2 = cwtS1;
cwtS2.cfs = zeros(size(cwtS1.cfs));
cwtS2.cfs(13:15,:) = cwtS1.cfs(13:15,:);
xrec = icwtft(cwtS2);
subplot(2,1,1);
plot(t,ynoise);
title('Sum of Disjoint Sinusoids in Noise');
subplot(2,1,2);
plot(t,xrec,'b'); hold on; axis([0 1 -4 4]);
plot(t,y,'r');
legend('Reconstructed Signal','Original Signal',...
    'Location','NorthWest');
xlabel('Seconds'); ylabel('Amplitude');
```



## Alternatives

- `cwt` — Computes the CWT using convolutions. `cwt` supports a wider choice of analyzing wavelets than `cwtft`, but may be more computationally expensive. The output of `cwt` is not compatible with the inverse CWT implemented with `icwtft`. To use `icwtft`, obtain the CWT with `cwtft`.

## More About

### Wavelet Definitions

### Morlet Wavelet

Both non-analytic and analytic Morlet wavelets are supported. The analytic Morlet wavelet, `'morl'`, is defined in the Fourier domain by:

$$\breve{\Psi}(s\omega) = \pi^{-1/4} e^{-(s\omega-\omega_0)^2/2} U(s\omega)$$

where $U(\omega)$ is the Heaviside step function [5].

The non-analytic Morlet wavelet, `'morlex'`, is defined in the Fourier domain by:

$$\breve{\Psi}(s\omega) = \pi^{-1/4} e^{-(s\omega-\omega_0)^2/2}$$

`'morl0'` defines a non-analytic Morlet wavelet in the Fourier domain with exact zero mean:

$$\breve{\Psi}(s\omega) = \pi^{-1/4} \{e^{-(s\omega-\omega_0)^2/2} - e^{-\omega_0^2/2}\}$$

The default value of $\omega_0$ is 6.

The scale-to-frequency Fourier factor for the Morlet wavelet is:

$$\frac{4\pi s}{\omega_0 + \sqrt{2 + \omega_0^2}}$$

The default smallest scale for the Morlet wavelets is `2*dt` where `dt` is the sampling period.

The default spacing between scales for the Morlet wavelets is `ds=0.4875`.

The default number of scales for the Morlet wavelets is `fix(log2(length(sig))/ds)+1`.

### *m*-th Order Derivative of Gaussian Wavelets

In the Fourier domain, the *m*-th order derivative of Gaussian wavelets, `'dog'`, are defined by:

$$\hat{\Psi}(s\omega) = -\frac{1}{\sqrt{\Gamma(m+1/2)}}(js\omega)^m e^{-(s\omega)^2/2}$$

where Γ( ) denotes the gamma function [5].

The derivative must be an even order. The default order of the derivative is 2, which is also known as the Mexican hat wavelet .

The scale-to-frequency Fourier factor for the DOG wavelet is:

$$\frac{2\pi s}{\sqrt{m+\frac{1}{2}}}$$

The default smallest scale for the DOG wavelet is `2*dt` where `dt` is the sampling period.

The default spacing between scales for the DOG wavelet is `ds=0.4875`.

The default number of scales for the DOG wavelet is `max([fix(log2(length(sig))/ds),1])`.

### Paul Wavelet

The Fourier transform of the analytic Paul wavelet, `'paul'`, of order *m* is:

$$\hat{\Psi}(s\omega) = 2^m \sqrt{m(2m-1)!}\,(s\omega)^m\, e^{-s\omega} U(sw)$$

where $U(\omega)$ is the Heaviside step function [5].

The default order of the Paul wavelet is 4.

The scale-to-frequency Fourier factor for the Paul wavelet is:

$$\frac{4\pi s}{2m+1}$$

The default smallest scale for the Paul wavelet is `2*dt` where `dt` is the sampling period.

The default spacing between scales for the Paul wavelet is `ds=0.4875`.

The default number of scales for the Paul wavelet is `fix(log2(length(sig))/ds)+1`.

### Algorithms

`cwtft` implements the following algorithm:

- Obtain the discrete Fourier transform (DFT) of the signal using `fft`.
- Obtain the DFT of the analyzing wavelet at the appropriate angular frequencies. Scale the DFT of the analyzing wavelet at different scales to ensure different scales are directly comparable.

- Take the product of the signal DFT and the wavelet DFT over all the scales. Invert the DFT to obtain the CWT coefficients.

For a mathematical motivation for the FFT-based algorithm see “DFT-Based Continuous Wavelet Transform”.

- “Continuous Wavelet Transform”
- “DFT-Based Continuous Wavelet Transform”
- “Inverse Continuous Wavelet Transform”

## References

[1] Daubechies, I. *Ten Lectures on Wavelets*, Philadelphia, PA: Society for Industrial and Applied Mathematics (SIAM), 1992.

[2] Farge, M. “Wavelet Transforms and Their Application to Turbulence”, *Ann. Rev. Fluid. Mech.*, 1992, 24, 395–457.

[3] Mallat, S. *A Wavelet Tour of Signal Processing*, San Diego, CA: Academic Press, 1998.

[4] Sun,W. “Convergence of Morlet's Reconstruction Formula”, *preprint*, 2010.

[5] Torrence, C. and G.P. Compo. “A Practical Guide to Wavelet Analysis”, *Bull. Am. Meteorol. Soc.*, 79, 61–78, 1998.

## See Also

`cwt` | `icwtft`

## cwtftinfo

Valid analyzing wavelets for FFT-based CWT

## Syntax

```
cwtftinfo
```

## Description

`cwtftinfo` displays expressions for the Fourier transforms of valid analyzing wavelets for use with `cwtft`.

## Examples

Display a list of Fourier transforms for all valid analyzing wavelets.

```
cwtftinfo
```

## More About

### Wavelet Definitions

#### Morlet Wavelet

Both non-analytic and analytic Morlet wavelets are supported. The analytic Morlet wavelet, `'morl'`, is defined in the Fourier domain by:

$$\breve{\Psi}(s\omega) = \pi^{-1/4} e^{-(s\omega-\omega_0)^2/2} U(s\omega)$$

where $U(\omega)$ is the Heaviside step function.

The non-analytic Morlet wavelet, `'morlex'`, is defined in the Fourier domain by:

$$\breve{\Psi}(s\omega) = \pi^{-1/4} e^{-(s\omega-\omega_0)^2/2}$$

`'morl0'` defines a non-analytic Morlet wavelet in the Fourier domain with exact zero mean:

$$\breve{\Psi}(s\omega) = \pi^{-1/4} \{e^{-(s\omega-\omega_0)^2/2} - e^{-\omega_0^2/2}\}$$

The default value of $\omega_0$ is 6.

The scale-to-frequency Fourier factor for the Morlet wavelet is:

$$\frac{4\pi s}{\omega_0 + \sqrt{2+\omega_0^2}}$$

### *m*-th Order Derivative of Gaussian Wavelets

In the Fourier domain, the *m*-th order derivative of Gaussian wavelets, `'dog'`, is defined by:

$$\hat{\Psi}(s\omega) = -\frac{1}{\sqrt{\Gamma(m+1/2)}}(js\omega)^m e^{-(s\omega)^2/2}$$

The derivative must be an even order. The default order of the derivative is 2, which is also known as the *Mexican hat* wavelet.

Because the unit imaginary, *j*, is always raised to an even power, the Fourier transform is real-valued.

The scale-to-frequency Fourier factor for the DOG wavelet is:

$$\frac{2\pi s}{\sqrt{m+\frac{1}{2}}}$$

### Paul Wavelet

The Fourier transform of the Paul wavelet, `'paul'`, of order *m* is:

$$\hat{\Psi}(s\omega) = 2^m \sqrt{m(2m-1)!}\,(s\omega)^m\, e^{-s\omega} U(sw)$$

where $U(\omega)$ is the Heaviside step function. The Paul wavelet is analytic.

The scale-to-frequency Fourier factor for the Paul wavelet is:

$$\frac{4\pi s}{2m+1}$$

The default order of the Paul wavelet is 4.

- “Continuous Wavelet Transform”
- “DFT-Based Continuous Wavelet Transform”
- “Inverse Continuous Wavelet Transform”

## References

[1] Daubechies, I. *Ten Lectures on Wavelets*, Philadelphia, PA: Society for Industrial and Applied Mathematics (SIAM), 1992.

[2] Farge, M. *Wavelet Transforms and Their Application to Turbulence*, Ann. Rev. Fluid. Mech., 1992, 24, 395–457.

[3] Mallat, S. *A Wavelet Tour of Signal Processing*, San Diego, CA: Academic Press, 1998.

[4] Torrence, C. and G.P. Compo *A Practical Guide to Wavelet Analysis*, Bull. Am. Meteorol. Soc., 79, 61–78, 1998.

## See Also

`cwtft` | `icwtft`

# cwtftinfo2

Supported 2-D CWT wavelets and Fourier transforms

## Syntax

```
cwtftinfo2
cwtftinfo2(wname)
```

## Description

`cwtftinfo2` lists the supported 2-D continuous wavelet transform (CWT) wavelets and corresponding parameters for use with `cwtft2`.

`cwtftinfo2(wname)` displays the equation for the 2-D Fourier transform of the wavelet, wname. The figure with the 2-D Fourier transform of the analyzing wavelet has a drop-down list from which you can select other wavelets.

## Examples

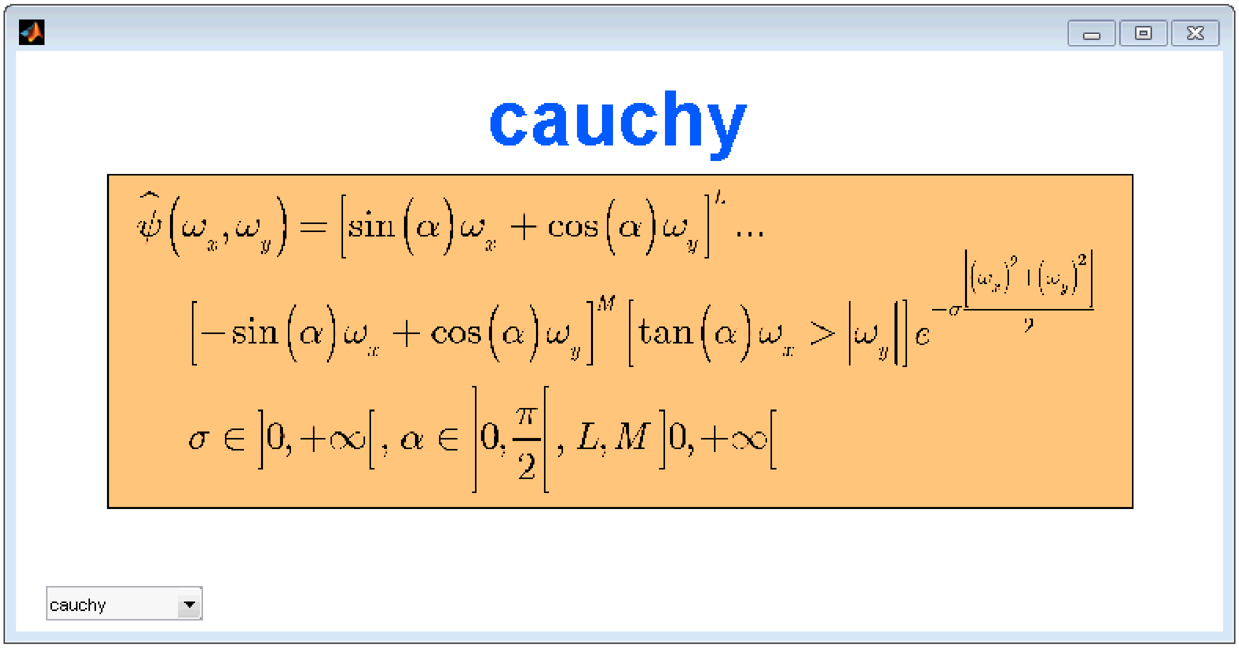
### Available Wavelets with Parameters

```
cwtftinfo2
```

### Display the Expression for the 2-D Fourier Transform

Display the expression for the 2-D Fourier transform of the Cauchy wavelet.

```
cwtftinfo2('cauchy')
```



$$\hat{\psi}(\omega_x,\omega_y) = \left[\sin(\alpha)\omega_x + \cos(\alpha)\omega_y\right]^L \ldots$$

$$\left[-\sin(\alpha)\omega_x + \cos(\alpha)\omega_y\right]^M \left[\tan(\alpha)\omega_x > \left|\omega_y\right|\right] e^{-\sigma\frac{\left|(\omega_x)^2+(\omega_y)^2\right|}{2}}$$

$$\sigma \in \left]0,+\infty\right[,\ \alpha \in \left]0,\frac{\pi}{2}\right[,\ L, M \left]0,+\infty\right[$$

After displaying the Fourier transform for any wavelet, you can use the drop-down list in the bottom left to view the Fourier transform for any supported wavelet.

## Input Arguments

### `wname` — Wavelet name

string

Wavelet name, specified as a string. The following table lists the supported wavelets for the 2-D CWT and associated parameters:

| Wavelet name | Parameters |
|---|---|
| `'morl'` | `{'Omega0',6;'Sigma',1;'Epsilon',1}` |
| `'mexh'` | `{'p',2;'sigmax',1;'sigmay',1}` |
| `'paul'` | `{'p',4}` |
| `'dog'` | `{'alpha',1.25}` |

| Wavelet name | Parameters |
|---|---|
| `'cauchy'` | `{'alpha','pi/6';'L',4;'M',4;'sigma',1}` |
| `'escauchy'` | `{'alpha','pi/6';'L',4;'M',4;'sigma',1}` |
| `'gaus'` | `{'p',1;'sigmax',1;'sigmay',1}` |
| `'wheel'` | `{'sigma',2}` |
| `'fan'` | `{'Omega0X',5.336;'Sigma',1;'Epsilon',1;'J',` |
| `'pethat'` | None |
| `'dogpow'` | `{'alpha',1.25;'p',2}` |
| `'esmorl'` | `{'Omega0',6;'Sigma',1;'Epsilon',1}` |
| `'esmexh'` | `{'Sigma',1;'Epsilon',0.5}` |
| `'gaus2'` | `{'p',1;'sigmax',1;'sigmay',1}` |
| `'gaus3'` | `{'A',1;'B',1;'p',1;'sigmax',1;'sigmay',1}` |
| `'isodog'` | `{'alpha',1.25,1.25}` |
| `'dog2'` | `{'alpha',1.25,1.25}` |
| `'isomorl'` | `{'Omega0',6;'Sigma',1}` |
| `'rmorl'` | `{'Omega0',6;'Sigma',1;'Epsilon',1}` |
| `'endstop1'` | `{'Omega0',6}` |
| `'endstop2'` | `{'Omega0',6;'Sigma',1}` |
| `'gabmexh'` | `{'Omega0',5.336;'Epsilon',1}` |
| `'sinc'` | `{'Ax',1;'Ay',1;'p',1;'Omega0X',0;'Omega0Y',` |

Example: `cwtftinfo2('paul')`

Data Types: `char`

## See Also

`cwtft2`

## cwtft2

2-D continuous wavelet transform

### Syntax

```
cwtstruct = cwtft2(x)
cwtstruct = cwtft2(x,'plot')
cwtstruct = cwtft2(x,Name,Value)
```

### Description

`cwtstruct = cwtft2(x)` returns the 2-D continuous wavelet transform (CWT) of the 2-D matrix, x. `cwtft2` uses a Fourier transform-based algorithm in which the 2-D Fourier transforms of the input data and analyzing wavelet are multiplied together and inverted.

`cwtstruct = cwtft2(x,'plot')` plots the data and the 2-D CWT.

`cwtstruct = cwtft2(x,Name,Value)` uses additional options specified by one or more Name,Value pair arguments.

### Examples

#### Compare Isotropic and Anisotropic Wavelets

This example shows how an isotropic wavelet does not discern the orientation of features while an anisotropic wavelet does. The example uses the Mexican hat isotropic wavelet and the directional (anisotropic) Cauchy wavelet.

Load and view the hexagon image.

```
Im = imread('hexagon.jpg');
imagesc(Im); colormap(jet);
```

Obtain the scale-one 2-D CWT with both the Mexican hat and Cauchy wavelets. Specify a vector of angles going from 0 to 15π/8 in π/8 increments.

```
cwtcauchy = cwtft2(Im,'wavelet','cauchy','scales',1,...
    'angles',0:pi/8:2*pi-pi/8);

cwtmexh = cwtft2(Im,'wavelet','mexh','scales',1,...
    'angles',0:pi/8:2*pi-pi/8);
```

Visualize the scale-one 2-D CWT coefficient magnitudes at each angle.

```
angz = {'0', 'pi/8', 'pi/4', '3pi/8', 'pi/2', '5pi/8', '3pi/4', ...
    '7pi/8','pi', '9pi/8', '5pi/4', '11pi/8', '3pi/2', ...
    '13pi/8' '7pi/4', '15pi/8'};
for angn = 1:length(angz)
    subplot(211)
    imagesc(abs(cwtmexh.cfs(:,:,1,1,angn)));
    title(['Mexican hat at ' angz(angn) 'radians']);
    subplot(212)
    imagesc(abs(cwtcauchy.cfs(:,:,1,1,angn)));
    title(['Cauchy wavelet at ' angz(angn) 'radians']);
    pause(1);
end
```

### Plot 2-D CWT

Load an image of a woman, obtain the 2-D CWT using the Morlet wavelet, and plot the CWT coefficients.

```
load woman;
cwtmorl = cwtft2(X,'scales',1:4,'angles',0:pi/2:3*pi/2,'plot');
```

### 2-D CWT with Morlet Wavelet

Obtain the 2-D CWT of the star image using the default Morlet wavelet, scales `2^(0:5)`, and an angle of 0.

```
Im = imread('star.jpg');
cwtout = cwtft2(Im);
```

- “Two-Dimensional CWT of Noisy Pattern”
- “2-D Continuous Wavelet Transform App”

## Input Arguments

### x — Input data

array

Input data, specified as a 2-D matrix or 3-D array. If the input data is a 3-D array, the input matrix is a truecolor image.

Example: `X = imread('stars.jpg');`

Data Types: `double` | `uint8`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name,Value arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

Example: `'wavelet','paul','scales',2^(0:5)` specifies to use the Paul wavelet and a vector of scales.

**`'angles'` — Angles**
0 (default) | scalar | vector

Angles in radians, specified as a comma-separated pair consisting of `'angles'` and either a scalar or a vector.

Example: `'angles',[0 pi/2 pi]`

**`'norm'` — Normalization**
`'L2'` (default) | `'L1'` | `'L0'`

Normalization used in the 2-D CWT, specified as a comma-separated pair consisting of `'norm'` and one of these strings:

- `'L2'` — The Fourier transform of the analyzing wavelet at a given scale is multiplied by the corresponding scale. `'L2'` is the default normalization.
- `'L1'` — The Fourier transform of the analyzing wavelet is multiplied by 1 at all scales.
- `'L0'` — The Fourier transform of the analyzing wavelet at a given scale is multiplied by the square of the corresponding scale.

Example: `'norm','L1'`

**`'scales'` — Scales**
`2^(0:5)` (default) | scalar | vector

Scales, specified as a comma-separated pair consisting of `'scales'` and either a positive real-valued scalar or a vector of positive real numbers.

Example: `'scales',2^(1:6)`

**`'wavelet'` — Analyzing wavelet**
`'morl'` (default) | string | structure | cell array

Analyzing wavelet, specified as a comma-separated pair consisting of `'wavelet'` and a string, a structure, or a cell array. `cwtftinfo2` provides a comprehensive list of supported wavelets and associated parameters.

If you specify `'wavelet'` as a structure, the structure must contain two fields:

- `name` — the string corresponding to a supported wavelet.
- `param` — a cell array with the parameters of the wavelet.

If you specify `'wavelet'` as a cell array, `wav`, the cell array must contain two elements:

- `wav{1}` — the string corresponding to a supported wavelet.
- `wav{2}` — a cell array with the parameters of the wavelet.

Example: `'wavelet',{'morl',{6,1,1}}`

Example: `'wavelet',struct('name','paul','param',{'p',2})`

## Output Arguments

**`cwtstruct` — 2-D CWT**
structure

The 2-D CWT, returned as a structure with the following fields:

**`wav` — Analyzing wavelet and parameters**
structure

Analyzing wavelet and parameters, returned as a structure with the following fields:

- `wname` — name
- `param` — parameters

### `wav_norm` — Normalization constants
matrix

Normalization constants, returned as a $M$-by-$N$ matrix where $M$ is the number of scales and $N$ is the number of angles.

### `cfs` — CWT coefficients
array

CWT coefficients, returned as an N-D array. The row and column dimensions of the array equal the row and column dimensions of the input data. The third page of the array is equal to 1 or 3 depending on whether the input data is a grayscale or truecolor image. The fourth page of the array is equal to the number of scales and the fifth page of the array is equal to the number of angles.

### `scales` — Scales
vector

Scales for the 2-D CWT, returned as a row vector.

### `angles` — Angles
vector

Angles for the 2-D CWT, returned as a row vector.

### `meanSIG` — Mean
scalar

Mean of the input data, returned as a scalar

## See Also
`cwtftinfo2`

# cwtext

Real or complex continuous 1-D wavelet coefficients using extension parameters

## Syntax

```
COEFS = cwtext(S,SCALES,'wname')
COEFS = cwtext(S,SCALES,'wname',PropName1,ProVal1, ...)
EXTMODE = struct('Mode',ModeVAL,'Side',SideVAL,'Len',LenVAL);
EXTMODE = {ModeVAL,SideVAL,LenVAL};
COEFS = cwtext(...,'PlotMode',PLOTMODE)
```

## Description

`COEFS = cwtext(S,SCALES,'wname')` computes the continuous wavelet coefficients of the vector S at real, positive SCALES, using a wavelet named `'wname'`. The signal S is real; the wavelet can be real or complex.

`COEFS = cwtext(S,SCALES,'wname',PropName1,ProVal1, ...)` computes and plots the continuous wavelet transform coefficients using extra parameters. Valid values for PropName are:

- `'ExtMode'`
- `'ExtSide'`
- `'ExtLen'`
- `'PlotMode'`
- `'xlim'`

The continuous wavelet transform coefficients are computed using the extension parameters:

- `'ExtMode'`
- `'ExtSide'`
- `'ExtLen'`

Valid values for *`ExtMode`* are:

- 'zpd' (zero padding)
- 'sp0' (smooth extension of order 0)
- 'sp1' (smooth extension of order 1)

etc.

Valid values for *ExtSide* are:

- `ExtSide = 'l'` (or `'u'`) for left (or up) extension
- `ExtSide = 'r'` (or `'d'`) for right (or down) extension
- `ExtSide = 'b'` for extension on both sides
- `ExtSide = 'n'` null extension

For the complete list of valid values for `ExtMode` and `ExtSide`, see `wextend`.

`ExtLen` is the length of extension.

Default values for extension parameters are `'zpd'` and `'b'`. `ExtLen` is computed using the maximum of SCALES.

Instead of three parameters, use the following syntaxes:

```
EXTMODE = struct('Mode',ModeVAL,'Side',SideVAL,'Len',LenVAL);
```

```
EXTMODE = {ModeVAL,SideVAL,LenVAL};
```

`COEFS = cwtext(...,'PlotMode',PLOTMODE)` computes and plots the continuous wavelet transform coefficients. Coefficients are colored using `PLOTMODE`:

- `PLOTMODE = 'lvl'` (By scale)
- `PLOTMODE = 'glb'` (All scales)
- `PLOTMODE = 'abslvl'` or `'lvlabs'` (Absolute value and By scale)
- `PLOTMODE = 'absglb'` or `'glbabs'` (Absolute value and All scales)

You get 3-D plots (surfaces) using the same keywords listed above for the `PLOTMODE` parameter, preceded by `'3D'`, for example, `PLOTMODE = '3Dlvl'`.

When `PLOTMODE = 'scal'` or `'scalCNT'` the continuous wavelet transform coefficients and the corresponding scalogram (percentage of energy for each coefficient) are computed.

When `PLOTMODE` is `'scal'`, a scaled image of scalogram is displayed. When `PLOTMODE` is `'scalCNT'`, a contour representation of scalogram is displayed.

If the `XLIM` parameter is given, the continuous wavelet transform coefficients are colored using `PLOTMODE` and `XLIM`.

`XLIM = [x1 x2]` with `1 <= x1 < x2 <= length(S)`.

For each given scale `a` within the vector `SCALES`, the wavelet coefficients `C(a,b)` are computed for `b = 1` to `ls = length(S)`, and are stored in `COEFS(i,:)` if `a = SCALES(i)`.

Output argument `COEFS` is a `la`-by-`ls` matrix where `la` is the length of `SCALES`. `COEFS` is a real or complex matrix depending on the wavelet type.

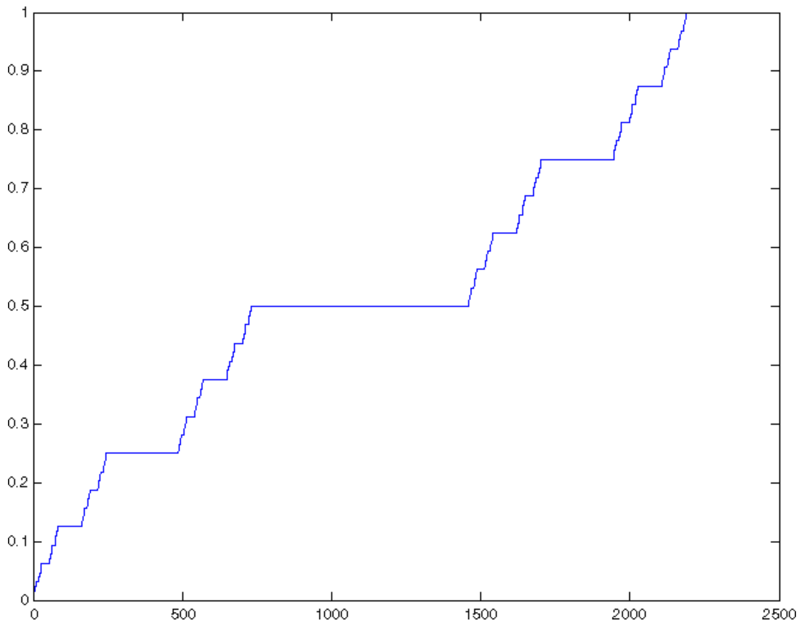
Examples of valid use are as follows:

```
t = linspace(-1,1,512);
s = 1-abs(t);
c = cwtext(s,1:32,'cgau4');
c = cwtext(s,[64 32 16:-2:2],'morl');
c = cwtext(s,[3 18 12.9 7 1.5],'db2');
c = cwtext(s,1:32,'sym2','plotMode','lvl');
c = cwtext(s,1:64,'sym4','plotMode','abslvl','xlim',[100 400]);

[c,Sc] = cwtext(s,1:64,'sym4','plotMode','scal');
[c,Sc] = cwtext(s,1:64,'sym4','plotMode','scalCNT');
[c,Sc] = cwtext(s,1:64,'sym4','plotMode','scalCNT', ...
                              'extMode','sp1');

c = cwtext(s,1:64,'sym4','plotMode','lvl','extMode','sp0');
c = cwtext(s,1:64,'sym4','plotMode','lvl','extMode','sp1');
c = cwtext(s,1:64,'sym4','plotMode','lvl', ...
                        'extMode',{'sp1','b',300});

ext = struct('Mode','sp1','Side','b','Len',300);
c = cwtext(s,1:64,'sym4','plotMode','lvl','extMode',ext);
```

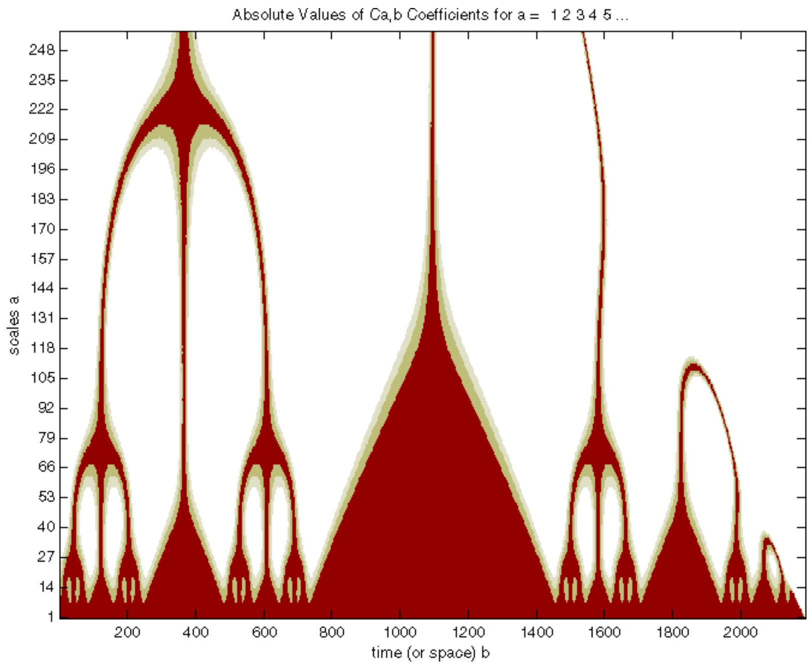
## Examples

This example demonstrates the difference between a continuous wavelet transform which deals with signal extension and one which does not.

```
% Load and plot the signal
load wcantor
plot(wcantor)
```

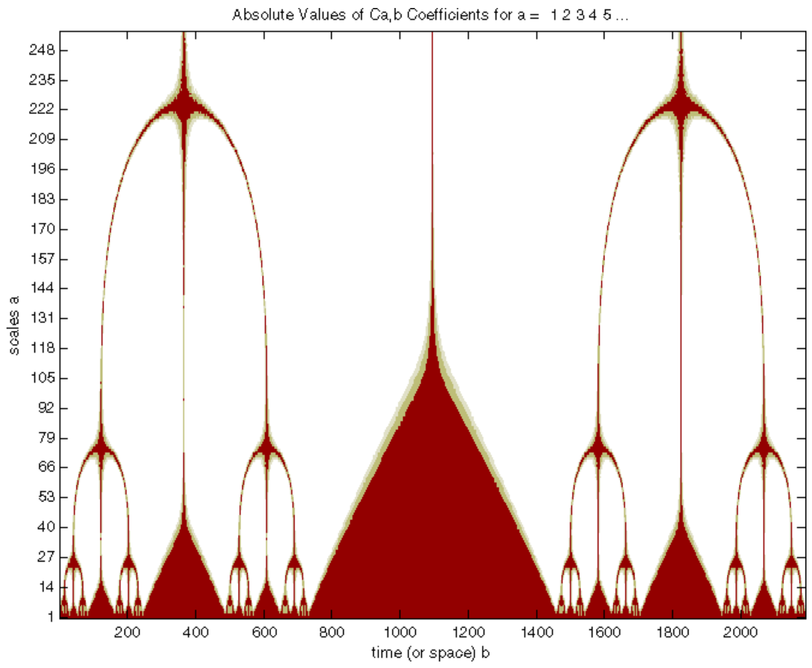


```
% Compute and plot the coefficients
cwt(wcantor,(1:256),'mexh','absglb');
colormap(pink(4))
```



In this figure above, which is produced by the `cwt` function, the values of coefficients are tremendously affected by the boundary effect due to the discontinuity of the signal on the right. The default (zero-padding) extension mode on the right explains this important discontinuity because the last value is 1. On the left there is no effect because the first value is 0.

```
% Compute and plot the coefficients with adapted extension mode
figure;
cwtext(wcantor,(1:256),'mexh','extmode','sp0','extLen',2000, ...
                              'plotMode','absglb');
colormap(pink(4))
```



In this figure, produced by the `cwtext` function, the suitable extension mode of the signal is very efficient, giving as it can be seen, a good result.

## See Also

`cwt` | `wavedec` | `wavefun` | `waveinfo` | `wcodemat`

# dbaux

Daubechies wavelet filter computation

## Syntax

```
W = dbaux(N,SUMW)
W = dbaux(N)
W = dbaux(N,0)
```

## Description

`W = dbaux(N,SUMW)` is the order *N* Daubechies scaling filter such that `sum(W) = SUMW`. Possible values for *N* are 1, 2, 3, ...

**Note** Instability may occur when *N* is too large.

`W = dbaux(N)` is equivalent to `W = dbaux(N,1)`

`W = dbaux(N,0)` is equivalent to `W = dbaux(N,1)`

## Daubechies' Extremal Phase Scaling Filter with Specified Sum

This example shows to determine the Daubechies' extremal phase scaling filter with a specified sum. The two most common values for the sum are $\sqrt{2}$ and 1.

Construct two versions of the `'db4'` scaling filter. One scaling filter sums to $\sqrt{2}$ and the other version sums to 1.

```
NumVanishingMoments = 4;
h = dbaux(NumVanishingMoments,sqrt(2));
m0 = dbaux(NumVanishingMoments,1);
```

The filter with sum equal to $\sqrt{2}$ is the synthesis (reconstruction) filter returned by `wfilters` and used in the discrete wavelet transform.

```
[LoD,HiD,LoR,HiR] = wfilters('db4');
max(abs(LoR-h))
```

```
ans =

   4.2613e-13
```

For orthogonal wavelets, the analysis (decomposition) filter is the time-reverse of the synthesis filter.

```
max(abs(LoD-fliplr(h)))
```

```
ans =

   4.2613e-13
```

## Limitations

The computation of the `dbN` Daubechies scaling filter requires the extraction of the roots of a polynomial of order `4N`. Instability may occur when `N` is too large.

## More About

### Algorithms

The algorithm used is based on a result obtained by Shensa (see “References”), showing a correspondence between the “Lagrange à trous” filters and the convolutional squares of the Daubechies wavelet filters.

The computation of the order $N$ Daubechies scaling filter $w$ proceeds in two steps: compute a “Lagrange à trous” filter $P$, and extract a square root. More precisely:

- P the associated “Lagrange à trous” filter is a symmetric filter of length 4N-1. P is defined by

$P = [a(N)\ 0\ a(N\text{-}1)\ 0 \ldots 0\ a(1)\ 1\ a(1)\ 0\ a(2)\ 0 \ldots 0\ a(N)]$

- where

$$a(k) = \frac{\displaystyle\prod_{\substack{i=-N+1 \\ i \neq k}}^{N} \left(\frac{1}{2} - i\right)}{\displaystyle\prod_{\substack{i=-N+1 \\ i \neq k}}^{N} (k - i)} \quad \text{for } k = 1, \ldots, N$$

- Then, if $w$ denotes db$N$ Daubechies scaling filter of sum $\sqrt{2}$, $w$ is a square root of $P$:

  $P$ = `conv(wrev(`$w$`),`$w$`)` where $w$ is a filter of length $2N$.

  The corresponding polynomial has $N$ zeros located at $-1$ and $N-1$ zeros less than 1 in modulus.

Note that other methods can be used; see various solutions of the spectral factorization problem in Strang-Nguyen (p. 157).

## References

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics, SIAM Ed.

Shensa, M.J. (1992), “The discrete wavelet transform: wedding the a trous and Mallat Algorithms,” *IEEE Trans. on Signal Processing*, vol. 40, 10, pp. 2464-2482.

Strang, G.; T. Nguyen (1996), *Wavelets and Filter Banks*, Wellesley-Cambridge Press.

## See Also

`dbwavf` | `wfilters`

## dbwavf

Daubechies wavelet filter

### Syntax

`F = dbwavf(W)`

### Description

`F = dbwavf(W)` returns the scaling filter associated with Daubechies wavelet specified by the string *W* where `W = 'dbN'`. Possible values for `N` are 1, 2, 3, ..., 45.

### Examples

```
% Set Daubechies wavelet name.
wname = 'db4';

% Compute the corresponding scaling filter.
f = dbwavf(wname)

f =
Columns 1 through 7
0.1629  0.5055  0.4461 -0.0198 -0.1323  0.0218  0.0233
Column 8
-0.0075
```

### See Also

`dbaux` | `waveinfo` | `wfilters`

# ddencmp

Default values for denoising or compression

## Syntax

```
[THR,SORH,KEEPAPP,CRIT] = ddencmp(IN1,IN2,X)
[THR,SORH,KEEPAPP] = ddencmp(IN1,'wv',X)
[THR,SORH,KEEPAPP,CRIT] = ddencmp(IN1,'wp',X)
```

## Description

`ddencmp` returns default values for denoising or compression for the critically-sampled discrete wavelet or wavelet packet transform.

You can use `ddencmp` for 1-D signals or 2-D images.

`[THR,SORH,KEEPAPP,CRIT] = ddencmp(IN1,IN2,X)` returns default values for denoising or compression, using wavelets or wavelet packets, of an input vector or matrix *X*, which can be a one- or two-dimensional signal. `THR` is the threshold, `SORH` is for soft or hard thresholding, `KEEPAPP` allows you to keep approximation coefficients, and `CRIT` (used only for wavelet packets) is the entropy name (see `wentropy` for more information).

`IN1` is `'den'` for denoising or `'cmp'` for compression.

`IN2` is `'wv'` for wavelet or `'wp'` for wavelet packet.

For wavelets (three output arguments):

`[THR,SORH,KEEPAPP] = ddencmp(IN1,'wv',X)` returns default values for denoising (if `IN1 = 'den'`) or compression (if `IN1 = 'cmp'`) of *X*. These values can be used for `wdencmp`.

For wavelet packets (four output arguments):

`[THR,SORH,KEEPAPP,CRIT] = ddencmp(IN1,'wp',X)` returns default values for denoising (if `IN1 = 'den'`) or compression (if `IN1 = 'cmp'`) of *X*. These values can be used for `wdencmp`.

## Examples

### Default Global Threshold for Wavelet Denoising

Determine the default global denoising threshold for an N(0,1) white noise input.

Create an N(0,1) white noise input. Set the random number generator to the default initial settings for reproducible results.

```
dwtmode('per');
rng default;
x = randn(512,1);
```

Use `ddencmp` to obtain the default global threshold for wavelet denoising. Demonstrate that the threshold is equal to the universal threshold of Donoho and Johnstone scaled by a robust estimate of the variance.

```
[thr,sorh,keepapp] = ddencmp('den','wv',x);
[A,D] = dwt(x,'db1');
noiselev = median(abs(D))/0.6745;
thresh = sqrt(2*log(length(x)))*noiselev;
```

Compare the value of the variable `thr` to the value of `thresh`.

### Default Global Threshold for Wavelet Packet Compression

Determine the default global compression threshold for an N(0,1) white noise input.

Create an N(0,1) white noise input. Set the random number generator to the default initial settings for reproducible results.

```
dwtmode('per');
rng default;
x = randn(512,1);
```

Use `ddencmp` with the `'cmp'` and `'wp'` input arguments to return the default global compression threshold for a wavelet packet transform.

```
[thr,sorh,keepapp,crit] = ddencmp('den','wp',x);
```

## References

Donoho, D.L. (1995), “De-noising by soft-thresholding,” *IEEE*, *Trans. on Inf. Theory*, 41, 3, pp. 613–627.

Donoho, D.L.; I.M. Johnstone (1994), “Ideal spatial adaptation by wavelet shrinkage,” *Biometrika*, vol 81, pp. 425–455.

Donoho, D.L.; I.M. Johnstone (1994), “Ideal de-noising in an orthonormal basis chosen from a library of bases,” *C.R.A.S. Paris*, *Ser. I*, t. 319, pp. 1317–1322.

## See Also

`wdencmp` | `wenergy` | `wpdencmp`

## dddtree

Dual-tree and double-density 1-D wavelet transform

### Syntax

```
wt = dddtree(typetree,x,level,fdf,df)
wt = dddtree(typetree,x,level,fname)
wt = dddtree(typetree,x,level,fname1,fname2)
```

### Description

`wt = dddtree(typetree,x,level,fdf,df)` returns the typetree discrete wavelet transform (DWT) of the 1-D input signal, x, down to level, level. The wavelet transform uses the decomposition (analysis) filters, fdf, for the first level and the analysis filters, df, for subsequent levels. Supported wavelet transforms are the critically sampled DWT, double-density, dual-tree complex, and dual-tree double-density complex wavelet transform. The critically sampled DWT is a filter bank decomposition in an orthogonal or biorthogonal basis (nonredundant). The other wavelet transforms are oversampled filter banks.

`wt = dddtree(typetree,x,level,fname)` uses the filters specified by fname to obtain the wavelet transform. Valid filter specifications depend on the type of wavelet transform. See `dtfilters` for details.

`wt = dddtree(typetree,x,level,fname1,fname2)` uses the filters specified in fname1 for the first stage of the dual-tree wavelet transform and the filters specified in fname2 for subsequent stages of the dual-tree wavelet transform. Specifying different filters for stage 1 is valid and necessary only when typetree is `'cplxdt'` or `'cplxdddt'`.

### Examples

#### Complex Dual-Tree Wavelet Transform

Obtain the complex dual-tree wavelet transform of the noisy Doppler signal. The FIR filters in the first and subsequent stages result in an approximately analytic wavelet as required.

Create the first-stage analysis filters for the two trees.

```
  Faf{1} = [0         0
   -0.0884   -0.0112
    0.0884    0.0112
    0.6959    0.0884
    0.6959    0.0884
    0.0884   -0.6959
   -0.0884    0.6959
    0.0112   -0.0884
    0.0112   -0.0884
         0         0];
Faf{2} = [ 0.0112  0
    0.0112         0
   -0.0884   -0.0884
    0.0884   -0.0884
    0.6959    0.6959
    0.6959   -0.6959
    0.0884    0.0884
   -0.0884    0.0884
         0    0.0112
         0   -0.0112];
```

Create the analysis filters for subsequent stages of the multiresolution analysis.

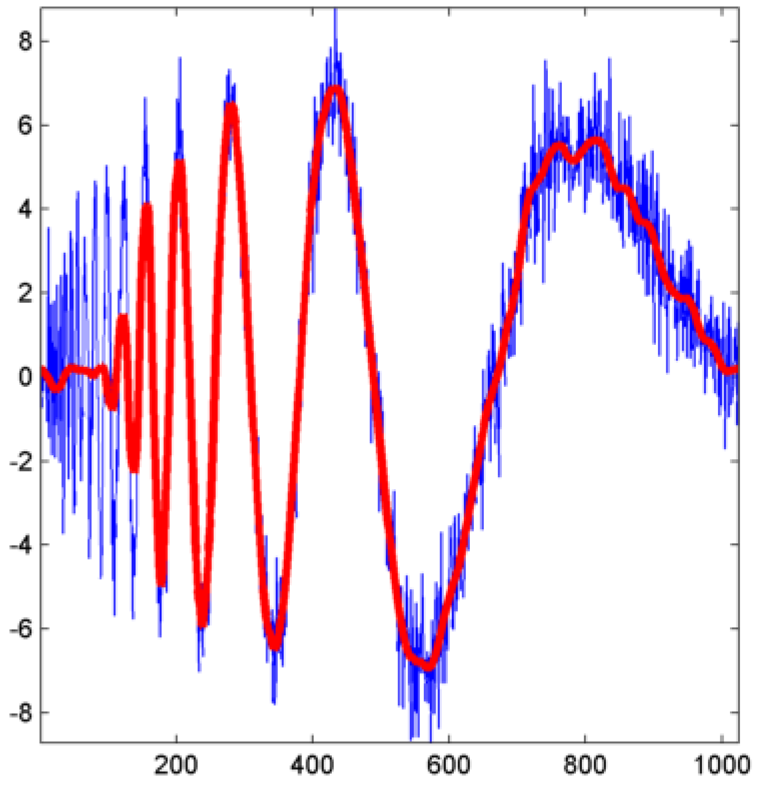
```
af{1} = [ 0.0352         0
         0         0
   -0.0883   -0.1143
    0.2339         0
    0.7603    0.5875
    0.5875   -0.7603
         0    0.2339
   -0.1143    0.0883
         0         0
         0   -0.0352];

af{2} = [0   -0.0352
         0         0
   -0.1143    0.0883
         0    0.2339
    0.5875   -0.7603
    0.7603    0.5875
    0.2339         0
   -0.0883   -0.1143
         0         0
    0.0352         0];
```

Load the noisy Doppler signal and obtain the complex dual-tree wavelet transform down to level 4.

```
load noisdopp;
wt = dddtree('cplxdt',noisdopp,4,Faf,af);
```

Plot an approximation based on the level-four approximation coefficients.

```
xapp = dddtreecfs('r',wt,'scale',{5});
plot(noisdopp);  hold on;
plot(cell2mat(xapp),'r','linewidth',3);
axis tight;
```



### Double-Density Wavelet Transform

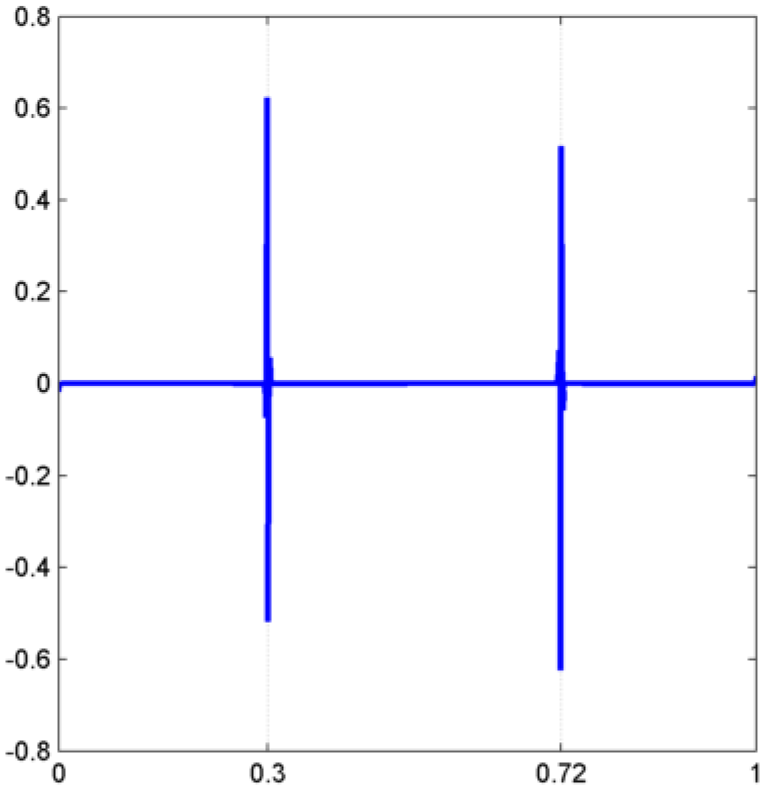
Obtain the double-density wavelet transform of a signal with two discontinuities. Use the level-one detail coefficients to localize the discontinuities.

Create a signal consisting of a 2-Hz sine wave with a duration of 1 second. The sine wave has discontinuities at 0.3 and 0.72 seconds.

```
N = 1024;
t = linspace(0,1,1024);
x = 4*sin(4*pi*t);
x = x - sign(t - .3) - sign(.72 - t);
plot(t,x); xlabel('t'); ylabel('x');
title('Original Signal');
```

Obtain the double-density wavelet transform of the signal, reconstruct an approximation based on the level-one detail coefficients, and plot the result.

```
wt = dddtree('ddt',x,1,'filters1');
wt.cfs{2} = zeros(1,512);
xrec = idddtree(wt);
plot(t,xrec,'linewidth',2)
set(gca,'xtick',[0 0.3 0.72 1]); set(gca,'xgrid','on');
```



**First-Level Detail Coefficients Approximation — Complex Dual-Tree**

Obtain the complex dual-tree wavelet transform of a signal with two discontinuities. Use the first-level detail coefficients to localize the discontinuities.

Create a signal consisting of a 2-Hz sine wave with a duration of 1 second. The sine wave has discontinuities at 0.3 and 0.72 seconds.

```
N = 1024;
```

```
t = linspace(0,1,1024);
x = 4*sin(4*pi*t);
x = x - sign(t - .3) - sign(.72 - t);
plot(t,x); xlabel('t'); ylabel('x');
title('Original Signal');
```

Obtain the dual-tree wavelet transform of the signal, reconstruct an approximation based on the level-one detail coefficients, and plot the result.

```
wt = dddtree('cplxdt',x,1,'FSfarras','qshift06');
wt.cfs{2} = zeros(1,512,2);
xrec = idddtree(wt);
plot(t,xrec,'linewidth',2)
set(gca,'xtick',[0 0.3 0.72 1]); set(gca,'xgrid','on');
```

- "Analytic Wavelets Using the Dual-Tree Wavelet Transform"

## Input Arguments

### `typetree` — Type of wavelet decomposition
`'dwt'` | `'ddt'` | `'cplxdt'` | `'cplxdddt'`

Type of wavelet decomposition, specified as one of `'dwt'`, `'ddt'`, `'cplxdt'`, or `'cplxdddt'`. The type, `'dwt'`, gives a critically sampled (nonredundant) discrete wavelet transform. The other decomposition types produce oversampled wavelet transforms. `'ddt'` produces a double-density wavelet transform. `'cplxdt'` produces a dual-tree complex wavelet transform. `'cplxdddt'` produces a double-density dual-tree complex wavelet transform.

### `x` — Input signal
vector

Input signal, specified as an even-length row or column vector. If $L$ is the value of the level of the wavelet decomposition, $2^L$ must divide the length of x. Additionally, the length of the signal must be greater than or equal to the product of the maximum length of the decomposition (analysis) filters and $2^{(L-1).}$

Data Types: `double`

### `level` — Level of wavelet decomposition
positive integer

Level of the wavelet decomposition, specified as an integer. If $L$ is the value of level, $2^L$ must divide the length of x . Additionally, the length of the signal must be greater than or equal to the product of the maximum length of the decomposition (analysis) filters and $2^{(L-1)}$.

Data Types: `double`

**`fdf` — Level-one analysis filters**
matrix | cell array

The level-one analysis filters, specified as a matrix or cell array of matrices. Specify fdf as a matrix when typetree is `'dwt'` or `'ddt'`. The size and structure of the matrix depend on the typetree input as follows:

- `'dwt'` — This is the critically sampled discrete wavelet transform. In this case, fdf is a two-column matrix with the lowpass (scaling) filter in the first column and the highpass (wavelet) filter in the second column.
- `'ddt'` — This is the double-density wavelet transform. The double-density DWT is a three-channel perfect reconstruction filter bank. fdf is a three-column matrix with the lowpass (scaling) filter in the first column and the two highpass (wavelet) filters in the second and third columns. In the double-density wavelet transform, the single lowpass and two highpass filters constitute a three-channel perfect reconstruction filter bank. This is equivalent to the three filters forming a tight frame. You cannot arbitrarily choose the two wavelet filters in the double-density DWT. The three filters together must form a tight frame.

Specify fdf as a 1-by-2 cell array of matrices when typetree is a dual-tree transform, `'cplxdt'` or `'cplxdddt'`. The size and structure of the matrix elements depend on the typetree input as follows:

- For the dual-tree complex wavelet transform, `'cplxdt'`, `fdf{1}` is a two-column matrix containing the lowpass (scaling) filter and highpass (wavelet) filters for the first tree. The scaling filter is the first column and the wavelet filter is the second column. `fdf{2}` is a two-column matrix containing the lowpass (scaling) and highpass (wavelet) filters for the second tree. The scaling filter is the first column and the wavelet filter is the second column.
- For the double-density dual-tree complex wavelet transform, `'cplxdddt'`, `fdf{1}` is a three-column matrix containing the lowpass (scaling) and two highpass (wavelet) filters for the first tree and `fdf{2}` is a three-column matrix containing the lowpass (scaling) and two highpass (wavelet) filters for the second tree.

Data Types: `double`

### `df` — Analysis filters for levels > 1

matrix | cell array

Analysis filters for levels > 1, specified as a matrix or cell array of matrices. Specify df as a matrix when typetree is `'dwt'` or `'ddt'`. The size and structure of the matrix depend on the typetree input as follows:

- `'dwt'` — This is the critically sampled discrete wavelet transform. In this case, df is a two-column matrix with the lowpass (scaling) filter in the first column and the highpass (wavelet) filter in the second column. For the critically sampled orthogonal or biorthogonal DWT, the filters in df and fdf must be identical.
- `'ddt'` — This is the double-density wavelet transform. The double-density DWT is a three-channel perfect reconstruction filter bank. df is a three-column matrix with the lowpass (scaling) filter in the first column and the two highpass (wavelet) filters in the second and third columns. In the double-density wavelet transform, the single lowpass and two highpass filters must constitute a three-channel perfect reconstruction filter bank. This is equivalent to the three filters forming a tight frame. For the double-density DWT, the filters in df and fdf must be identical.

Specify df as a 1-by-2 cell array of matrices when typetree is a dual-tree transform, `'cplxdt'` or `'cplxdddt'`. For dual-tree transforms, the filters in fdf and df must be different. The size and structure of the matrix elements in the cell array depend on the typetree input as follows:

- For the dual-tree complex wavelet transform, `'cplxdt'`, `df{1}` is a two-column matrix containing the lowpass (scaling) and highpass (wavelet) filters for the first tree. The scaling filter is the first column and the wavelet filter is the second column. `df{2}` is a two-column matrix containing the lowpass (scaling) and highpass (wavelet) filters for the second tree. The scaling filter is the first column and the wavelet filter is the second column.
- For the double-density dual-tree complex wavelet transform, `'cplxdddt'`, `df{1}` is a three-column matrix containing the lowpass (scaling) and two highpass (wavelet) filters for the first tree and `df{2}` is a three-column matrix containing the lowpass (scaling) and two highpass (wavelet) filters for the second tree.

Data Types: `double`

### `fname` — Filter name

string

Filter name, specified as a string. For the critically sampled DWT, specify any valid orthogonal or biorthogonal wavelet filter. See `wfilters` for details. For the double-density wavelet transform, `'ddt'`, valid choices are `'filters1'` and `'filters2'`. For the complex dual-tree wavelet transform, valid choices are `'dtfP'` with P = 1, 2, 3, 4. For the double-density dual-tree wavelet transform, the only valid choice is `'dddtf1'`. See `dtfilters` for more details on valid filter strings for the oversampled wavelet filter banks.

Data Types: `char`

**`fname1` — First-stage filter name**
string

First-stage filter name, specified as a string. Specifying a different filter for the first stage is valid and necessary only in the dual-tree transforms, `'cplxdt'` and `'cplxddt'`. In the complex dual-tree wavelet transform, you can use any valid wavelet filter for the first stage. In the double-density dual-tree wavelet transform, the first-stage filters must form a three-channel perfect reconstruction filter bank.

Data Types: `char`

**`fname2` — Filter name for stages > 1**
string

Filter name for stages > 1, specified as a string. You must specify a first-level filter that is different from the wavelet and scaling filters in subsequent levels when using the dual-tree wavelet transforms, `'cplxdt'` or `'cplxdddt'`. See `dtfilters` for valid choices.

Data Types: `char`

## Output Arguments

**`wt` — Wavelet transform**
structure

Wavelet transform, returned as a structure with these fields:

**`type` — Type of wavelet decomposition (filter bank)**
`'dwt'` | `'ddt'` | `'cplxdt'` | `'cplxdddt'`

Type of wavelet decomposition (filter bank) used in the analysis, returned as one of `'dwt'`, `'ddt'`, `'cplxdt'`, or `'cplxdddt'`. The type, `'dwt'`, gives a critically sampled discrete wavelet transform. The other types correspond to oversampled wavelet transforms. `'ddt'` is a double-density wavelet transform, `'cplxdt'` is a dual-tree complex wavelet transform, and `'cplxdddt'` is a double-density dual-tree complex wavelet transform.

**`level` — Level of the wavelet decomposition**
positive integer

Level of wavelet decomposition, returned as a positive integer.

**`filters` — Decomposition (analysis) and reconstruction (synthesis) filters**
structure

Decomposition (analysis) and reconstruction (synthesis) filters, returned as a structure with these fields:

**`Fdf` — First-stage analysis filters**
matrix | cell array

First-stage analysis filters, returned as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage analysis filters for the corresponding tree.

**`Df` — Analysis filters for levels > 1**
matrix | cell array

Analysis filters for levels > 1, returned as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the analysis filters for the corresponding tree.

**`Frf` — First-level reconstruction filters**
matrix | cell array

First-level reconstruction filters, returned as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage synthesis filters for the corresponding tree.

### `Rf` — Reconstruction filters for levels > 1

matrix | cell array

Reconstruction filters for levels > 1, returned as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the synthesis filters for the corresponding tree.

### `cfs` — Wavelet transform coefficients

cell array of matrices

Wavelet transform coefficients, returned as a 1-by-(level+1) cell array of matrices. The size and structure of the matrix elements of the cell array depend on the type of wavelet transform, typetree, as follows:

- `'dwt'` — `cfs{j}`
  - j = 1,2,... level is the level.
  - `cfs{level+1}` are the lowpass, or scaling, coefficients.
- `'ddt'` — `cfs{j}(:,:,k)`
  - j = 1,2,... level is the level.
  - k = 1,2 is the wavelet filter.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'cplxdt'` — `cfs{j}(:,:,m)`
  - j = 1,2,... level is the level.

    - m = 1,2 are the real and imaginary parts.
    - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'cplxdddt'` — `cfs{j}(:,:,k,m)`
    - j = 1,2,... level is the level.
    - k = 1,2 is the wavelet filter.
    - m = 1,2 are the real and imaginary parts.
    - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.

## More About

- "Critically Sampled and Oversampled Wavelet Filter Banks"

## See Also

`dddtree2` | `dddtreecfs` | `dtfilters` | `idddtree`

# dddtreecfs

Extract dual-tree/double-density wavelet coefficients or projections

## Syntax

```
out = dddtreecfs(outputtype,wt,outputspec,outputindices)
out = dddtreecfs(outputtype,wt,outputspec,outputindices,'plot')
```

## Description

`out = dddtreecfs(outputtype,wt,outputspec,outputindices)` extracts the coefficients or subspace projections from the 1-D or 2-D wavelet decomposition, wt. If outputtype equals `'e'`, out contains wavelet or scaling coefficients. If outputtype equals `'r'`, out contains wavelet or scaling subspace projections (reconstructions).

`out = dddtreecfs(outputtype,wt,outputspec,outputindices,'plot')` plots the signal or image reconstruction or specified analysis coefficients. You can include the `'plot'` option anywhere after the wt input.

## Examples

### Reconstruction from 1-D Complex Dual-Tree Wavelet Transform

Obtain the complex dual-tree wavelet transform of the 1-D noisy Doppler signal. Reconstruct an approximation based on the level-three detail coefficients

Load the noisy Doppler signal. Obtain the complex dual-tree transform down to level 3.

```
load noisdopp;
wt = dddtree('cplxdt',noisdopp,3,'dtf1');
```

Plot a reconstruction of the original signal based on the level-three detail coefficients

```
xr = dddtreecfs('r',wt,'scale',{3},'plot');
```

### Coefficients from 1-D Complex Dual-Tree Wavelet Transform

Load the noisy Doppler signal. Obtain the complex dual-tree transform down to level 3.

```
load noisdopp;
wt = dddtree('cplxdt',noisdopp,3,'dtf1');
```

Create a cell array of vectors to obtain the second- and third-level detail coefficients from each of the wavelet filter bank trees.

```
outputindices = {[2 1]; [2 2]; [3 1]; [3 2]};
```

The first element of each vector in the cell array denotes the level, or stage. The second element denotes the tree.

Extract the detail coefficients.

```
out = dddtreecfs('e',wt,'ind',outputindices);
```

`out` is a 1-by-4 cell array. The cell array elements contain the wavelet coefficients corresponding to the elements in `outputindices`. For example, `out{1}` contains the level-two detail coefficients from the first tree.

### 1-D Complex Dual-Tree Wavelet Transform Structure

Load the noisy Doppler signal. Obtain the complex dual-tree transform down to level 3.

```
load noisdopp;
wt = dddtree('cplxdt',noisdopp,3,'dtf1');
```

Create a cell array of vectors to obtain the second- and third-level detail coefficients from each of the wavelet filter bank trees.

```
outputindices = {[2 1]; [2 2]; [3 1];[3 2]};
```

The first element of each vector in the cell array denotes the level, or stage. The second element denotes the tree.

Create a structure array identical to the `wt` output of `dddtree` with all the coefficients equal to zero except the first- and second-level detail coefficients.

```
out = dddtreecfs('e',wt,'cumind',outputindices);
```

### Extract Diagonal Features from Image

Use the complex dual-tree wavelet transform to isolate diagonal features in an image at +45 and –45 degrees.

Load and display the `xbox` image.

```
load xbox;
imagesc(xbox)
```

Obtain the complex dual-tree wavelet transform down to level 3.

```
fdf = dtfilters('FSfarras');
df = dtfilters('qshift10');
wt = dddtree2('cplxdt',xbox,3,fdf,df);
```

Isolate the +45 and -45 diagonal image features in the level-one wavelet coefficients. Plot the result.

```
out = dddtreecfs('e',wt,'ind',{[1 3 1 2]; [1 3 2 2]},'plot');
```

## Input Arguments

### `outputtype` — Output type

`'e'` | `'r'`

Output type, specified as `'e'` or `'r'`. Use `'e'` to obtain the scaling or wavelet coefficients. Use `'r'` to obtain a projection, or reconstruction, onto the appropriate scaling or wavelet subspace.

### `wt` — Wavelet transform

structure

Wavelet transform, specified as a structure. The structure array is the output of `dddtree` or `dddtree2`.

### `outputspec` — Output specification

`'lowpass'` | `'scale'` | `'ind'` | `'cumind'`

Output specification, specified as one of `'lowpass'`, `'scale'`, `'ind'`, or `'cumind'`. The output specifications are defined as follows:

- `'lowpass'` — Outputs the lowpass, or scaling, coefficients or a signal/image approximation based on the scaling coefficients. If you set the output specification to `'lowpass'`, do not specify outputindices. If the outputtype is `'e'`, out is a structure array with fields identical to the input structure array wt except that all wavelet (detail) coefficients are equal to zero. If the outputtype is `'r'`, out is a signal or image approximation based on the scaling coefficients. The signal or image approximation is equal in size to the original input to `dddtree` or `dddtree2`.

- `'scale'` — Outputs the coefficients or a signal/image approximation based on the scales specified in outputindices. If the outputtype is `'e'`, out is a cell array of structure arrays. The fields of the structure arrays in out are identical to the fields of the input structure array wt. The coefficients in the `cfs` field are all equal to zero except the coefficients corresponding to the scales in outputindices. If the outputtype is `'r'`, out is a signal or image approximation based on the scales in outputindices. The signal or image approximation is equal in size to the original input to `dddtree` or `dddtree2`.
- `'ind'` — Outputs the coefficients or a signal/image approximation based on the tree-position indices specified in outputindices. If the outputtype is `'e'`, out is a cell array of vectors or matrices containing the coefficients specified by the tree-position indices in outputindices. If the outputtype is `'r'`, out is a cell array of vectors or matrices containing signal or image approximations based on the corresponding tree-position indices in outputindices.
- `'cumind'` — Outputs the coefficients or a signal/image approximation based on the tree-position indices specified in outputindices. If the outputtype is `'e'`, out is a structure array. The fields of the structure array are identical to the fields of the input structure array wt. The coefficients in the `cfs` field are all equal to zero except the coefficients corresponding to the tree positions in outputindices. If the outputtype is `'r'`, out is a signal or image approximation based on the coefficients corresponding to the tree-position indices in outputindices.

Example: `'ind',{[1 1]; [1 2]}`

### `outputindices` — Output indices

cell array

Output indices, specified as a cell array with scalar or vector elements. If outputspec equals `'scale'`, a scalar element selects the corresponding element in the `cfs` field of wt. If outputspec equals `'ind'` or `'cumind'`, the elements of outputspec are row vectors. The first element of the row vector corresponds to the element in the `cfs` field of wt. Subsequent elements in the row vector correspond to the indices of the array contained in the cell array element.

Example: `'scale',{1;2;3}`

## Output Arguments

### `out` — Signal or image reconstruction or coefficients

cell array | structure | vector | matrix

Signal or image reconstruction or coefficients, returned as a vector, matrix, structure array, cell array of vectors or matrices, or cell array of structure arrays. The form of out depends on the value of outputspec and outputindices.

## See Also

`dddtree` | `dddtree2` | `plotdt`

# dddtree2

Dual-tree and double-density 2-D wavelet transform

## Syntax

```
wt = dddtree2(typetree,x,level,fdf,df)
wt = dddtree2(typetree,x,level,fname)
wt = dddtree2(typetree,x,level,fname1,fname2)
```

## Description

`wt = dddtree2(typetree,x,level,fdf,df)` returns the typetree discrete wavelet transform of the 2-D input image, x, down to level, level. The wavelet transform uses the decomposition (analysis) filters, fdf, for the first level and the analysis filters, df, for subsequent levels. Supported wavelet transforms are the critically sampled DWT, double-density, real oriented dual-tree, complex oriented dual-tree, real oriented dual-tree double-density, and complex oriented dual-tree double-density wavelet transform. The critically sampled DWT is a filter bank decomposition in an orthogonal or biorthogonal basis (nonredundant). The other wavelet transforms are oversampled filter banks with differing degrees of directional selectivity.

`wt = dddtree2(typetree,x,level,fname)` uses the filters specified by fname to obtain the wavelet transform. Valid filter specifications depend on the type of wavelet transform. See `dtfilters` for details.

`wt = dddtree2(typetree,x,level,fname1,fname2)` uses the filters specified in fname1 for the first stage of the dual-tree wavelet transform and the filters specified in fname2 for subsequent stages of the dual-tree wavelet transform. Specifying different filters for stage 1 is valid and necessary only when typetree is `'realdt'`, `'cplxdt'`, `'realdddt'`, or `'cplxdddt'`.

## Examples

### Real Oriented Dual-Tree Wavelets

Visualize the six directional wavelets of the real oriented dual-tree wavelet transform.

Create the first-stage analysis filters for the two trees.

```
  Faf{1} = [0         0
   -0.0884   -0.0112
    0.0884    0.0112
    0.6959    0.0884
    0.6959    0.0884
    0.0884   -0.6959
   -0.0884    0.6959
    0.0112   -0.0884
    0.0112   -0.0884
         0         0];
Faf{2} = [ 0.0112  0
    0.0112         0
   -0.0884   -0.0884
    0.0884   -0.0884
    0.6959    0.6959
    0.6959   -0.6959
    0.0884    0.0884
   -0.0884    0.0884
         0    0.0112
         0   -0.0112];
```

Create the analysis filters for subsequent stages of the multiresolution analysis.

```
af{1} = [ 0.0352         0
         0         0
   -0.0883   -0.1143
    0.2339         0
    0.7603    0.5875
    0.5875   -0.7603
         0    0.2339
   -0.1143    0.0883
         0         0
         0   -0.0352];

af{2} = [0   -0.0352
         0         0
   -0.1143    0.0883
         0    0.2339
    0.5875   -0.7603
    0.7603    0.5875
    0.2339         0
   -0.0883   -0.1143
         0         0
    0.0352         0];
```

Obtain the real dual-tree wavelet transform of an image of zeros down to level 4.

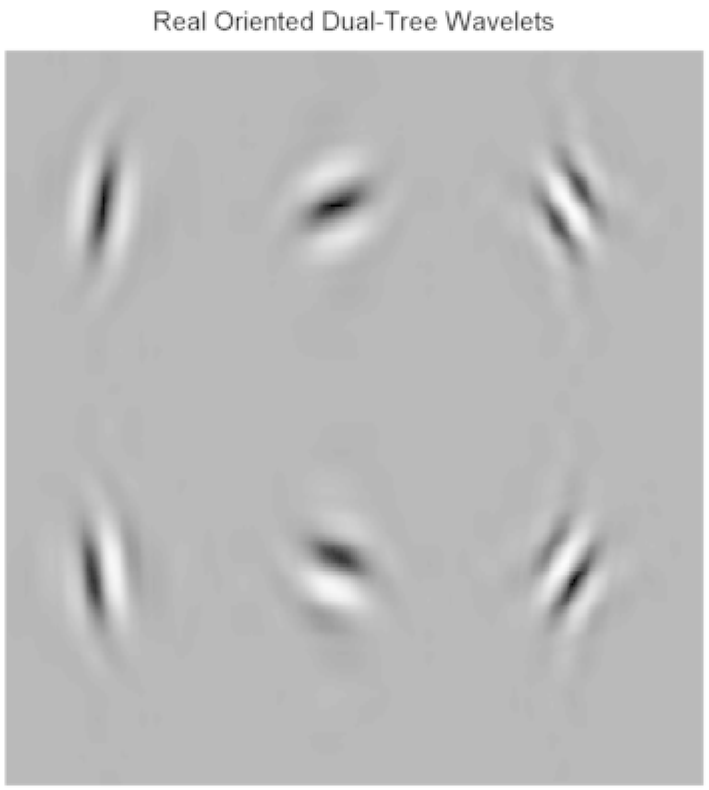
```
J = 4;
L = 3*2^(J+1);
N = L/2^J;
x = zeros(2*L,3*L);
wt = dddtree2('realdt',x,J,Faf,af);
```

Insert a 1 in one position of the six subbands and invert the wavelet transform.

```
wt.cfs{4}(N/2,N/2+0*N,1,1) = 1;
wt.cfs{4}(N/2,N/2+1*N,2,1) = 1;
wt.cfs{4}(N/2,N/2+2*N,3,1) = 1;
wt.cfs{4}(N/2+N,N/2+0*N,1,2) = 1;
wt.cfs{4}(N/2+N,N/2+1*N,2,2) = 1;
wt.cfs{4}(N/2+N,N/2+2*N,3,2) = 1;
xrec = idddtree2(wt);
```

Visualize the six directional wavelets.

```
imagesc(xrec);
colormap gray; axis off;
title('Real Oriented Dual-Tree Wavelets')
```

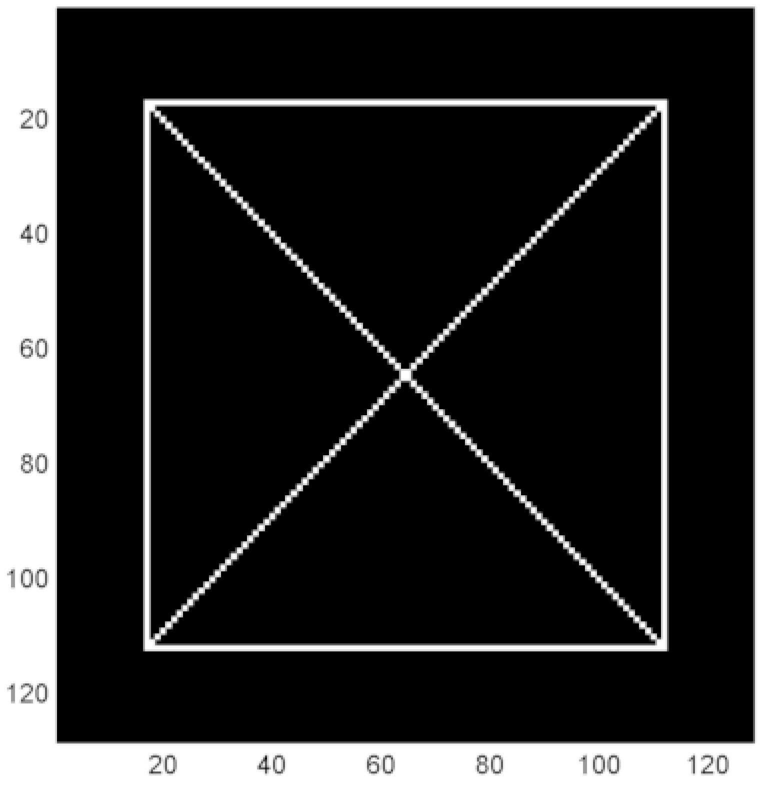


### Double-Density Wavelet Transform

Obtain the double-density wavelet transform of an image.

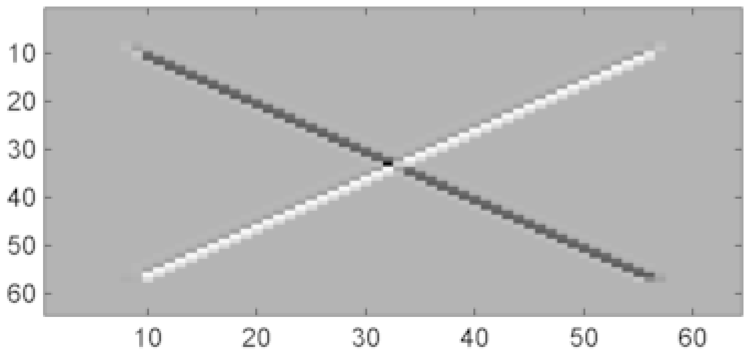
Load the image and obtain the double-density wavelet transform.

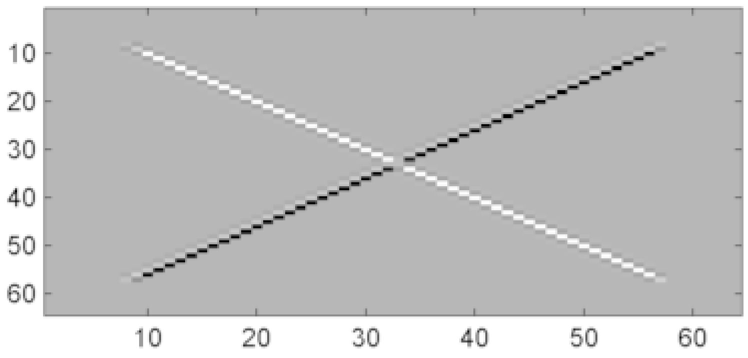
```
load xbox;
imagesc(xbox); colormap gray;
wt = dddtree2('ddt',xbox,1,'filters1');
```



Visualize the diagonal details in the two wavelet HH subbands.

```
HH1 = wt.cfs{1}(:,:,5);
HH2 = wt.cfs{1}(:,:,8);
subplot(211)
imagesc(HH1);
colormap gray;
subplot(212);
imagesc(HH2);
```



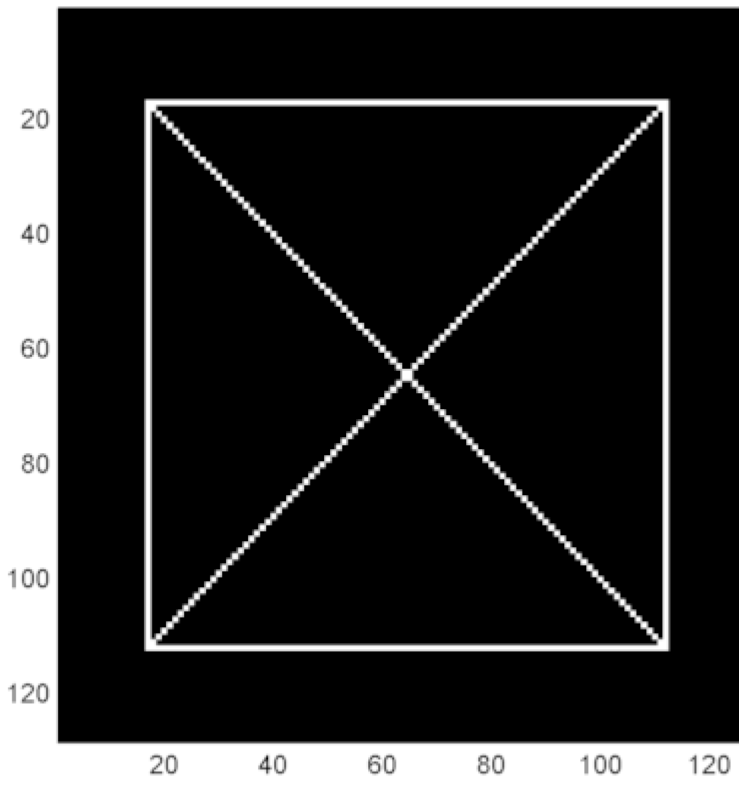


### Complex Dual-Tree Wavelet Transform

Obtain the complex dual-tree wavelet transform of an image. Show that the complex dual-tree wavelet transform can detect the two different diagonal directions.

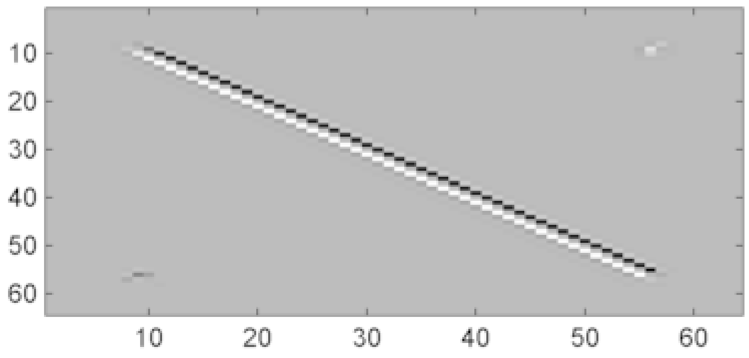
Load the image and obtain the complex dual-tree wavelet transform.

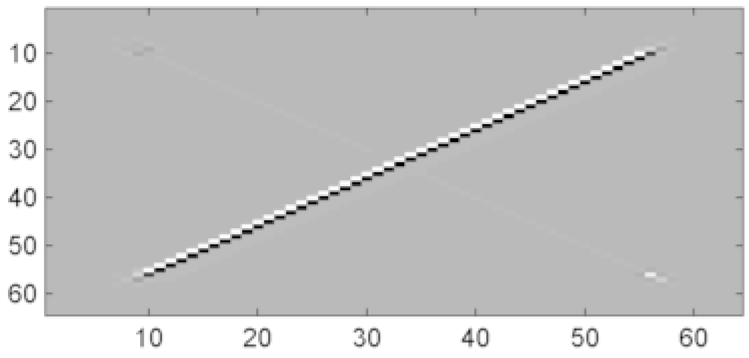
```
load xbox;
imagesc(xbox); colormap gray;
wt = dddtree2('cplxdt',xbox,1,'FSfarras','qshift10');
```



Obtain and display the imaginary parts of the 2 trees.

```
waveletcfs = wt.cfs{1};
subplot(211)
imagesc(waveletcfs(:,:,3,1,2));
colormap gray;
subplot(212)
imagesc(waveletcfs(:,:,3,2,2));
```





- "Analytic Wavelets Using the Dual-Tree Wavelet Transform"

## Input Arguments

### `typetree` — Type of wavelet decomposition

`'dwt'` | `'ddt'` | `'realdt'` | `'cplxdt'` | `'realdddt'` | `'cplxdddt'`

Type of wavelet decomposition, specified as one of `'dwt'`, `'ddt'`, `'realdt'`, `'cplxdt'`, `'realdddt'`, or `'cplxdddt'`. The type, `'dwt'`, produces a critically sampled (nonredundant) discrete wavelet transform. The other decomposition types produce oversampled wavelet transforms. `'ddt'` produces a double-density wavelet transform with one scaling and two wavelet filters for both row and column filtering. The double-density wavelet transform uses the same filters at all stages. `'realdt'` and `'cplxdt'` produce oriented dual-tree wavelet transforms consisting of two and four separable wavelet transforms. `'realdddt'` and `'cplxdddt'` produce double-density dual-tree wavelet transforms. The dual-tree wavelet transforms use different filters for the first stage (level).

### `x` — Input image

matrix

Input image, specified as a matrix with even-length row and column dimensions. Both the row and column dimensions must be divisible by $2^L$, where $L$ is the level of the wavelet transform. Additionally, the minimum of the row and column dimensions of the image must be greater than or equal to the product of the maximum length of the decomposition (analysis) filters and $2^{(L-1).}$

Data Types: `double`

### `level` — Level of wavelet decomposition

integer

Level of the wavelet decomposition, specified as a positive integer. If $L$ is the value of level, $2^L$ must divide both the row and column dimensions of x. Additionally, the minimum of the row and column dimensions of the image must be greater than or equal to the product of the maximum length of the decomposition (analysis) filters and $2^{(L-1)}$.

### `fdf` — Level-one analysis filters

matrix | cell array

The level-one analysis filters, specified as a matrix or cell array of matrices. Specify fdf as a matrix when typetree is `'dwt'` or `'ddt'`. The size and structure of the matrix depend on the typetree input as follows:

- `'dwt'` — This is the critically sampled discrete wavelet transform. In this case, fdf is a two-column matrix with the lowpass (scaling) filter in the first column and the highpass (wavelet) filter in the second column.
- `'ddt'` — This is the double-density wavelet transform. The double-density DWT is a three-channel perfect reconstruction filter bank. fdf is a three-column matrix with the lowpass (scaling) filter in the first column and the two highpass (wavelet) filters in the second and third columns. In the double-density wavelet transform, the single lowpass and two highpass filters constitute a three-channel perfect reconstruction filter bank. This is equivalent to the three filters forming a tight frame. You cannot arbitrarily choose the two wavelet filters in the double-density DWT. The three filters together must form a tight frame.

Specify fdf as a 1-by-2 cell array of matrices when typetree is a dual-tree transform, `'realdt'`, `'cplxdt'`, `'realdddt'`, or `'cplxdddt'`. The size and structure of the matrix elements in the cell array depend on the typetree input as follows:

- For the dual-tree complex wavelet transforms, `'realdt'` and `'cplxdt'`, `fdf{1}` is an *N*-by-2 matrix containing the lowpass (scaling) and highpass (wavelet) filters for the first tree and `fdf{2}` is an *N*-by-2 matrix containing the lowpass (scaling) and highpass (wavelet) filters for the second tree.
- For the double-density dual-tree complex wavelet transforms, `'realdddt'` and `'cplxdddt'`, `fdf{1}` is an *N*-by-3 matrix containing the lowpass (scaling) and two highpass (wavelet) filters for the first tree and `fdf{2}` is an *N*-by-3 matrix containing the lowpass (scaling) and two highpass (wavelet) filters for the second tree.

### `df` — Analysis filters for levels > 1

matrix | cell array

Analysis filters for levels > 1, specified as a matrix or cell array of matrices. Specify df as a matrix when typetree is `'dwt'` or `'ddt'`. The size and structure of the matrix depend on the typetree input as follows:

- `'dwt'` — This is the critically sampled discrete wavelet transform. In this case, df is a two-column matrix with the lowpass (scaling) filter in the first column and the highpass (wavelet) filter in the second column. For the critically sampled orthogonal or biorthogonal DWT, the filters in df and fdf must be identical.
- `'ddt'` — This is the double-density wavelet transform. The double-density DWT is a three-channel perfect reconstruction filter bank. df is a three-column matrix with the lowpass (scaling) filter in the first column and the two highpass (wavelet) filters in the second and third columns. In the double-density wavelet transform, the single lowpass and two highpass filters constitute a three-channel perfect reconstruction filter bank. This is equivalent to the three filters forming a tight frame. For the double-density DWT, the filters in df and fdf must be identical.

Specify df as a 1-by-2 cell array of matrices when typetree is a dual-tree transform, `'realdt'`, `'cplxdt'`, `'realdddt'`, or `'cplxdddt'`. For dual-tree transforms, the filters in fdf and df must be different. The size and structure of the matrix elements in the cell array depend on the typetree input as follows:

- For the dual-tree wavelet transforms, `'realdt'` and `'cplxdt'`, `df{1}` is an *N*-by-2 matrix containing the lowpass (scaling) and highpass (wavelet) filters for the first tree and `df{2}` is an *N*-by-2 matrix containing the lowpass (scaling) and highpass (wavelet) filters for the second tree.

- For the double-density dual-tree complex wavelet transforms, `'realdddt'` and `'cplxdddt'`, `df{1}` is an *N*-by-3 matrix containing the lowpass (scaling) and two highpass (wavelet) filters for the first tree and `df{2}` is an *N*-by-3 matrix containing the lowpass (scaling) and two highpass (wavelet) filters for the second tree.

#### `fname` — Filter name

string

Filter name, specified as a string. For the critically sampled DWT, specify any valid orthogonal or biorthogonal wavelet filter. See `wfilters` for details. For the redundant wavelet transforms, see `dtfilters` for valid filter names.

#### `fname1` — First-stage filter name

string

First-stage filter name, specified as a string. Specifying a first-level filter that is different from the wavelet and scaling filters in subsequent levels is valid and necessary only with the dual-tree wavelet transforms, `'realdt'`, `'cplxdt'`, `'realdddt'`, and `'cplxdddt'`.

#### `fname2` — Filter name for stages > 1

string

Filter name for stages > 1, specified as a string. Specifying a different filter for stages > 1 is valid and necessary only with the dual-tree wavelet transforms, `'realdt'`, `'cplxdt'`, `'realdddt'`, and `'cplxdddt'`.

## Output Arguments

#### `wt` — Wavelet transform

structure

Wavelet transform, returned as a structure with these fields:

#### `type` — Type of wavelet decomposition (filter bank)

`'dwt'` | `'ddt'` | `'realdt'` | `'cplxdt'` | `'realdddt'` | `'cplxdddt'`

Type of wavelet decomposition used in the analysis returned as one of `'dwt'`, `'ddt'`, `'realdt'`, `'cplxdt'`, `'realdddt'`, or `'cplxdddt'`. `'dwt'` is the critically sampled DWT. `'ddt'` produces a double-density wavelet transform with one scaling and two wavelet filters for both row and column filtering. `'realdt'` and `'cplxdt'` produce oriented dual-tree wavelet transforms consisting of 2 and 4 separable wavelet transforms. `'realdddt'` and `'cplxdddt'` produce double-density dual-tree wavelet transforms consisting of two and four separable wavelet transforms.

**`level` — Level of wavelet decomposition**
positive integer

Level of wavelet decomposition, returned as a positive integer.

**`filters` — Decomposition (analysis) and reconstruction (synthesis) filters**
structure

Decomposition (analysis) and reconstruction (synthesis) filters, returned as a structure with these fields:

**`Fdf` — First-stage analysis filters**
matrix | cell array

First-stage analysis filters, returned as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage analysis filters for the corresponding tree.

**`Df` — Analysis filters for levels > 1**
matrix | cell array

Analysis filters for levels > 1, returned as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the analysis filters for the corresponding tree.

**`Frf` — First-level reconstruction filters**
matrix | cell array

First-level reconstruction filters, returned as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage synthesis filters for the corresponding tree.

### `Rf` — Reconstruction filters for levels > 1

matrix | cell array

Reconstruction filters for levels > 1, returned as an N-by-2 or N-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two N-by-2 or N-by-3 matrices for dual-tree wavelet transforms. The matrices are N-by-3 for the double-density wavelet transforms. For an N-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an N-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage analysis filters for the corresponding tree.

### `cfs` — Wavelet transform coefficients

cell array of matrices

Wavelet transform coefficients, specified as a 1-by-(level+1) cell array of matrices. The size and structure of the matrix elements of the cell array depend on the type of wavelet transform, typetree as follows:

- `'dwt'` — `cfs{j}(:,:,d)`
  - j = 1,2,... level is the level.
  - d = 1,2,3 is the orientation.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'ddt'` — `cfs{j}(:,:,d)`
  - j = 1,2,... level is the level.
  - d = 1,2,3,4,5,6,7,8 is the orientation.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'realddt'` — `cfs{j}(:,:,d,k)`

  - j = 1,2,... level is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'cplxdt'` — `cfs{j}(:,:,d,k,m)`
  - j = 1,2,... level is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - m = 1,2 are the real and imaginary parts.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'realdddt'` — `cfs{j}(:,:,d,k)`
  - j = 1,2,... level is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'cplxdddt'` — `cfs{j}(:,:,d,k,m)`
  - j = 1,2,... level is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - m = 1,2 are the real and imaginary parts.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.

## More About

- “Critically Sampled and Oversampled Wavelet Filter Banks”

## See Also

`dddtree` | `dddtreecfs` | `dtfilters` | `idddtree2`

# depo2ind

Node depth-position to node index

## Syntax

## Description

`depo2ind` is a tree-management utility.

For a tree of order `ORD`, `N = depo2ind(ORD,[D P])` computes the indices `N` of the nodes whose depths and positions are encoded within `[D,P]`.

The nodes are numbered from left to right and from top to bottom. The root index is 0.

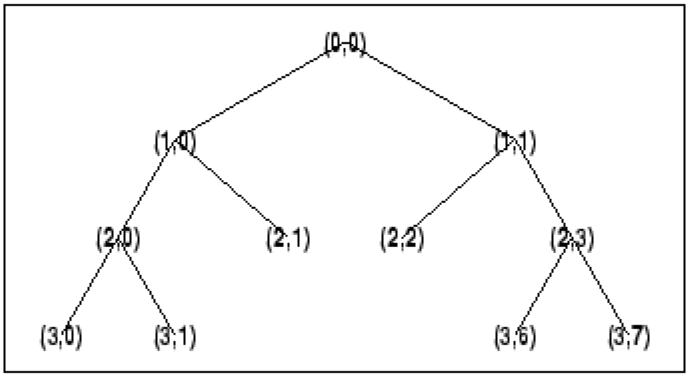
$D$ and $P$ are column vectors. The values of depths $D$ and positions $P$ must be such that $D \geq 0$ and $0 \leq P \leq ORD^{D-1}$.

Output indices $N$ are such that $0 \leq N < (ORD^{max(D)}-1)/ORD-1$.

Note that for a column vector `X`, we have `depo2ind(0,X) = X`.

## Examples

```
% Create initial tree.
ord = 2;
t = ntree(ord,3);     % binary tree of depth 3.
t = nodejoin(t,5);
t = nodejoin(t,4);
plot(t)
```



```
% List t nodes (Depth_Position).
aln_depo = allnodes(t,'deppos')
aln_depo =
    0     0
    1     0
    1     1
    2     0
    2     1
    2     2
    2     3
    3     0
    3     1
    3     6
    3     7

% Switch from Depth_Position to index.
aln_ind = depo2ind(ord,aln_depo)
aln_ind =
    0
    1
    2
    3
    4
    5
    6
    7
    8
   13
   14
```

## See Also

`ind2depo`

## detcoef

1-D detail coefficients

### Syntax

```
D = detcoef(C,L,N)
D = detcoef(C,L)
```

### Description

`detcoef` is a one-dimensional wavelet analysis function.

`D = detcoef(C,L,N)` extracts the detail coefficients at level *N* from the wavelet decomposition structure `[C,L]`. See `wavedec` for more information on *C* and *L*.

Level `N` must be an integer such that $1 \le N \le NMAX$ where `NMAX` = `length(L)-2`.

`D = detcoef(C,L)` extracts the detail coefficients at last level `NMAX`.

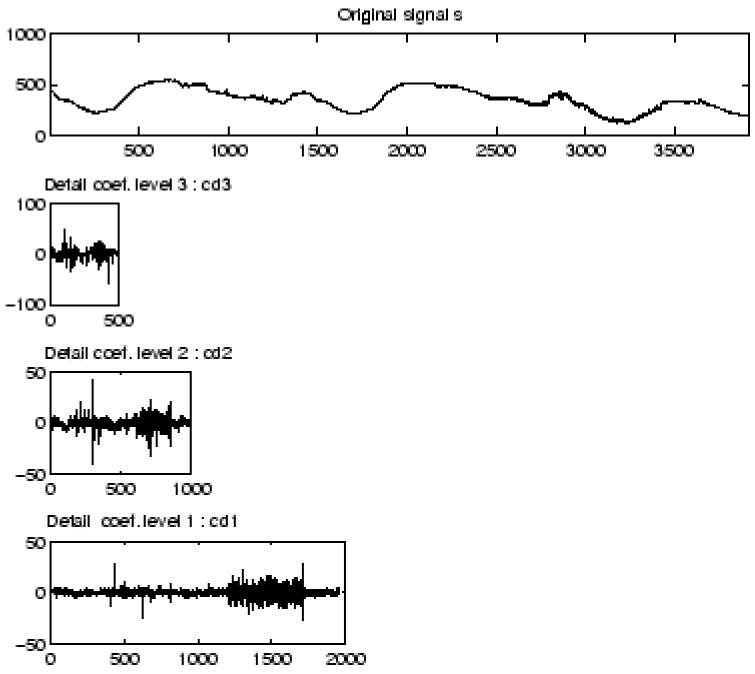
If *N* is a vector of integers such that 1 ≤ `N(j)` ≤ `NMAX`:

- `DCELL = detcoef(C,L,N,'cells')` returns a cell array where `DCELL{j}` contains the coefficients of detail `N(j)`.
- If `length(N) > 1`, `DCELL = detcoef(C,L,N)` is equivalent to `DCELL = detcoef(C,L,N,'cells')`.
- `DCELL = detcoef(C,L,'cells')` is equivalent to `DCELL = detcoef(C,L,[1:NMAX])`.
- `[D1, ... ,Dp] = detcoef(C,L,[N(1), ... ,N(p)])` extracts the details coefficients at levels `[N(1), ... ,N(p)]`.

### Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load original one-dimensional signal.
```

```
load leleccum;
s = leleccum(1:3920);

% Perform decomposition at level 3 of s using db1.
[c,l] = wavedec(s,3,'db1');

% Extract detail coefficients at levels
% 1, 2 and 3, from wavelet decomposition
% structure [c,l].
[cd1,cd2,cd3] = detcoef(c,l,[1 2 3]);

% Using some plotting commands,
% the following figure is generated.
```



## See Also

appcoef | wavedec

## detcoef2

2-D detail coefficients

### Syntax

```
D = detcoef2(O,C,S,N)
```

### Description

`detcoef2` is a two-dimensional wavelet analysis function.

`D = detcoef2(O,C,S,N)` extracts from the wavelet decomposition structure `[C,S]` the horizontal, vertical, or diagonal detail coefficients for `O = 'h'`(or `'v'` or `'d'`, respectively), at level *N*, where *N* must be an integer such that `1 ≤ N ≤ size(S,1)-2`. See `wavedec2` for more information on `C` and `S`.

`[H,V,D] = detcoef2('all',C,S,N)` returns the horizontal `H`, vertical `V`, and diagonal `D` detail coefficients at level `N`.

`D = detcoef2('compact',C,S,N)` returns the detail coefficients at level `N`, stored row-wise.

`detcoef2('a',C,S,N)` is equivalent to `detcoef2('all',C,S,N)`.

`detcoef2('c',C,S,N)` is equivalent to `detcoef2('compact',C,S,N)`.

### Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load original image.
load woman;

% X contains the loaded image.
% Perform decomposition at level 2
% of X using db1.
[c,s] = wavedec2(X,2,'db1');
sizex = size(X)
sizex =
    256    256

sizec = size(c)
sizec =
    1    65536

val_s = s
val_s =
     64     64
     64     64
    128    128
    256    256

% Extract details coefficients at level 2
% in each orientation, from wavelet decomposition
% structure [c,s].
[chd2,cvd2,cdd2] = detcoef2('all',c,s,2);
sizecd2 = size(chd2)
sizecd2 =
     64     64

% Extract details coefficients at level 1
% in each orientation, from wavelet decomposition
% structure [c,s].
[chd1,cvd1,cdd1] = detcoef2('all',c,s,1);
sizecd1 = size(chd1)
sizecd1 =
    128    128
```

## More About

### Tips

If `C` and `S` are obtained from an indexed image analysis or a truecolor image analysis, `D` is an `m`-by-`n` matrix or an `m`-by-`n`-by-3 array, respectively.

For more information on image formats, see the `image` and `imfinfo` reference pages.

## See Also

`appcoef2` | `wavedec2`

# disp

WPTREE information

## Syntax

```
disp(T)
```

## Description

`disp(T)` displays the content of the WPTREE object *T*.

## Examples

```
% Compute a wavelet packets tree
x = rand(1,1000);
t = wpdec(x,2,'db2');
disp(t)

 Wavelet Packet Object Structure
=================================
 Size of initial data       : [1 1000]
 Order                      : 2
 Depth                      : 2
 Terminal nodes             : [3  4  5  6]
--------------------------------------------------
 Wavelet Name               : db2
 Low Decomposition filter   : [-0.1294  0.2241  0.8365  0.483]
 High Decomposition filter  : [ -0.483  0.8365 -0.2241 -0.1294]
 Low Reconstruction filter  : [  0.483  0.8365  0.2241 -0.1294]
 High Reconstruction filter : [-0.1294 -0.2241  0.8365 -0.483]
--------------------------------------------------
 Entropy Name               : shannon
 Entropy Parameter          : 0
--------------------------------------------------
```

## See Also

`get` | `read` | `set` | `write`

# displs

Display lifting scheme

## Syntax

```
S = displs(LS,FRM)
```

## Description

`S = displs(`*`LS`*`,`*`FRM`*`)` returns a string describing the lifting scheme *`LS`*. The format string *`FRM`* (see `sprintf`) builds S.

`displs(LS)` is equivalent to DISPLS(LS,'%12.8f')

For more information about lifting schemes, see `lsinfo`.

## Examples

```
% Start from the Haar wavelet and get the
% corresponding lifting scheme.
lshaar = liftwave('haar');

% Visualize the obtained lifting scheme.
displs(lshaar);

lshaar = {...
'd'             [ -1.00000000]  [0]
'p'             [  0.50000000]  [0]
[  1.41421356]  [  0.70710678]  []
};

% Add a primal ELS to the lifting scheme.
els = {'p',[-0.125 0.125],0};
lsnew = addlift(lshaar,els);
displs(lsnew);

lsnew = {...
'd'             [ -1.00000000]              [0]
'p'             [  0.50000000]              [0]
'p'             [ -0.12500000  0.12500000]  [0]
[  1.41421356]  [  0.70710678]              []
};
```

## See Also

`lsinfo`

## drawtree

Draw wavelet packet decomposition tree (GUI)

### Syntax

```
drawtree(T)
F = drawtree(T)
drawtree(T,F)
```

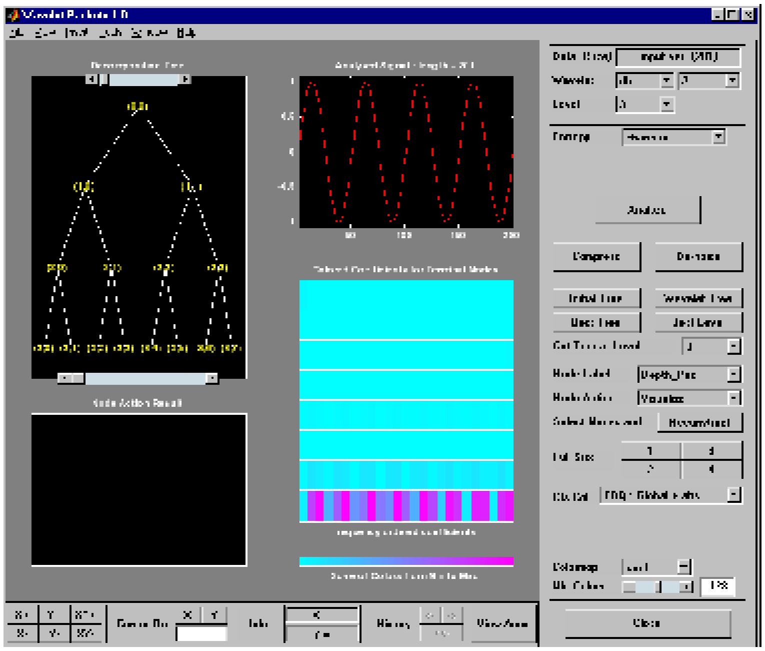
### Description

`drawtree(T)` draws the wavelet packet tree *T*, and `F = drawtree(T)` also returns the figure's handle.

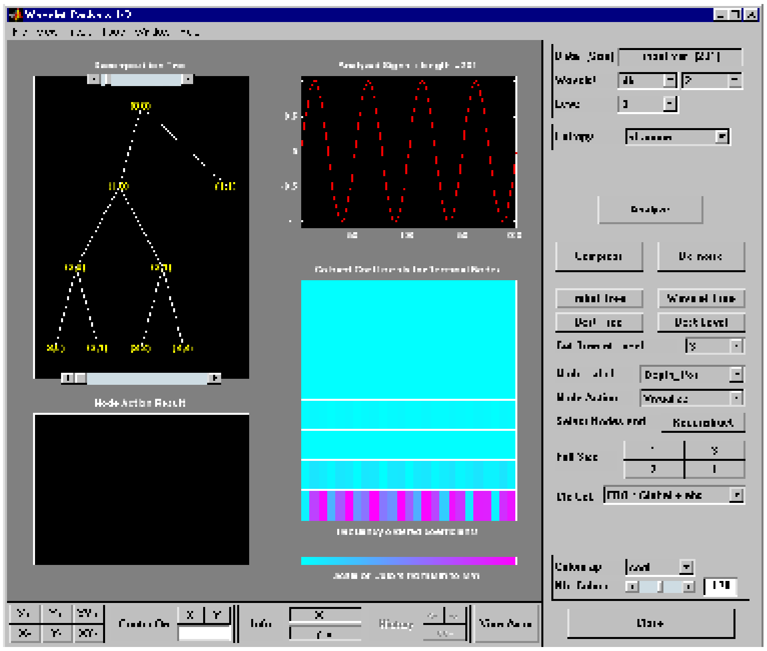
For an existing figure F produced by a previous call to the drawtree function, `drawtree(T,F)` draws the wavelet packet tree T in the figure whose handle is F.

### Examples

```
x   = sin(8*pi*[0:0.005:1]);
t   = wpdec(x,3,'db2');
fig = drawtree(t);
```



```
%----------------------------------------
% Use command line function to modify t.
%----------------------------------------
t   = wpjoin(t,2);
drawtree(t,fig);
```



## See Also

`readtree`

# dtfilters

Analysis and synthesis filters for oversampled wavelet filter banks

## Syntax

```
df = dtfilters(name)
[df,rf] = dtfilters(name)
```

## Description

`df = dtfilters(name)` returns the decomposition (analysis) filters corresponding to the string, name.

`[df,rf] = dtfilters(name)` returns the reconstruction (synthesis) filters corresponding to the string, name.

## Examples

### Filters for Complex Dual-Tree Wavelet Transform

Obtain valid filters for the complex dual-tree wavelet transform. The transform uses Farras nearly symmetric filters for the first stage and Kingsbury Q-shift filters with 10 taps for subsequent stages.

Load the noisy Doppler signal. Obtain the filters for the first and subsequent stages of the complex dual-tree wavelet transform. Demonstrate perfect reconstruction using the complex dual-tree wavelet transform.

```
load noisdopp;
df = dtfilters('dtf2');
dt = dddtree('cplxdt',noisdopp,5,df{1},df{2});
xrec = idddtree(dt);
max(abs(noisdopp-xrec))
```

### Filters for Double-Density Wavelet Transform

Obtain valid filters for the double-density wavelet transform.

Load the noisy Doppler signal. Obtain the filters for the double-density wavelet transform. The double-density wavelet transform uses the same filters at all stages. Demonstrate perfect reconstruction using the double-density wavelet transform.

```
df = dtfilters('filters1');
load noisdopp;
dt = dddtree('ddt',noisdopp,5,df,df);
xrec = idddtree(dt);
max(abs(noisdopp-xrec))
```

## Input Arguments

### `name` — Filter name

`'dtf1'` | `'dddtf1'` | `'self1'` | `'self2'` | ...

Filter name, specified as a string. Valid entries for name are:

- Any valid orthogonal or biorthogonal wavelet name. See `wfilters` for details. An orthogonal or biorthogonal wavelet is only valid when the filter bank type is `'dwt'`, or when you use the filter as the first stage in a complex dual-tree transform, `'realdt'` or `'cplxdt'`. An orthogonal or biorthogonal wavelet filter is not a valid filter if you have a double-density, `'ddt'` or dual-tree double-density, `'realdddt'` or `'cplxdddt'`, filter bank. An orthogonal or biorthogonal wavelet filter is not a valid filter for complex dual-tree filter banks for stages greater than 1.
- `'dtfP'` — With P equal to 1, 2, 3, or 4 returns the first-stage Farras filters (`'FSfarras'`) and Kingsbury Q-shift filters (`'qshiftN'` for subsequent stages. This input is only valid for a dual-tree transform, `'realdt'` or `'cplxdt'`. Setting P= 1, 2, 3, or 4 specifies the Kingsbury Q-shift filters with N = 6, 10, 14, or 18 taps respectively.
- `'dddtf1'` — Returns the filters for the first and subsequent stages of the double-density dual-tree transform. This input is only valid for the double-density dual-tree transforms, `'realdddt'` and `'cplxdddt'`.
- `'self1'` — Returns 10-tap filters for the double-density wavelet transform. This option is only valid for double-density wavelet transforms, `'ddt'`, `'realdddt'`, and `'cplxdddt'`.
- `'self2'` — Returns 16-tap filters for the double-density wavelet transform. This option is only valid for double-density wavelet transforms, `'ddt'`, `'realdddt'`, and `'cplxdddt'`.

- `'filters1'` — Returns 6-tap filters for the double-density wavelet transform, `'ddt'`.
- `'filters2'` — Returns 12-tap filters for the double-density wavelet transform, `'ddt'`.
- `'farras'` — Farras nearly symmetric filters for a two-channel perfect reconstruction filter bank. This option is only valid for an orthogonal critically sampled wavelet transform, `'dwt'`.
- he `'FSfarras'` — Farras nearly symmetric first-stage filters for a dual-tree wavelet transform.
- `'qshiftN'` — Kingsbury Q-shift N-tap filters with N = 6,10,14, or 18. The Kingsbury Q-shift filters are most commonly used in dual-tree wavelet transforms for stages greater than 1.
- `'doubledualfilt'` — Filters for one stage of the double-density dual-tree wavelet transforms, `'realdddt'` or `'cplxdddt'`.

## Output Arguments

### `df` — Decomposition (analysis) filters

matrix | cell array

Decomposition (analysis) filters, returned as a matrix or cell array of matrices.

### `rf` — Reconstruction (synthesis) filters

matrix | cell array

Reconstruction (synthesis) filters, returned as a matrix or cell array of matrices.

## See Also

`dddtree` | `dddtree2`

# dtree

DTREE constructor

## Syntax

```
T = dtree(ORD,D,X)
T = dtree(ORD,D,X,U)
[T,NB] = dtree(...)
[T,NB] = dtree('PropName1',PropValue1,'PropName2',PropValue2,...)
```

## Description

`T = dtree(ORD,D,X)` returns a complete data tree (`DTREE`) object of order *ORD* and depth *D*. The data associated with the tree *T* is *X*.

With `T = dtree(ORD,D,X,U)` you can set a user data field.

`[T,NB] = dtree(...)` returns also the number of terminal nodes (leaves) of *T*.

`[T,NB] = dtree('PropName1',PropValue1,'PropName2',PropValue2,...)` is the most general syntax to construct a `DTREE` object.

The valid choices for `'PropName'` are

| | |
|---|---|
| `'order'` | Order of the tree |
| `'depth'` | Depth of the tree |
| `'data'` | Data associated to the tree |
| `'spsch'` | Split scheme for nodes |
| `'ud'` | User data field |

The split scheme field is an order `ORD` by 1 logical array. The root of the tree can be split and it has `ORD` children. If `spsch(j) = 1`, you can split the `j`-th child. Each node that you can split has the same property as the root node.

For more information on object fields, type `help dtree/get`.

Class DTREE (Parent class: NTREE)

## Fields

| | |
|---|---|
| dtree | Parent object |
| allNI | All nodes information |
| terNI | Terminal nodes information |

## Examples

```
% Create a data tree.
x = [1:10];
t = dtree(3,2,x);
t = nodejoin(t,2);
```

### See Also

ntree | wtbo

# dwt

Single-level discrete 1-D wavelet transform

## Syntax

```
[cA,cD] = dwt(X,'wname')
[cA,cD] = dwt(X,Lo_D,Hi_D)
[cA,cD] = dwt(...,'mode',MODE)
```

## Description

The `dwt` command performs a single-level one-dimensional wavelet decomposition with respect to either a particular wavelet (`'wname'`, see `wfilters` for more information) or particular wavelet decomposition filters (`Lo_D` and `Hi_D`) that you specify.

`[cA,cD] = dwt(X,'wname')` computes the approximation coefficients vector `cA` and detail coefficients vector `cD`, obtained by a wavelet decomposition of the vector `X`. The string `'wname'` contains the wavelet name.

`[cA,cD] = dwt(X,Lo_D,Hi_D)` computes the wavelet decomposition as above, given these filters as input:

- `Lo_D` is the decomposition low-pass filter.
- `Hi_D` is the decomposition high-pass filter.

`Lo_D` and `Hi_D` must be the same length.

Let `lx` = the length of `X` and `lf` = the length of the filters `Lo_D` and `Hi_D`; then `length(cA) = length(cD) = la` where `la = ceil(lx/2)`, if the DWT extension mode is set to periodization. For the other extension modes, `la = floor(lx+lf-1)/2`.

For more information about the different Discrete Wavelet Transform extension modes, see `dwtmode`.

`[cA,cD] = dwt(...,'mode',MODE)` computes the wavelet decomposition with the extension mode `MODE` that you specify. `MODE` is a string containing the desired extension mode.

Example:

```
[cA,cD] = dwt(x,'db1','mode','sym');
```

## Examples

### DWT Using Wavelet Name

Obtain the level-1 DWT of the noisy Doppler signal using a wavelet name.

```
load noisdopp;
[A,D] = dwt(noisdopp,'sym4');
```

### DWT Using Wavelet and Scaling Filters

Obtain the level-1 DWT of the noisy Doppler signal using wavelet and scaling filters.

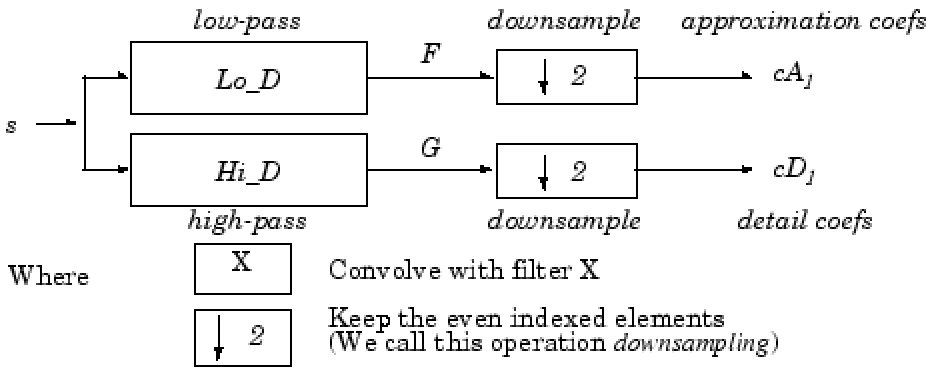
```
load noisdopp;
[Lo_D,Hi_D] = wfilters('bior3.5','d');
[A,D] = dwt(noisdopp,Lo_D,Hi_D);
```

## More About

### Algorithms

Starting from a signal *s* of length *N*, two sets of coefficients are computed: approximation coefficients $CA_1$, and detail coefficients $CD_1$. These vectors are obtained by convolving *s* with the low-pass filter Lo_D for approximation and with the high-pass filter Hi_D for detail, followed by dyadic decimation.

More precisely, the first step is



The length of each filter is equal to $2L$. For signal of length $N$, the signals $F$ and $G$ are of length $N + 2L - 1$, and then the coefficients $CA_1$ and $CD_1$ are of length

$$\left\lfloor \frac{N-1}{2} + L \right\rfloor.$$

To deal with signal-end effects involved by a convolution-based algorithm, a global variable managed by `dwtmode` is used. This variable defines the kind of signal extension mode used. The possible options include zero-padding (used in the previous example) and symmetric extension, which is the default mode.

**Note** For the same input, this `dwt` function and the DWT block in the Signal Processing Toolbox™ do not produce the same results. The blockset is designed for real-time implementation while Wavelet Toolbox™ software is designed for analysis, so they produce handle boundary conditions and filter states differently.

To make the `dwt` function output match the DWT block output, set the function boundary condition to zero-padding by typing `dwtmode('zpd')` at the MATLAB® command prompt. To match the latency of the DWT block, which is implemented using FIR filters, add zeros to the input of the `dwt` function. The number of zeros you add must be equal to half the filter length.

## References

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics. SIAM Ed.

Mallat, S. (1989), “A theory for multiresolution signal decomposition: the wavelet representation,” *IEEE Pattern Anal. and Machine Intell.*, vol. 11, no. 7, pp. 674–693.

Meyer, Y. (1990), *Ondelettes et opérateurs*, Tome 1, Hermann Ed. (English translation: *Wavelets and operators*, Cambridge Univ. Press. 1993.)

### See Also

`dwtmode` | `idwt` | `wavedec` | `waveinfo`

# dwt2

Single-level discrete 2-D wavelet transform

## Syntax

```
[cA,cH,cV,cD] = dwt2(X,'wname')
[cA,cH,cV,cD] = dwt2(X,Lo_D,Hi_D)
[cA,cH,cV,cD] = dwt2(...,'mode',MODE)
```

## Description

The `dwt2` command performs a single-level two-dimensional wavelet decomposition with respect to either a particular wavelet (`'wname'`, see `wfilters` for more information) or particular wavelet decomposition filters (`Lo_D` and `Hi_D`) you specify.

`[cA,cH,cV,cD] = dwt2(X,'wname')` computes the approximation coefficients matrix `cA` and details coefficients matrices `cH`, `cV`, and `cD` (horizontal, vertical, and diagonal, respectively), obtained by wavelet decomposition of the input matrix `X`. The `'wname'` string contains the wavelet name.

`[cA,cH,cV,cD] = dwt2(X,Lo_D,Hi_D)` computes the two-dimensional wavelet decomposition as above, based on wavelet decomposition filters that you specify.

- `Lo_D` is the decomposition low-pass filter.
- `Hi_D` is the decomposition high-pass filter.

`Lo_D` and `Hi_D` must be the same length.

Let `sx = size(X)` and `lf =` the length of filters; then
`size(cA) = size(cH) = size(cV) = size(cD) = sa` where `sa = ceil(sx/2)`, if the DWT extension mode is set to periodization. For the other extension modes, `sa = floor((sx+lf-1)/2)`.

For information about the different Discrete Wavelet Transform extension modes, see `dwtmode`.

`[cA,cH,cV,cD] = dwt2(...,'mode',MODE)` computes the wavelet decomposition with the extension mode `MODE` that you specify.

`MODE` is a string containing the desired extension mode.

An example of valid use is

```
[cA,cH,cV,cD] = dwt2(x,'db1','mode','sym');
```

## 2-D DWT of Image

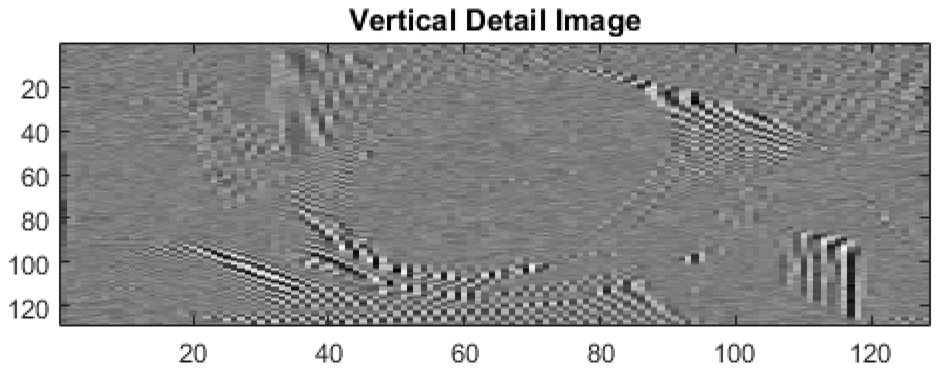
This example shows how to obtain the 2-D DWT of an image.

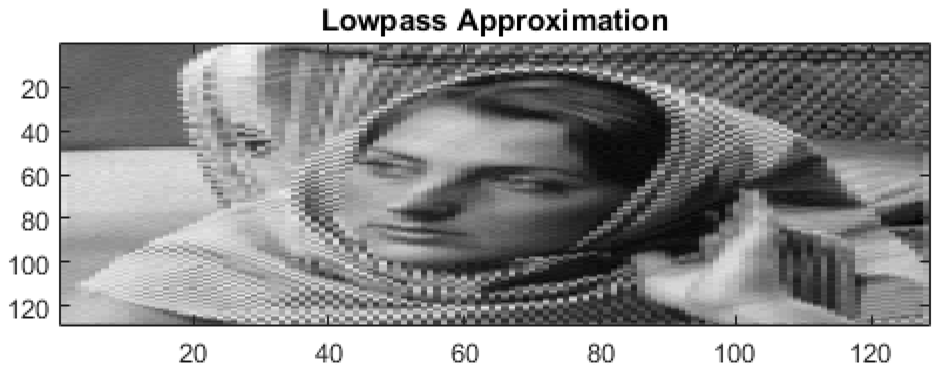
Load the "woman" image and obtain the 2-D DWT using the 'sym4' wavelet. Use the periodic extension mode.

```
load woman;
wname = 'sym4';
[CA,CH,CV,CD] = dwt2(X,wname,'mode','per');
```

Display the vertical detail image and the lowpass approximation.

```
subplot(211)
imagesc(CV); title('Vertical Detail Image');
colormap gray;
subplot(212)
imagesc(CA); title('Lowpass Approximation');
```





## More About

### Tips

When X represents an indexed image, then X, as well as the output arrays cA,cH,cV,cD are `m`-by-`n` matrices. When X represents a truecolor image, it is an `m`-by-`n`-by-3 array, where each `m`-by-`n` matrix represents a red, green, or blue color plane concatenated along the third dimension.

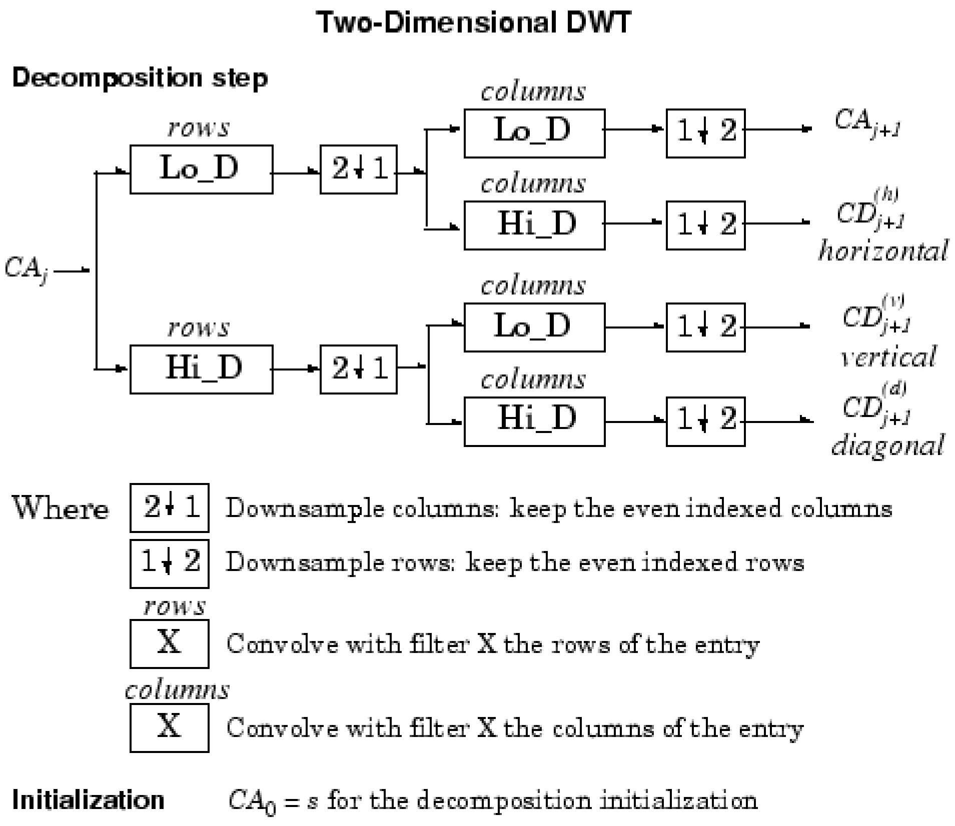
For more information on image formats, see the `image` and `imfinfo` reference pages.

## Algorithms

For images, there exist an algorithm similar to the one-dimensional case for two-dimensional wavelets and scaling functions obtained from one- dimensional ones by tensorial product.

This kind of two-dimensional DWT leads to a decomposition of approximation coefficients at level *j* in four components: the approximation at level *j* + 1, and the details in three orientations (horizontal, vertical, and diagonal).

The following chart describes the basic decomposition steps for images:


Two-Dimensional DWT
Decomposition step
CA_j
rows
Lo_D
2↓1
rows
Hi_D
2↓1
columns
Lo_D
1↓2
CA_{j+1}
columns
Hi_D
1↓2
CD_{j+1}^{(h)} horizontal
columns
Lo_D
1↓2
CD_{j+1}^{(v)} vertical
columns
Hi_D
1↓2
CD_{j+1}^{(d)} diagonal
Where
2↓1 Downsample columns: keep the even indexed columns
1↓2 Downsample rows: keep the even indexed rows
rows X Convolve with filter X the rows of the entry
columns X Convolve with filter X the columns of the entry
Initialization
CA_0 = s for the decomposition initialization

---

**Note** To deal with signal-end effects involved by a convolution-based algorithm, a global variable managed by `dwtmode` is used. This variable defines the kind of signal extension mode used. The possible options include zero-padding (used in the previous example) and symmetric extension, which is the default mode.

## References

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics. SIAM Ed.

Mallat, S. (1989), “A theory for multiresolution signal decomposition: the wavelet representation,” *IEEE Pattern Anal. and Machine Intell.*, vol. 11, no. 7, pp. 674–693.

Meyer, Y. (1990), *Ondelettes et opérateurs*, Tome 1, Hermann Ed. (English translation: *Wavelets and operators*, Cambridge Univ. Press. 1993.)

### See Also

`dwtmode` | `idwt2` | `wavedec2` | `waveinfo`

## dwt3

Single-level discrete 3-D wavelet transform

### Syntax

```
WT = dwt3(X,'wname')
WT = dwt3(X,'wname','mode','ExtM')
WT = dwt3(X,W,...)
WT = dwt3(X,WF,...)
```

### Description

`dwt3` performs a single-level three-dimensional wavelet decomposition using either a particular wavelet (`'wname'`) or the wavelet decomposition and reconstruction filters you specify. The decomposition also uses the specified DWT extension mode (see `dwtmode`).

`WT = dwt3(X,'wname')` returns the 3-D wavelet transform of the 3-D array `X` `'wname'` is a string containing the wavelet name. The default extension mode is `'sym'`. For more information on *`wname`*, see `wfilters`.

`WT = dwt3(X,'wname','mode','ExtM')` uses the extension mode `'ExtM'`.

`WT` is a structure with the following fields shown in the table.

| | |
|---|---|
| `sizeINI` | Size of the three-dimensional array `X`. |
| `mode` | Name of the wavelet transform extension mode. |
| `filters` | Structure with four fields: `LoD`, `HiD`, `LoR`, `HiR`, which are the filters used for DWT. |
| `dec` | 2 x 2 x 2 cell array containing the coefficients of the decomposition.<br><br>`dec{i,j,k}`, `i,j,k = 1` or `2` contains the coefficients obtained by low-pass filtering (for `i` or `j` or `k = 1`) or high-pass filtering (for `i` or `j` or `k = 2`) |

`WT = dwt3(X,W,...)` specify three wavelets, one for each direction. `W = {'wname1','wname2','wname3'}` or `W` is a structure with 3 fields `'w1'`, `'w2'`, `'w3'` containing strings that are the names of wavelets.

`WT = dwt3(X,WF,...)` specify four filters, two for decomposition, and two for reconstructionm or 3 x 4 filters (one quadruplet by direction). `WF` is either a cell array (1 x 4) or (3 x 4) : `{LoD,HiD,LoR,HiR}` or a structure with the four fields `'LoD','HiD','LoR','HiR'`.

## Examples

```
% Define the original 3-D data.
X = reshape(1:64,4,4,4)

X(:,:,1) =

     1     5     9    13
     2     6    10    14
     3     7    11    15
     4     8    12    16

X(:,:,2) =

    17    21    25    29
    18    22    26    30
    19    23    27    31
    20    24    28    32

X(:,:,3) =

    33    37    41    45
    34    38    42    46
    35    39    43    47
    36    40    44    48

X(:,:,4) =

    49    53    57    61
    50    54    58    62
    51    55    59    63
    52    56    60    64
```

```
% Perform single level decomposition of X using db1.
wt = dwt3(X,'db1')

wt =

    sizeINI: [4 4 4]
    filters: [1x1 struct]
       mode: 'sym'
        dec: {2x2x2 cell}

% Decompose X using db2.
[LoD,HiD,LoR,HiR] = wfilters('db2');
wt = dwt3(X,{LoD,HiD,LoR,HiR})

wt =

    sizeINI: [4 4 4]
    filters: [1x1 struct]
       mode: 'sym'
        dec: {2x2x2 cell}


% Decompose X using different wavelets, one for
% each orientation (db1, db2 and again db1).
WS = struct('w1','db1','w2','db2','w3','db1');
wt = dwt3(X,WS,'mode','per')

wt =

    sizeINI: [4 4 4]
    filters: [1x1 struct]
       mode: 'per'
        dec: {2x2x2 cell}

WF = wt.filters;

% Decompose X using the filters given by WF and
% set the extension mode to symmetric.
wtBIS = dwt3(X,WF,'mode','sym')

wtBIS =

    sizeINI: [4 4 4]
    filters: [1x1 struct]
```

```
       mode: 'sym'
        dec: {2x2x2 cell}
```

## See Also

`dwtmode` | `idwt3` | `wavedec3` | `waverec3` | `waveinfo` | `wfilters`

# dwtmode

Discrete wavelet transform extension mode

## Syntax

```
ST = dwtmode
ST = dwtmode('status')
dwtmode('mode')
```

## Description

`dwtmode` sets the signal or image extension mode for discrete wavelet and wavelet packet transforms. The extension modes represent different ways of handling the problem of border distortion in signal and image analysis. For more information, see "Border Effects" , in the User's Guide.

`dwtmode` or `dwtmode('status')` display the current mode.

`ST = dwtmode` or `ST = dwtmode('status')` display and returns in `ST` the current mode.

`ST = dwtmode('status','nodisp')` returns in `ST` the current mode and no text (status or warning) is displayed in the MATLAB Command Window.

`dwtmode('mode')` sets the DWT extension mode according to the value of `'mode'`:

| `'mode'` | DWT Extension Mode |
|---|---|
| `'sym'` or `'symh'` | Symmetric-padding (half-point): boundary value symmetric replication — default mode |
| `'symw'` | Symmetric-padding (whole-point): boundary value symmetric replication |
| `'asym'` or `'asymh'` | Antisymmetric-padding (half-point): boundary value antisymmetric replication |
| `'asymw'` | Antisymmetric-padding (whole-point): boundary value antisymmetric replication |

| '*mode*' | DWT Extension Mode |
|---|---|
| 'zpd' | Zero-padding |
| 'spd' or 'sp1' | Smooth-padding of order 1 (first derivative interpolation at the edges) |
| 'sp0' | Smooth-padding of order 0 (constant extension at the edges) |
| 'ppd' | Periodic-padding (periodic extension at the edges) |

For more information on symmetric extension modes see “References”.

The DWT associated with these five modes is slightly redundant. But, the IDWT ensures a perfect reconstruction for any of the five previous modes whatever is the extension mode used for DWT.

dwtmode('per') sets the DWT mode to periodization.

This mode produces the smallest length wavelet decomposition. But, the extension mode used for IDWT must be the same to ensure a perfect reconstruction.

Using this mode, dwt and dwt2 produce the same results as the obsolete functions dwtper and dwtper2, respectively.

All functions and GUI tools involving the DWT (1-D & 2-D) or Wavelet Packet transform (1-D & 2-D) use the specified DWT extension mode.

dwtmode updates a global variable allowing the use of these six signal extensions. The extension mode should only be changed using this function. Avoid changing the global variable directly.

The default mode is loaded from the file DWTMODE.DEF (in the current path) if it exists. If not, the file DWTMODE.CFG (in the toolbox/wavelet/wavelet folder) is used.

dwtmode('save',MODE) saves MODE as the new default mode in the file DWTMODE.DEF (in the current folder). If a file with the same name already exists in the current folder, it is deleted before saving.

dwtmode('save') is equivalent to dwtmode('save',CURRENTMODE).

In these last two cases, the new default mode saved in the file DWTMODE.DEF will be active as default mode in the next MATLAB session.

## Examples

```
% If the DWT extension mode global variable does not
% exist, default is Symmetrization.
clear global
dwtmode

******************************************
**  DWT Extension Mode: Symmetrization  **
******************************************

% Display current DWT signal extension mode.
dwtmode

******************************************
**  DWT Extension Mode: Symmetrization  **
******************************************
% Change to Periodization extension mode.
dwtmode('per')

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!  WARNING: Change DWT Extension Mode  !
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

*****************************************
**  DWT Extension Mode: Periodization  **
*****************************************

% Display current DWT signal extension mode.
dwtmode

*****************************************
**  DWT Extension Mode: Periodization  **
*****************************************
```

**Note** You should change the extension mode only by using `dwtmode`. Avoid changing the global variable directly.

## References

Strang, G.; T. Nguyen (1996), *Wavelets and filter banks*, Wellesley- Cambridge Press.

## See Also

`idwt` | `idwt2` | `dwt` | `dwt2` | `wextend`

## dyaddown

Dyadic downsampling

### Syntax

```
Y = dyaddown(X,EVENODD)
Y = dyaddown(X)
Y = dyaddown(X,EVENODD,'type')
Y = dyaddown(X,'type',EVENODD)
Y = dyaddown(X)
Y = dyaddown(X,'type')
Y = dyaddown(X,0,'type')
Y = dyaddown(X,EVENODD)
Y = dyaddown(X,EVENODD,'c')
```

### Description

`Y = dyaddown(X,EVENODD)` where *X* is a *vector*, returns a version of *X* that has been downsampled by 2. Whether `Y` contains the even- or odd-indexed samples of *X* depends on the value of positive integer `EVENODD`:

- If `EVENODD` is even, then `Y(k) = X(2k)`.
- If `EVENODD` is odd, then `Y(k) = X(2k+1)`.

`Y = dyaddown(X)` is equivalent to `Y = dyaddown(X,0)` (even-indexed samples).

`Y = dyaddown(X,EVENODD,'type')` or `Y = dyaddown(X,'type',EVENODD)`, where *X* is a *matrix*, returns a version of *X* obtained by suppressing one out of two:

| | |
|---|---|
| Columns of *X* | If `'type'= 'c'` |
| Rows of *X* | If `'type'= 'r'` |
| Rows and columns of *X* | If `'type'= 'm'` |

according to the parameter *`EVENODD`*, which is as above.

If you omit the *EVENODD* or `'type'` arguments, `dyaddown` defaults to `EVENODD = 0` (even-indexed samples) and `'type'= 'c'` (columns).

`Y = dyaddown(X)` is equivalent to `Y = dyaddown(X,0,'c')`.
`Y = dyaddown(X,'type')` is equivalent to `Y = dyaddown(X,0,'type')`.
`Y = dyaddown(X,EVENODD)` is equivalent to `Y = dyaddown(X,EVENODD,'c')`.

## Examples

```
% For a vector.
s = 1:10
s =
   1   2   3   4   5   6   7   8   9  10

dse = dyaddown(s)   % Downsample elements with even indices.
dse =
   2   4   6   8  10
% or equivalently
dse = dyaddown(s,0)
dse =
   2   4   6   8  10

dso = dyaddown(s,1) % Downsample elements with odd indices.
dso =
   1   3   5   7   9

% For a matrix.
s = (1:3)'*(1:4)
s =
   1   2   3   4
   2   4   6   8
   3   6   9  12

dec = dyaddown(s,0,'c') % Downsample columns with even indices.
dec =
   2   4
   4   8
   6  12

der = dyaddown(s,1,'r') % Downsample rows with odd indices.
der =
 1   2   3   4
 3   6   9  12

dem = dyaddown(s,1,'m') % Downsample rows and columns
                        % with odd indices.
dem =
     1     3
     3     9
```

## References

Strang, G.; T. Nguyen (1996), *Wavelets and Filter Banks*, Wellesley-Cambridge Press.

### See Also

`dyadup`

# dyadup

Dyadic upsampling

## Syntax

```
Y = dyadup(X,EVENODD)
Y = dyadup(X)
Y = dyadup(X,EVENODD,'type')
Y = dyadup(X,'type',EVENODD)
Y = dyadup(X)
Y = dyaddown(X,1,'c')
Y = dyadup(X,'type')
Y = dyadup(X,1,'type')
Y = dyadup(X,EVENODD)
Y = dyadup(X,EVENODD,'c')
```

## Description

`dyadup` implements a simple zero-padding scheme very useful in the wavelet reconstruction algorithm.

`Y = dyadup(X,EVENODD)`, where *X* is a *vector*, returns an extended copy of vector *X* obtained by inserting zeros. Whether the zeros are inserted as even- or odd-indexed elements of *Y* depends on the value of positive integer `EVENODD`:

- If *EVENODD* is even, then `Y(2k–1) = X(k), Y(2k) = 0`.
- If *EVENODD* is odd, then `Y(2k–1) = 0, Y(2k) = X(k)`.

`Y = dyadup(X)` is equivalent to `Y = dyadup(X,1)` (odd-indexed samples).

`Y = dyadup(X,EVENODD,'type')` or `Y = dyadup(X,'type',EVENODD)`, where *X* is a *matrix*, returns extended copies of *X* obtained by inserting

| | |
|---|---|
| Columns in *X* | If `'type'= 'c'` |
| Rows in *X* | If `'type'= 'r'` |

| | |
|---|---|
| Rows and columns in *X* | If '*type*'= 'm' |

according to the parameter *EVENODD*, which is as above.

If you omit the *EVENODD* or '*type*' arguments, `dyadup` defaults to `EVENODD = 1` (zeros in odd-indexed positions) and '*type*'= 'c' (insert columns).

`Y = dyadup(X)` is equivalent to `Y = dyaddown(X,1,'c')`.

`Y = dyadup(X,'type')` is equivalent to `Y = dyadup(X,1,'type')`.
`Y = dyadup(X,EVENODD)` is equivalent to `Y = dyadup(X,EVENODD,'c')`.

## Examples

```
% For a vector.
s = 1:5
s =
    1 2 3 4 5

dse = dyadup(s) % Upsample elements at odd indices.
dse =
    0 1 0 2 0 3 0 4 0 5 0

% or equivalently
dse = dyadup(s,1)
dse =
    0 1 0 2 0 3 0 4 0 5 0

dso = dyadup(s,0) % Upsample elements at even indices.
dso =
    1 0 2 0 3 0 4 0 5

% For a matrix.
s = (1:2)'*(1:3)
s =
    1 2 3
    2 4 6

der = dyadup(s,1,'r') % Upsample rows at even indices.
der =
    0 0 0
    1 2 3
    0 0 0
    2 4 6
    0 0 0

doc = dyadup(s,0,'c') % Upsample columns at odd indices.
doc =
    1 0 2 0 3
    2 0 4 0 6
dem = dyadup(s,1,'m') % Upsample rows and columns
                      % at even indices.
dem =
     0     0     0     0     0     0     0
     0     1     0     2     0     3     0
     0     0     0     0     0     0     0
     0     2     0     4     0     6     0
     0     0     0     0     0     0     0

% Using default values for dyadup and dyaddown, we have:
% dyaddown(dyadup(s)) = s.
s = 1:5
s =
    1 2 3 4 5

uds = dyaddown(dyadup(s))
uds =
    1 2 3 4 5

% In general reversed identity is false.
```

## References

Strang, G.; T. Nguyen (1996), *Wavelets and Filter Banks*, Wellesley-Cambridge Press.

### See Also

`dyaddown`

## entrupd

Entropy update (wavelet packet)

### Syntax

```
T = entrupd(T,ENT)
T = entrupd(T,ENT,PAR)
```

### Description

`entrupd` is a one- or two-dimensional wavelet packet utility.

`T = entrupd(`*`T`*`,`*`ENT`*`)` or `T = entrupd(`*`T`*`,`*`ENT`*`,`*`PAR`*`)` returns for a given wavelet packet tree *T*, the updated tree using the entropy function *ENT* with the optional parameter *PAR* (see `wenergy` for more information).

### Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load signal.
load noisdopp; x = noisdopp;

% Decompose x at depth 2 with db1 wavelet packets
% using shannon entropy.
t = wpdec(x,2,'db1','shannon');

% Read entropy of all the nodes.
nodes = allnodes(t);
ent = read(t,'ent',nodes);
ent'
ent =
    1.0e+04 *
    -5.8615 -6.8204 -0.0350 -7.7901 -0.0497 -0.0205 -0.0138

% Update nodes entropy.
t = entrupd(t,'threshold',0.5);
```

```
nent = read(t,'ent');
nent'
nent =
    937 488 320 241 175 170 163
```

## See Also

wenergy | wpdec | wpdec2

## fbspwavf

Complex frequency B-spline wavelet

### Syntax

```
[PSI,X] = fbspwavf(LB,UB,N,M,FB,FC)
```

### Description

`[PSI,X] = fbspwavf(LB,UB,N,M,FB,FC)` returns values of the complex frequency B-Spline wavelet defined by the order parameter *M* (*M* is an integer such that $1 \leq M$), a bandwidth parameter `FB`, and a wavelet center frequency *FC*.

The function `PSI` is computed using the explicit expression

```
PSI(X) = (FB^0.5)*((sinc(FB*X/M).^M).*exp(2*i*pi*FC*X))
```

on an *N* point regular grid in the interval `[LB,UB]`.

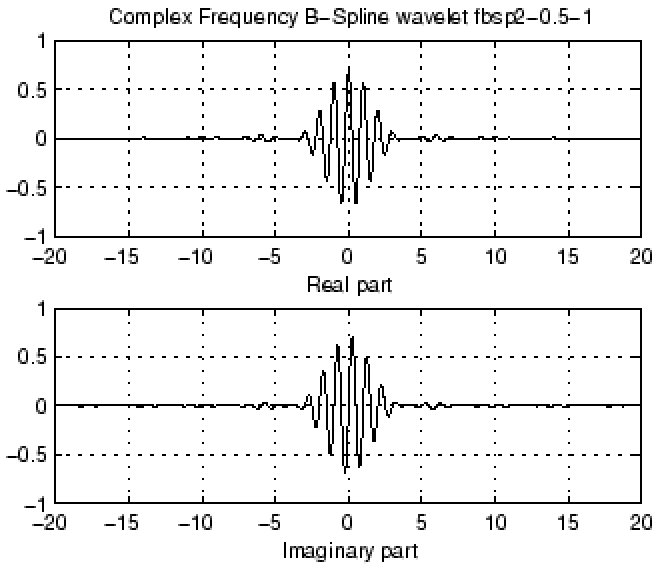
*FB* and *FC* must be such that `FC > 0` and > `FB > 0`.

Output arguments are the wavelet function `PSI` computed on the grid *X*.

### Examples

```
% Set order, bandwidth and center frequency parameters.
m = 2; fb = 0.5; fc = 1;

% Set effective support and grid parameters.
lb = -20; ub = 20; n = 1000;

% Compute complex Frequency B-Spline wavelet fbsp2-0.5-1.
[psi,x] = fbspwavf(lb,ub,n,m,fb,fc);

% Plot complex Frequency B-Spline wavelet.
subplot(211)
plot(x,real(psi))
```

```
title('Complex Frequency B-Spline wavelet fbsp2-0.5-1')
xlabel('Real part'), grid
subplot(212)
plot(x,imag(psi))
xlabel('Imaginary part'), grid
```



## References

Teolis, A. (1998), *Computational signal processing with wavelets*, Birkhauser, p. 63.

## See Also

`waveinfo`

# filt2ls

Transform quadruplet of filters to lifting scheme

## Syntax

```
LS = filt2ls(LoD,HiD,LoR,HiR)
```

## Description

`LS = filt2ls(LoD,HiD,LoR,HiR)` returns the lifting scheme `LS` associated with the four input filters `LoD`, `HiD`, `LoR`, and `HiR` that verify the perfect reconstruction condition.

## Examples

```
[LoD,HiD,LoR,HiR] = wfilters('db2')

LoD =

   -0.1294    0.2241    0.8365    0.4830

HiD =

   -0.4830    0.8365   -0.2241   -0.1294

LoR =

    0.4830    0.8365    0.2241   -0.1294

HiR =

   -0.1294   -0.2241    0.8365   -0.4830

LS = filt2ls(LoD,HiD,LoR,HiR);
displs(LS);

LS = {...
'd'              [ -1.73205081]              [0]
'p'             [ -0.06698730  0.43301270]  [1]
'd'             [  1.00000000]              [-1]
[  1.93185165]  [  0.51763809]              []
};

LSref = liftwave('db2');
displs(LSref);

LSref = {...
'd'             [ -1.73205081]              [0]
'p'             [ -0.06698730  0.43301270]  [1]
'd'             [  1.00000000]              [-1]
[  1.93185165]  [  0.51763809]              []
};
```

## See Also

`ls2filt` | `lsinfo`

## gauswavf

Gaussian wavelet

### Syntax

```
[PSI,X] = gauswavf(LB,UB,N,P)
[PSI,X] = gauswavf(LB,UB,N)
[PSI,X] = gauswavf(LB,UB,N,1)
```

### Description

`[PSI,X] = gauswavf(LB,UB,N,P)` returns values of the *P*-th derivative of the Gaussian function on an *N* point regular grid for the interval `[LB,UB]`. *Cp* is such that the 2-norm of the *P*-th derivative of *F* is equal to 1.

For `P > 8`, Symbolic Math Toolbox software is required.

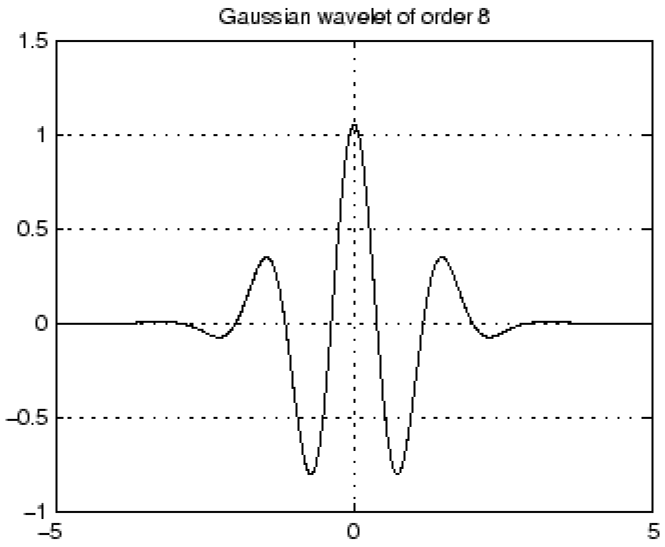
Output arguments are the wavelet function `PSI` computed on the grid `X`.

`[PSI,X] = gauswavf(LB,UB,N)` is equivalent to
`[PSI,X] = gauswavf(LB,UB,N,1)`.

These wavelets have an effective support of `[-5 5]`.

### Examples

```
% Set effective support and grid parameters.
lb = -5; ub = 5; n = 1000;

% Compute Gaussian wavelet of order 8.
[psi,x] = gauswavf(lb,ub,n,8);

% Plot Gaussian wavelet of order 8.
plot(x,psi),
title('Gaussian wavelet of order 8'), grid
```



## See Also

`waveinfo`

## get

WPTREE contents

### Syntax

```
[FieldValue1,FieldValue2, ...] =
get(T,'FieldName1','FieldName2', ...)
[FieldValue1,FieldValue2, ...] = get(T)
```

### Description

`[FieldValue1,FieldValue2, ...] = get(T,'FieldName1','FieldName2', ...)` returns the content of the specified fields for the WPTREE object T.

For the fields that are objects or structures, you can get the subfield contents, giving the name of these subfields as `'FieldName'` values. (See "Examples" below.)

`[FieldValue1,FieldValue2, ...] = get(T)` returns all the field contents of the tree *T*.

The valid choices for `'FieldName'` are

| | |
|---|---|
| `'dtree'` | DTREE parent object |
| `'wavInfo'` | Structure (wavelet information) |

The fields of the wavelet information structure, `'wavInfo'`, are also valid for `'FieldName'`:

| | |
|---|---|
| `'wavName'` | Wavelet name |
| `'Lo_D'` | Low Decomposition filter |
| `'Hi_D'` | High Decomposition filter |
| `'Lo_R'` | Low Reconstruction filter |
| `'Hi_R'` | High Reconstruction filter |

| | |
|---|---|
| `'entInfo'` | Structure (entropy information) |

The fields of the entropy information structure, `'entInfo'`, are also valid for `'`*`FieldName`*`'`:

| | |
|---|---|
| `'entName'` | Entropy name |
| `'entPar'` | Entropy parameter |

Or fields of DTREE parent object:

| | |
|---|---|
| `'ntree'` | NTREE parent object |
| `'allNI'` | All nodes information |
| `'terNI'` | Terminal nodes information |

Or fields of NTREE parent object:

| | |
|---|---|
| `'wtbo'` | WTBO parent object |
| `'order'` | Order of the tree |
| `'depth'` | Depth of the tree |
| `'spsch'` | Split scheme for nodes |
| `'tn'` | Array of terminal nodes of the tree |

Or fields of WTBO parent object:

| | |
|---|---|
| `'wtboInfo'` | Object information |
| `'ud'` | Userdata field |

## Examples

```
% Compute a wavelet packets tree
x = rand(1,1000);
t = wpdec(x,2,'db2');
o = get(t,'order');
[o,tn] = get(t,'order','tn');
[o,allNI,tn] = get(t,'order','allNI','tn');
[o,wavInfo,allNI,tn] = get(t,'order','wavInfo','allNI','tn');
```

```
[o,tn,Lo_D,EntName] = get(t,'order','tn','Lo_D','EntName');
[wo,nt,dt] = get(t,'wtbo','ntree','dtree');
```

## See Also

disp | read | set | write

# icwtft

Inverse CWT

## Syntax

```
xrec = icwtft(cwtstruct)
xrec = icwtft(cwtstruct,'plot')
xrec = icwtft(cwtstruct,'signal',SIG,'plot')
```

## Description

`xrec = icwtft(cwtstruct)` returns the inverse continuous wavelet transform of the CWT coefficients contained in the `cfs` field of the structure array cwtstruct. Obtain the structure array cwtstruct as the output of `cwtft`.

`xrec = icwtft(cwtstruct,'plot')` plots the reconstructed signal.

`xrec = icwtft(cwtstruct,'signal',SIG,'plot')` places a radio button in the bottom left corner of the plot. Enabling the radio button superimposes the plot of the input signal SIG on the plot of the reconstructed signal. By default the radio button is not enabled and only the reconstructed signal is plotted.

## Input Arguments

### `cwtstruct`

Structure array containing six fields.

- `dt` — The sampling period
- `cfs` — CWT coefficient matrix
- `scales` — Vector of scales
- `wav` — Analyzing wavelet used in the CWT

- `omega` — Angular frequencies used in the Fourier transform
- `meanSig` — Mean of the analyzed signal

cwtstruct is the output of `cwtft`.

## Output Arguments

**`xrec`**

Reconstructed signal

## Examples

Compute the CWT and inverse CWT of two sinusoids with disjoint support.

```
N = 1024;
t = linspace(0,1,N);
y = sin(2*pi*8*t).*(t<=0.5)+sin(2*pi*16*t).*(t>0.5);
dt = 0.05;
s0 = 2*dt;
ds = 0.4875;
NbSc = 20;
wname = 'morl';
sig = {y,dt};
sca = {s0,ds,NbSc};
wave = {wname,[]};
cwtsig = cwtft(sig,'scales',sca,'wavelet',wave);

% Compute inverse CWT and plot reconstructed signal with original
sigrec = icwtft(cwtsig,'signal',sig,'plot');
```

Select the radio button in the bottom left corner of the plot.

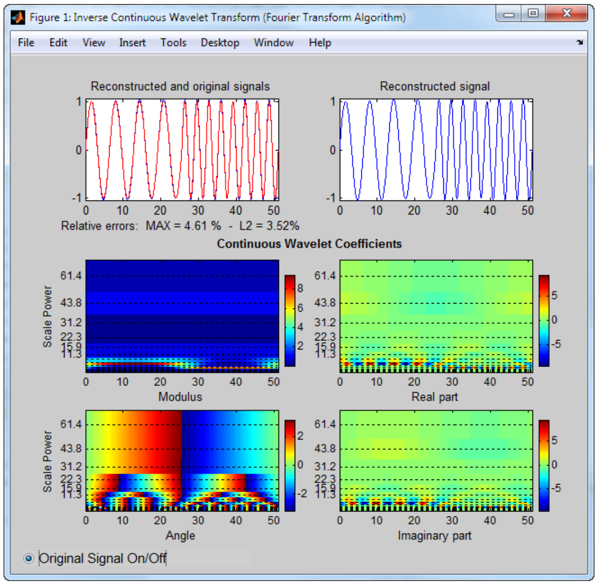
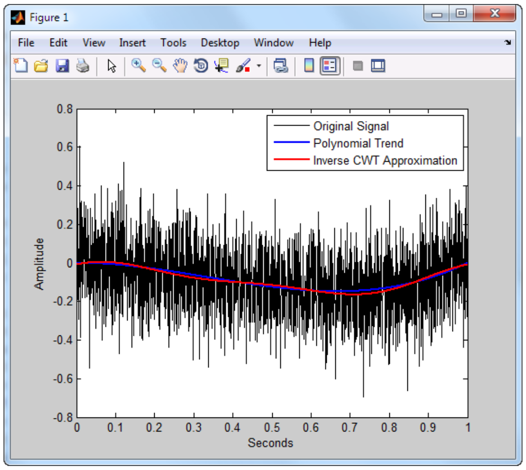


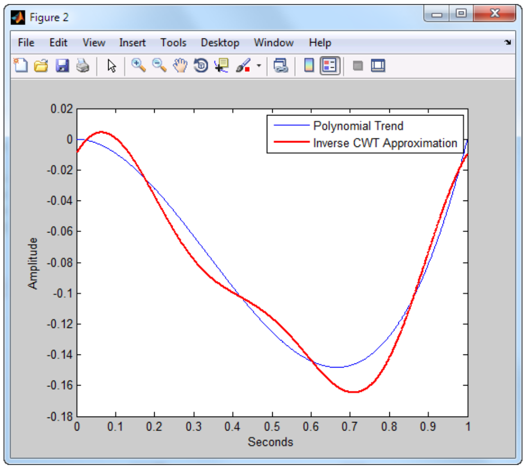
Figure 1: Inverse Continuous Wavelet Transform (Fourier Transform Algorithm)
File Edit View Insert Tools Desktop Window Help
Reconstructed and original signals
Reconstructed signal
Relative errors: MAX = 4.61 % - L2 = 3.52%
Continuous Wavelet Coefficients
Scale Power
Modulus
Real part
Angle
Imaginary part
Original Signal On/Off

Use the inverse CWT to approximate a trend in a time series. Construct a time series consisting of a polynomial trend, a sinewave (oscillatory component), and additive white Gaussian noise. Obtain the CWT of the input signal and use the inverse CWT based on only the coarsest scales to reconstruct an approximation to the trend. To obtain an accurate approximation based on select scales use the default power of two spacing for the scales in the continuous wavelet transform. See `cwtft` for details.

```
t = linspace(0,1,1e3);
% Polynomial trend
x = t.^3-t.^2;
% Periodic term
x1 = 0.25*cos(2*pi*250*t);
% Reset random number generator for reproducible results
rng default
y = x+x1+0.1*randn(size(t));
% Obtain CWT of input time series
cwty = cwtft({y,0.001},'wavelet','morl');
% Zero out all but the coarsest scale CWT coefficients
cwty.cfs(1:16,:) = 0;
% Reconstruct a signal approximation based on the coarsest scales
xrec = icwtft(cwty);
plot(t,y,'k'); hold on;
xlabel('Seconds'); ylabel('Amplitude');
```

```
plot(t,x,'b','linewidth',2);
plot(t,xrec,'r','linewidth',2);
legend('Original Signal','Polynomial Trend','Inverse CWT Approximation');
figure
plot(t,x,'b'); hold on;
xlabel('Seconds'); ylabel('Amplitude');
plot(t,xrec,'r','linewidth',2);
legend('Polynomial Trend','Inverse CWT Approximation');
```





You can also use the following syntax to plot the approximation. Select the radio button to view the original polynomial trend superimposed on the wavelet approximation.

```
% Input the polynomial trend as the value of 'signal'
xrec = icwtft(cwty,'signal',x,'plot');
```

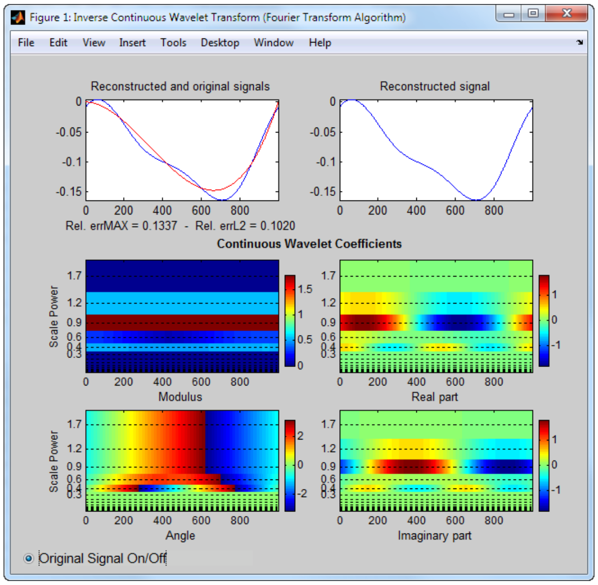


Figure 1: Inverse Continuous Wavelet Transform (Fourier Transform Algorithm)
File Edit View Insert Tools Desktop Window Help
Reconstructed and original signals
Reconstructed signal
Rel. errMAX = 0.1337 - Rel. errL2 = 0.1020
Continuous Wavelet Coefficients
Scale Power
Modulus
Real part
Angle
Imaginary part
Original Signal On/Off

## More About

### Inverse CWT

`icwtft` computes the inverse CWT based on a discretized version of the single integral formula due to Morlet. The Wavelet Toolbox Getting Started Guide contains a brief description of the theoretical foundation for the single integral formula in “Inverse Continuous Wavelet Transform”. The discretized version of this integral is presented in [5]

- “Continuous Wavelet Transform”
- “DFT-Based Continuous Wavelet Transform”
- “Inverse Continuous Wavelet Transform”

## References

[1] Daubechies, I. *Ten Lectures on Wavelets*, Philadelphia, PA: Society for Industrial and Applied Mathematics (SIAM), 1992.

[2] Farge, M. “Wavelet Transforms and Their Application to Turbulence”, *Ann. Rev. Fluid. Mech.*, 1992, 24, 395–457.

[3] Mallat, S. *A Wavelet Tour of Signal Processing*, San Diego, CA: Academic Press, 1998.

[4] Sun,W. “Convergence of Morlet's Reconstruction Formula”, *preprint*, 2010.

[5] Torrence, C. and G.P. Compo “A Practical Guide to Wavelet Analysis”, *Bull. Am. Meteorol. Soc.*, 79, 61–78, 1998.

### See Also

`cwt` | `cwtft`

# icwtlin

Inverse continuous wavelet transform (CWT) for linearly spaced scales

## Syntax

```
xrec = icwtlin(cwtstruct)
xrec = icwtlin(wav,meanSIG,cfs,scales,dt)
xrec = icwtin(...,'plot')
xrec = icwtlin (...,'signal',SIG,'plot')
xrec = icwtlin(...,Name,Value)
```

## Description

`xrec = icwtlin(cwtstruct)` returns the inverse continuous wavelet transform (CWT) of the CWT coefficients obtained at linearly-spaced scales.

**Note:** To use `icwtlin` you must:

- Use linearly-spaced scales in the CWT. `icwtlin` does not verify that the scales are linearly-spaced.
- Use one of the supported wavelets. See "Input Arguments" on page 1-198 for a list of supported wavelets.

`xrec = icwtlin(wav,meanSIG,cfs,scales,dt)` returns the inverse CWT of the coefficients in cfs. The inverse CWT is obtained using the wavelet wav, the linearly spaced scales scales, the sampling period dt, and the mean signal value meanSig.

`xrec = icwtin(...,'plot')` plots the reconstructed signal xrec along with the CWT coefficients and CWT moduli. If the analyzing wavelet is complex-valued, the plot includes the real and imaginary parts of the CWT coefficients.

`xrec = icwtlin (...,'signal',SIG,'plot')` places a radio button in the bottom-left corner of the plot. Enabling the radio button superimposes the plot of the input signal SIG on the plot of the reconstructed signal. SIG can be a structure array, a cell array, or a vector. If SIG is a structure array, there must be two fields: `val` and `period`. The `val` field contains the signal and the `period` field contains the sampling period. If SIG is a cell array, `SIG{1}` contains the signal and `SIG{2}` is the sampling period.

`xrec = icwtlin(...,Name,Value)` returns the inverse CWT transform with additional options specified by one or more Name,Value pair arguments.

## Input Arguments

### `cwtstruct`

A structure array that is the output of `cwtft` or constructed from the output of `cwt`. If you obtain cwtstruct from `cwtft`, the structure array contains six fields:

- cfs — CWT coefficient matrix
- scales — Vector of linearly spaced scales. The scale vector must be linearly-spaced to ensure accurate reconstruction. `icwtlin` does not check that the spacing of your scale vector is linear.
- wav — Analyzing wavelet. `icwtlin` uses this wavelet as the reconstruction wavelet. The supported wavelets are:
  - `'dog'` — An *m*-th order derivative of Gaussian wavelet where *m* is a positive even integer
  - `'morl'` — Analytic Morlet wavelet
  - `'morlex'` — Nonanalytic Morlet wavelet
  - `'morl0'` — Nonanalytic Morlet wavelet with exact zero mean
  - `'mexh'` — Mexican-hat wavelet. This argument represents a special case of the derivative of Gaussian wavelet with *m*=2.
  - `'paul'` — Paul wavelet
- omega — Angular frequencies used in the Fourier transform in radians/sample
- MeanSIG — Signal mean
- dt — Sampling period in seconds

If you create cwtstruct from the output of `cwt`, cwtstruct contains all of the preceding fields except omega.

Using `cwt` to obtain the CWT coefficients, the valid analyzing wavelets are:

- Coiflets — `'coif1'`,`'coif2'` ,`'coif3'` ,`'coif4'`, `'coif5'`
- Biorthogonal wavelets — `'bior2.2'`, `'bior2.4'`, `'bior2.6'`, `'bior2.8'`, `'bior4.4'`, `'bior5.5'`, `bior6.8`
- Reverse biorthogonal wavelets — `'rbio2.2'`, `'rbio2.4'`, `'rbio2.6'`, `'rbio2.8'`, `'rbio4.4'`, `'rbio5.5'`, `'rbio6.8'`
- Complex Gaussian wavelets — `'cgau2'`, `'cgau4'`, `'cgau6'`, `'cgau8'`

### Name-Value Pair Arguments

**`'IdxSc'`**

Vector of scales to use in the signal reconstruction. Specifying a subset of scales results in a scale-localized approximation of the analyzed signal.

## Output Arguments

**`xrec`**

Reconstructed signal. Signal approximation based on the input CWT coefficient matrix, analyzing wavelet, selected scales, and sampling period.

The purpose of the CWT inversion algorithm is not to produce a perfect reconstruction of the input signal. The inversion preserves time and scale-localized features in the reconstructed signal. The amplitude scaling in the reconstructed signal, however, can be significantly different. This difference in scaling can occur whether or not you use all the CWT coefficients in the inversion.

## Examples

Compute the inverse CWT of a sum of sine waves with disjoint support.

```
% Define the signal
N = 100;
t = linspace(0,1,N);
Y = sin(8*pi*t).*(t<=0.5) + sin(16*pi*t).*(t>0.5) ;

% Define parameters before analysis
```

```
dt = 0.001;
maxsca = 1; s0 = 2*dt; ds = 2*dt;
scales = s0:ds:maxsca;
wname = 'morl';
SIG = {Y,dt};
WAV = {wname,[]};

% Compute the CWT using cwtft with linear scales
cwtS = cwtft(SIG,'scales',scales,'wavelet',WAV);
% Compute inverse CWT using linear scales
Yrec = icwtlin(cwtS,'Signal',Y,'plot');
```

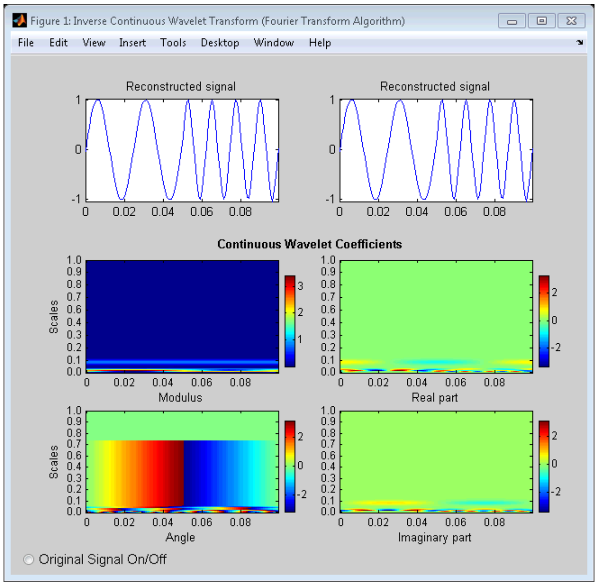


Figure 1: Inverse Continuous Wavelet Transform (Fourier Transform Algorithm)
File Edit View Insert Tools Desktop Window Help
Reconstructed signal
Reconstructed signal
Continuous Wavelet Coefficients
Scales
Modulus
Real part
Angle
Imaginary part
Original Signal On/Off

Reconstruct an approximation to a noisy Doppler signal based on thresholded coefficients. Use the universal threshold. Assume the sampling period is 0.05 seconds.

```
load noisdopp;
Y = noisdopp;
N = length(Y);

% Define parameters before analysis
% Assume sampling period is 0.05
dt = 0.05;
maxsca = 100; s0 = 2*dt; ds = 4*dt;
```

```
scales = s0:ds:maxsca;
wname = 'morl';
SIG = {Y,dt};
WAV = {wname,[]};

% Compute CWT
cwtS = cwtft(SIG,'scales',scales,'wavelet',WAV,'plot');

% Select subset of coefficients
cwtS1 = cwtS;
Hfreq = cwtS.cfs(1:10,:);
% Set threshold
thr = sqrt(2*log(N))*median(abs(Hfreq(:)))/0.6745;
newCFS = cwtS.cfs;
% Set coefficients smaller than threshold in absolute value to 0
newCFS(abs(newCFS)<thr) = 0;
cwtS1.cfs = newCFS;


% Reconstruction from the modified structure
YRDen = icwtlin(cwtS1,'signal',Y,'plot');
```

Enable the **Reconstructed Signal On/Off** radio button in the bottom-left corner.

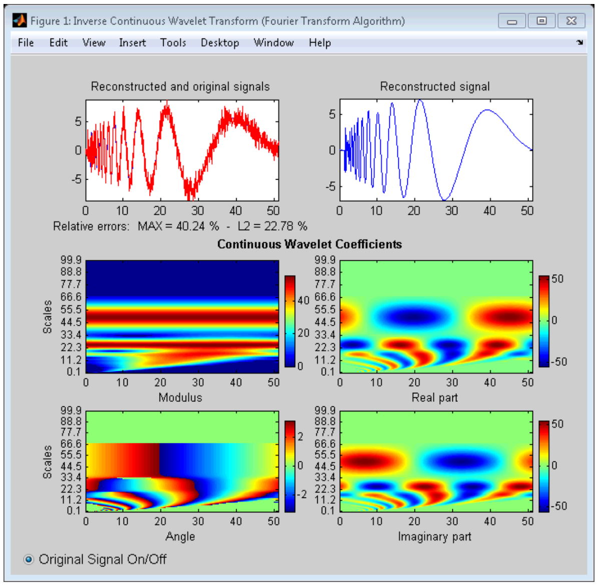


Figure 1: Inverse Continuous Wavelet Transform (Fourier Transform Algorithm)
File Edit View Insert Tools Desktop Window Help
Reconstructed and original signals
Reconstructed signal
Relative errors: MAX = 40.24 % - L2 = 22.78 %
Continuous Wavelet Coefficients
Scales
Modulus
Real part
Angle
Imaginary part
Original Signal On/Off

## Alternatives

- `icwtft` — Computes the inverse for the CWT obtained using `cwtft` with logarithmically spaced scales. If you use linearly spaced scales in `cwtft`, or you obtain the CWT with `cwt`, use `icwtlin` to compute the inverse.

## More About

### Algorithms

See [4] for a description of the inverse CWT algorithm for linearly spaced scales. The `icwtlin` function uses heuristic scaling factors for the analyzing wavelets. These scaling factors can result in significant differences in the amplitude scaling of the reconstructed signal.

- "Continuous Wavelet Transform"
- "DFT-Based Continuous Wavelet Transform"
- "Inverse Continuous Wavelet Transform"

## References

[1] Daubechies, I. *Ten Lectures on Wavelets*, Philadelphia, PA: Society for Industrial and Applied Mathematics (SIAM), 1992.

[2] Farge, M. "Wavelet Transforms and Their Application to Turbulence", *Ann. Rev. Fluid. Mech.*, 1992, 24, 395–457.

[3] Mallat, S. *A Wavelet Tour of Signal Processing*, San Diego, CA: Academic Press, 1998.

[4] Sun,W. "Convergence of Morlet's Reconstruction Formula", *preprint*, 2010.

[5] Torrence, C. and G.P. Compo. "A Practical Guide to Wavelet Analysis", *Bull. Am. Meteorol. Soc.*, 79, 61–78, 1998.

## See Also

`icwtft` | `cwtft` | `cwt`

# idddtree

Inverse dual-tree and double-density 1-D wavelet transform

## Syntax

```
xrec = idddtree(wt)
```

## Description

`xrec = idddtree(wt)` returns the inverse wavelet transform of the wavelet decomposition (analysis filter bank), wt. wt is the output of `dddtree`.

## Examples

### Perfect Reconstruction Using the Dual-Tree Double-Density Wavelet Filter Bank

Demonstrate perfect reconstruction of a signal using a dual-tree double-density wavelet transform.

Load the noisy Doppler signal. Obtain the dual-tree double-density wavelet transform down to level 5. Invert the transform and demonstrate perfect reconstruction.

```
load noisdopp;
wt = dddtree('cplxdddt',noisdopp,5,'FSdoubledualfilt','doubledualfilt');
xrec = idddtree(wt);
max(abs(noisdopp-xrec))
```

- “Analytic Wavelets Using the Dual-Tree Wavelet Transform”

## Input Arguments

### `wt` — Wavelet transform

structure

Wavelet transform, returned as a structure from `dddtree` with these fields:

### `type` — Type of wavelet decomposition (filter bank)

`'dwt'` | `'ddt'` | `'cplxdt'` | `'cplxdddt'`

Type of wavelet decomposition (filter bank), specified as one of `'dwt'`, `'ddt'`, `'cplxdt'`, or `'cplxdddt'`. The type,`'dwt'`, gives a critically sampled discrete wavelet transform. The other types are oversampled wavelet transforms. `'ddt'` is a double-density wavelet transform, `'cplxdt'` is a dual-tree complex wavelet transform, and `'cplxdddt'` is a double-density dual-tree complex wavelet transform.

### `level` — Level of wavelet decomposition

positive integer

Level of wavelet decomposition, specified as a positive integer.

### `filters` — Decomposition (analysis) and reconstruction (synthesis) filters

structure

Decomposition (analysis) and reconstruction (synthesis) filters, specified as a structure with these fields:

#### `Fdf` — First-stage analysis filters

matrix | cell array

First-stage analysis filters, specified as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage analysis filters for the corresponding tree.

#### `Df` — Analysis filters for levels > 1

matrix | cell array

Analysis filters for levels > 1, specified as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the analysis filters for the corresponding tree.

### `Frf` — First-level reconstruction filters

matrix | cell array

First-level reconstruction filters, specified as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage synthesis filters for the corresponding tree.

### `Rf` — Reconstruction filters for levels > 1

matrix | cell array

Reconstruction filters for levels > 1, specified as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are N-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the synthesis filters for the corresponding tree.

### `cfs` — Wavelet transform coefficients

cell array of matrices

Wavelet transform coefficients, specified as a 1-by-(level+1) cell array of matrices. The size and structure of the matrix elements of the cell array depend on the type of wavelet transform as follows:

- `'dwt'` — `cfs{j}`
  - j = 1,2,... level is the level.
  - `cfs{level+1}` are the lowpass, or scaling, coefficients.
- `'ddt'` — `cfs{j}(:,:,k)`
  - j = 1,2,... level is the level.

- k = 1,2 is the wavelet filter.
- `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.

- `'cplxdt'` — `cfs{j}(:,:,m)`
  - j = 1,2,... level is the level.
  - m = 1,2 are the real and imaginary parts.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'cplxdddt'` — `cfs{j}(:,:,k,m)`
  - j = 1,2 level is the level.
  - k = 1,2 is the wavelet filter.
  - m = 1,2 are the real and imaginary parts.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.

## Output Arguments

### `xrec` — Synthesized 1-D signal
vector

Synthesized 1-D signal, returned as a vector. The row or column orientation of xrec depends on the row or column orientation of the 1-D signal input to `dddtree`.

Data Types: `double`

## More About

- “Critically Sampled and Oversampled Wavelet Filter Banks”

### See Also
`dddtree` | `dddtreecfs` | `plotdt`

# idddtree2

Inverse dual-tree and double-density 2-D wavelet transform

## Syntax

```
xrec = idddtree2(wt)
```

## Description

`xrec = idddtree2(wt)` returns the inverse wavelet transform of the 2-D decomposition (analysis filter bank), wt. wt is the output of `dddtree2`.

## Examples

### Perfect Reconstruction Using the Complex Oriented Dual-Tree Wavelet Filter Bank

Demonstrate perfect reconstruction of an image using a complex oriented dual-tree wavelet transform.

Load the image and obtain the complex oriented dual-tree wavelet transform down to level 5 using `dddtree2`. Reconstruct the image using `idddtree2` and demonstrate perfect reconstruction.

```
load woman;
wt = dddtree2('cplxdt',X,5,'dtf2');
xrec = idddtree2(wt);
max(max(abs(X-xrec)))
```

- "Analytic Wavelets Using the Dual-Tree Wavelet Transform"

## Input Arguments

### `wt` — Wavelet transform

structure

Wavelet transform, returned as a structure from `dddtree2` with these fields:

### `type` — Type of wavelet decomposition (filter bank)

`'dwt'` | `'ddt'` | `'realdt'` | `'cplxdt'` | `'realdddt'` | `'cplxdddt'`

Type of wavelet decomposition (filter bank), specified as one of `'dwt'`, `'ddt'`, `'realdt'`, `'cplxdt'`, `'realdddt'`, or `'cplxdddt'`. `'dwt'` is the critically sampled DWT. `'ddt'` produces a double-density wavelet transform with one scaling and two wavelet filters for both row and column filtering. `'realdt'` and `'cplxdt'` produce oriented dual-tree wavelet transforms consisting of two and four separable wavelet transforms. `'realdddt'` and `'cplxdddt'` produce double-density dual-tree wavelet transforms consisting of two and four separable wavelet transforms.

### `level` — Level of the wavelet decomposition

positive integer

Level of the wavelet decomposition, specified as a positive integer.

### `filters` — Decomposition (analysis) and reconstruction (synthesis) filters

structure

Decomposition (analysis) and reconstruction (synthesis) filters, specified as a structure with these fields:

#### `Fdf` — First-stage analysis filters

matrix | cell array

First-stage analysis filters, specified as an $N$-by-2 or $N$-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two $N$-by-2 or $N$-by-3 matrices for dual-tree wavelet transforms. The matrices are $N$-by-3 for the double-density wavelet transforms. For an $N$-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an $N$-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage analysis filters for the corresponding tree.

#### `Df` — Analysis filters for levels > 1

matrix | cell array

Analysis filters for levels > 1, specified as an $N$-by-2 or $N$-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two $N$-by-2 or $N$-by-3 matrices for dual-tree wavelet transforms. The matrices are $N$-by-3 for the double-density wavelet transforms. For an $N$-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an $N$-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the analysis filters for the corresponding tree.

### `Frf` — First-level reconstruction filters

matrix | cell array

First-level reconstruction filters, specified as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage synthesis filters for the corresponding tree.

### `Rf` — Reconstruction filters for levels > 1

matrix | cell array

Reconstruction filters for levels > 1, specified as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage analysis filters for the corresponding tree.

### `cfs` — Wavelet transform coefficients

cell array of matrices

Wavelet transform coefficients, specified as a 1-by-(level+1) cell array of matrices. The size and structure of the matrix elements of the cell array depend on the type of wavelet transform as follows:

- `'dwt'` — `cfs{j}(:,:,d)`
  - j = 1,2,... level is the level.
  - d = 1,2,3 is the orientation.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'ddt'` — `cfs{j}(:,:,d)`
  - j = 1,2,... level is the level.

  - d = 1,2,3,4,5,6,7,8 is the orientation.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'realddt'` — `cfs{j}(:,:,d,k)`
  - j = 1,2,... level is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'cplxdt'` — `cfs{j}(:,:,d,k,m)`
  - j = 1,2,... level is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - m = 1,2 are the real and imaginary parts.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients..
- `'realdddt'` — `cfs{j}(:,:,d,k)`
  - j = 1,2,... level is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'cplxdddt'` — `cfs{j}(:,:,d,k,m)`
  - j = 1,2,... level is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - m = 1,2 are the real and imaginary parts.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.

## Output Arguments

### `xrec` — Synthesized 2-D image
matrix

Synthesized image, returned as a matrix.

Data Types: `double`

## More About

- “Critically Sampled and Oversampled Wavelet Filter Banks”

### See Also

`dddtree2` | `dddtreecfs`

# idwt

Single-level inverse discrete 1-D wavelet transform

## Syntax

```
X = idwt(cA,cD,'wname')
X = idwt(cA,cD,Lo_R,Hi_R)
X = idwt(cA,cD,'wname',L)
X = idwt(cA,cD,Lo_R,Hi_R,L)
idwt(cA,cD,'wname')
X = idwt(...,'mode',MODE)
X = idwt(cA,[],...)
X = idwt([],cD,...)
```

## Description

The `idwt` command performs a single-level one-dimensional wavelet reconstruction with respect to either a particular wavelet (`'wname'`, see `wfilters` for more information) or particular wavelet reconstruction filters (`Lo_R` and `Hi_R`) that you specify.

`X = idwt(cA,cD,'wname')` returns the single-level reconstructed approximation coefficients vector `X` based on approximation and detail coefficients vectors `cA` and `cD`, and using the wavelet `'wname'`.

`X = idwt(cA,cD,Lo_R,Hi_R)` reconstructs as above using filters that you specify.

- `Lo_R` is the reconstruction low-pass filter.
- `Hi_R` is the reconstruction high-pass filter.

`Lo_R` and `Hi_R` must be the same length.

Let `la` be the length of `cA` (which also equals the length of `cD`) and `lf` the length of the filters `Lo_R` and `Hi_R`; then `length(X) = LX` where `LX = 2*la` if the DWT extension mode is set to periodization. For the other extension modes `LX = 2*la-lf+2`.

For more information about the different Discrete Wavelet Transform extension modes, see `dwtmode`.

`X = idwt(cA,cD,'wname',L)` or `X = idwt(cA,cD,Lo_R,Hi_R,L)` returns the length-`L` central portion of the result obtained using `idwt(cA,cD,'wname')`. `L` must be less than `LX`.

`X = idwt(...,'mode',MODE)` computes the wavelet reconstruction using the specified extension mode `MODE`.

`X = idwt(cA,[],...)` returns the single-level reconstructed approximation coefficients vector `X` based on approximation coefficients vector `cA`.

`X = idwt([],cD,...)` returns the single-level reconstructed detail coefficients vector `X` based on detail coefficients vector `cD`.

## Examples

### Inverse DWT Using Orthogonal Wavelet

Demonstrate perfect reconstruction using `dwt` and `idwt` with an orthonormal wavelet.

```
load noisdopp;
[A,D] = dwt(noisdopp,'sym4');
x = idwt(A,D,'sym4');
max(abs(noisdopp-x))
```

### Inverse DWT Using Biorthgonal Wavelet

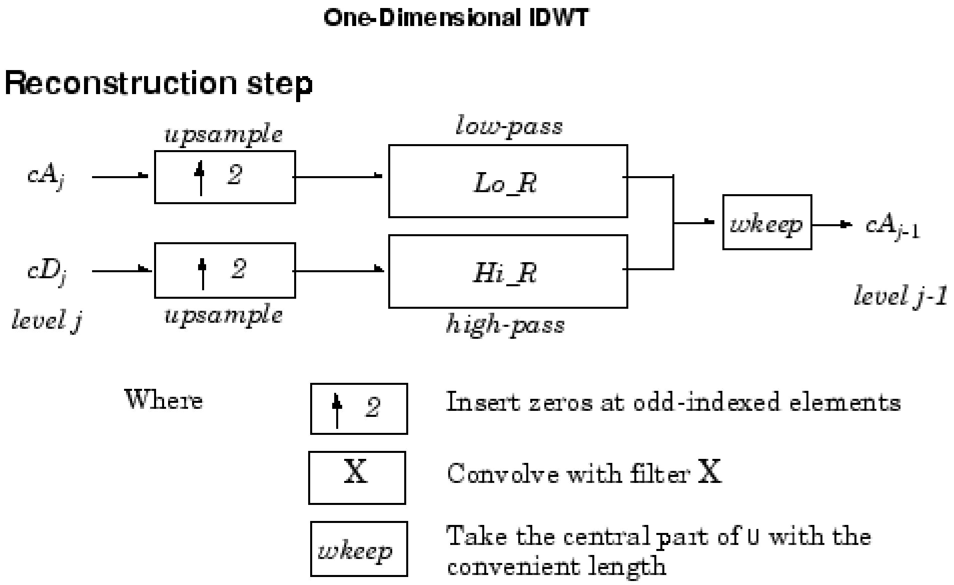
Demonstrate perfect reconstruction using `dwt` and `idwt` with a biorthogonal wavelet.

```
load noisdopp;
[Lo_D,Hi_D,Lo_R,Hi_R] = wfilters('bior3.5');
[A,D] = dwt(noisdopp,Lo_D,Hi_D);
x = idwt(A,D,Lo_R,Hi_R);
max(abs(noisdopp-x))
```

## More About

### Algorithms

Starting from the approximation and detail coefficients at level $j$, $cAj$ and $cD_j$, the inverse discrete wavelet transform reconstructs $cA_{j-1}$, inverting the decomposition step by inserting zeros and convolving the results with the reconstruction filters.



## References

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics. SIAM Ed.

Mallat, S. (1989), “A theory for multiresolution signal decomposition: the wavelet representation,” *IEEE Pattern Anal. and Machine Intell.*, vol. 11, no. 7, pp. 674–693.

Meyer, Y. (1990), *Ondelettes et opérateurs*, Tome 1, Hermann Ed. (English translation: *Wavelets and operators*, Cambridge Univ. Press. 1993.)

## See Also

`dwt` | `dwtmode` | `upwlev`

# idwt2

Single-level inverse discrete 2-D wavelet transform

## Syntax

```
X = idwt2(cA,cH,cV,cD,'wname')
X = idwt2(cA,cH,cV,cD,Lo_R,Hi_R)
X = idwt2(cA,cH,cV,cD,'wname',S)
X = idwt2(cA,cH,cV,cD,Lo_R,Hi_R,S)
idwt2(cA,cH,cV,cD,'wname')
X = idwt2(...,'mode',MODE)
X = idwt2(cA,[],[],[],...)
X = idwt2([],cH,[],[],...)
```

## Description

The `idwt2` command performs a single-level two-dimensional wavelet reconstruction with respect to either a particular wavelet (`'wname'`, see `wfilters` for more information) or particular wavelet reconstruction filters (`Lo_R` and `Hi_R`) that you specify.

`X = idwt2(cA,cH,cV,cD,'wname')` uses the wavelet `'wname'` to compute the single-level reconstructed approximation coefficients matrix *X*, based on approximation matrix `cA` and details matrices `cH`,`cV`, and `cD` (horizontal, vertical, and diagonal, respectively).

`X = idwt2(cA,cH,cV,cD,Lo_R,Hi_R)` reconstructs as above, using filters that you specify.

- `Lo_R` is the reconstruction low-pass filter.
- `Hi_R` is the reconstruction high-pass filter.

`Lo_R` and `Hi_R` must be the same length.

Let `sa = size(cA) = size(cH) = size(cV) = size(cD)` and `lf` the length of the filters; then `size(X) = SX`, where `SX = 2* SA`, if the DWT extension mode is set to periodization. For the other extension modes, `SX = 2*size(cA)-lf+2`.

For more information about the different Discrete Wavelet Transform extension modes, see `dwtmode`.

`X = idwt2(cA,cH,cV,cD,'`*`wname`*`',S)` and `X = idwt2(cA,cH,cV,cD,Lo_R,Hi_R,S)` return the size-`S` central portion of the result obtained using the syntax `idwt2(cA,cH,cV,cD,'`*`wname`*`')`. `S` must be less than `SX`.

`X = idwt2(...,'mode',MODE)` computes the wavelet reconstruction using the extension mode `MODE` that you specify.

`X = idwt2(cA,[],[],[],...)` returns the single-level reconstructed approximation coefficients matrix `X` based on approximation coefficients matrix `cA`.

`X = idwt2([],cH,[],[],...)` returns the single-level reconstructed detail coefficients matrix `X` based on horizontal detail coefficients matrix `cH`.

The same result holds for `X = idwt2([],[],cV,[],...)` and `X = idwt2([],[],[],cD,...)`, based on vertical and diagonal details.

More generally, `X = idwt2(AA,HH,VV,DD,...)` returns the single-level reconstructed matrix `X`, where `AA` can be `cA` or `[]`, and so on.

`idwt2` is the inverse function of `dwt2` in the sense that the abstract statement `idwt2(dwt2(X,'`*`wname`*`'),'`*`wname`*`')` would give back `X`.

## Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load original image.
load woman;

% X contains the loaded image.
sX = size(X);

% Perform single-level decomposition
% of X using db4.
[cA1,cH1,cV1,cD1] = dwt2(X,'db4');

% Invert directly decomposition of X
% using coefficients at level 1.
A0 = idwt2(cA1,cH1,cV1,cD1,'db4',sX);
```

```
% Check for perfect reconstruction.
max(max(abs(X-A0)))
ans =
    3.4176e-10
```

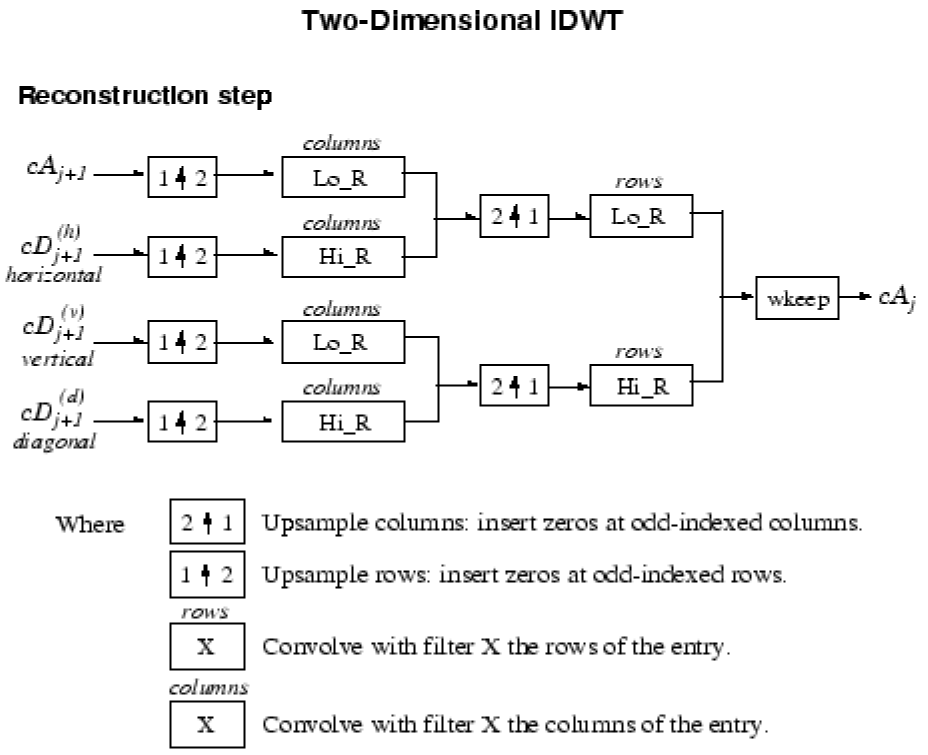
## More About

### Tips

If cA,cH,cV,cD are obtained from an indexed image analysis or a truecolor image analysis, they are `m`-by-`n` matrices or `m`-by-`n`-by-3 arrays, respectively.

For more information on image formats, see the `image` and `imfinfo` reference pages.

### Algorithms


Two-Dimensional IDWT
Reconstruction step
$cA_{j+1}$
$cD^{(h)}_{j+1}$ horizontal
$cD^{(v)}_{j+1}$ vertical
$cD^{(d)}_{j+1}$ diagonal
1 ↑ 2
columns
Lo_R
Hi_R
2 ↑ 1
rows
wkeep
$cA_j$
Where
2 ↑ 1 Upsample columns: insert zeros at odd-indexed columns.
1 ↑ 2 Upsample rows: insert zeros at odd-indexed rows.
rows X Convolve with filter X the rows of the entry.
columns X Convolve with filter X the columns of the entry.

## See Also

`dwt2` | `dwtmode` | `upwlev2`

# idwt3

Single-level inverse discrete 3-D wavelet transform

## Syntax

```
X = idwt3(WT)
C = idwt3(WT,TYPE)
```

## Description

The `idwt3` command performs a single-level three-dimensional wavelet reconstruction starting from a single-level three-dimensional wavelet decomposition.

`X = idwt3(WT)` computes the single-level reconstructed 3-D array X, based on the three-dimensional wavelet decomposition stored in the WT structure. This structure contains the following fields.

| | |
|---|---|
| `sizeINI` | Size of the three-dimensional array `X`. |
| `mode` | Name of the wavelet transform extension mode. |
| `filters` | Structure with 4 fields, `LoD`, `HiD`, `LoR`, `HiR`, which contain the filters used for DWT. |
| `dec` | 2 x 2 x 2 cell array containing the coefficients of the decomposition.<br><br>`dec{i,j,k}`, `i`,`j`,`k = 1` or `2` contains the coefficients obtained by low-pass filtering (for `i` or `j` or `k = 1`) or high-pass filtering (for `i` or `j` or `k = 2`). |

`C = idwt3(WT,TYPE)` computes the single-level reconstructed component based on the three-dimensional wavelet decomposition. Valid values for `TYPE` are:

- A group of three characters `'xyz'`, one per direction, with `'x'`,`'y'` and `'z'` selected in the set {`'a'`,`'d'`,`'l'`,`'h'`} or in the corresponding uppercase set {`'A'`,`'D'`,`'L'`,`'H'`}), where `'A'` (or `'L'`) specifies low-pass filter and `'D'` (or `'H'`) specifies high-pass filter.

- The char `'d'` (or `'h'` or `'D'` or `'H'`) which specifies the sum of all the components different from the low-pass component.

## Examples

```
% Define original 3D data.
X  = reshape(1:64,4,4,4);

% Decompose X using db1.
wt = dwt3(X,'db1');

% Reconstruct X from coefficients.
XR = idwt3(wt);

% Compute reconstructed approximation, i.e. the
% low-pass component.
A  = idwt3(wt,'aaa');

% Compute the sum of all the components different
% from the low-pass component.
D  = idwt3(wt,'d');

% Reconstruct the component associated with low-pass in the
% X and Z directions and high-pass in the Y direction.
ADA  = idwt3(wt,'ada');
```

## See Also

`dwt3` | `wavedec3` | `waverec3`

# ilwt

Inverse 1-D lifting wavelet transform

## Syntax

```
X = ilwt(AD_In_Place,W)
X = ilwt(CA,CD,W)
X = ilwt(AD_In_Place,W,LEVEL)
X = ILWT(CA,CD,W,LEVEL)
X = ilwt(AD_In_Place,W,LEVEL,'typeDEC',typeDEC)
X = ilwt(CA,CD,W,LEVEL,'typeDEC',typeDEC)
```

## Description

`ilwt` performs a 1-D lifting wavelet reconstruction with respect to a particular lifted wavelet that you specify.

`X = ilwt(AD_In_Place,W)` computes the reconstructed vector `X` using the approximation and detail coefficients vector `AD_In_Place` obtained by a lifting wavelet reconstruction. `W` is a lifted wavelet name (see `liftwave`).

`X = ilwt(CA,CD,W)` computes the reconstructed vector `X` using the approximation coefficients vector `CA` and detail coefficients vector `CD` obtained by a lifting wavelet reconstruction.

`X = ilwt(AD_In_Place,W,LEVEL)` or `X = ILWT(CA,CD,W,LEVEL)` computes the lifting wavelet reconstruction, at level `LEVEL`.

`X = ilwt(AD_In_Place,W,LEVEL,'typeDEC',typeDEC)` or `X = ilwt(CA,CD,W,LEVEL,'typeDEC',typeDEC)` with `typeDEC = 'w'` or `'wp'` computes the wavelet or the wavelet packet decomposition using lifting, at level `LEVEL`.

Instead of a lifted wavelet name, you may use the associated lifting scheme `LS`: `X = ilwt(...,LS,...)` instead of `X = ILWT(...,W,...)`.

For more information about lifting schemes, see `lsinfo`.

## Examples

```
% Start from the Haar wavelet and get the
% corresponding lifting scheme.
lshaar = liftwave('haar');

% Add a primal ELS to the lifting scheme.
els = {'p',[-0.125 0.125],0};
lsnew = addlift(lshaar,els);

% Perform LWT at level 1 of a simple signal.
x = 1:8;
[cA,cD] = lwt(x,lsnew);

% Perform integer LWT of the same signal.
lshaarInt = liftwave('haar','int2int');
lsnewInt = addlift(lshaarInt,els);
[cAint,cDint] = lwt(x,lsnewInt);

% Invert the two transforms.
xRec = ilwt(cA,cD,lsnew);
err = max(max(abs(x-xRec)))


err =

  4.4409e-016

xRecInt = ilwt(cAint,cDint,lsnewInt);
errInt = max(max(abs(x-xRecInt)))

errInt =

     0
```

## See Also

`lwt`

# ilwt2

Inverse 2-D lifting wavelet transform

## Syntax

```
X = ilwt2(AD_In_Place,W)
X = ilwt2(CA,CH,CV,CD,W)
X = ilwt2(AD_In_Place,W,LEVEL)
X = ILWT2(CA,CH,CV,CD,W,LEVEL)
X = ilwt2(AD_In_Place,W,LEVEL,'typeDEC',typeDEC)
X = ilwt2(CA,CH,CV,CD,W,LEVEL,'typeDEC',typeDEC)
```

## Description

`ilwt2` performs a 2-D lifting wavelet reconstruction with respect to a particular lifted wavelet that you specify.

`X = ilwt2(AD_In_Place,W)` computes the reconstructed matrix `X` using the approximation and detail coefficients matrix `AD_In_Place`, obtained by a lifting wavelet decomposition. `W` is a lifted wavelet name (see `liftwave`).

`X = ilwt2(CA,CH,CV,CD,W)` computes the reconstructed matrix `X` using the approximation coefficients vector `CA` and detail coefficients vectors `CH`, `CV`, and `CD` obtained by a lifting wavelet decomposition.

`X = ilwt2(AD_In_Place,W,LEVEL)` or `X = ILWT2(CA,CH,CV,CD,W,LEVEL)` computes the lifting wavelet reconstruction, at level `LEVEL`.

`X = ilwt2(AD_In_Place,W,LEVEL,'typeDEC',typeDEC)` or `X = ilwt2(CA,CH,CV,CD,W,LEVEL,'typeDEC',typeDEC)` with `typeDEC = 'w'` or `'wp'` computes the wavelet or the wavelet packet decomposition using lifting, at level `LEVEL`.

Instead of a lifted wavelet name, you may use the associated lifting scheme `LS`: `X = ilwt2(...,LS,...)` instead of `X = ilwt2(...,W,...)`.

For more information about lifting schemes, see `lsinfo`.

## Examples

```
% Start from the Haar wavelet and get the
% corresponding lifting scheme.
lshaar = liftwave('haar');

% Add a primal ELS to the lifting scheme.
els = {'p',[-0.125 0.125],0};
lsnew = addlift(lshaar,els);

% Perform LWT at level 1 of a simple image.
x = reshape(1:16,4,4);
[cA,cH,cV,cD] = lwt2(x,lsnew);

% Perform integer LWT of the same image.
lshaarInt = liftwave('haar','int2int');
lsnewInt = addlift(lshaarInt,els);
[cAint,cHint,cVint,cDint] = lwt2(x,lsnewInt);

% Invert the two transforms.
xRec = ilwt2(cA,cH,cV,cD,lsnew);
err = max(max(abs(x-xRec)))

err =

     0

xRecInt = ilwt2(cAint,cHint,cVint,cDint,lsnewInt);
errInt = max(max(abs(x-xRecInt)))

errInt =

     0
```

## More About

### Tips

If `AD_In_Place` or cA,cH,cV,cD are obtained from an indexed image analysis or a truecolor image analysis, they are `m`-by-`n` matrices or `m`-by-`n`-by-3 arrays, respectively.

For more information on image formats, see the `image` and `imfinfo` reference pages.

## See Also

`lwt2`

## ind2depo

Node index to node depth-position

### Syntax

```
[D,P] = ind2depo(ORD,[D P])
```

### Description

`ind2depo` is a tree-management utility.

For a tree of order `ORD`, `[D,P] = ind2depo(ORD,N)` computes the depths `D` and the positions `P` (at these depths `D`) for the nodes with indices `N`.

The nodes are numbered from left to right and from top to bottom. The root index is 0.

`N` must be a column vector of integers `(N ≥ 0)`.

Note that `[D,P] = ind2depo(ORD,[D P])`.

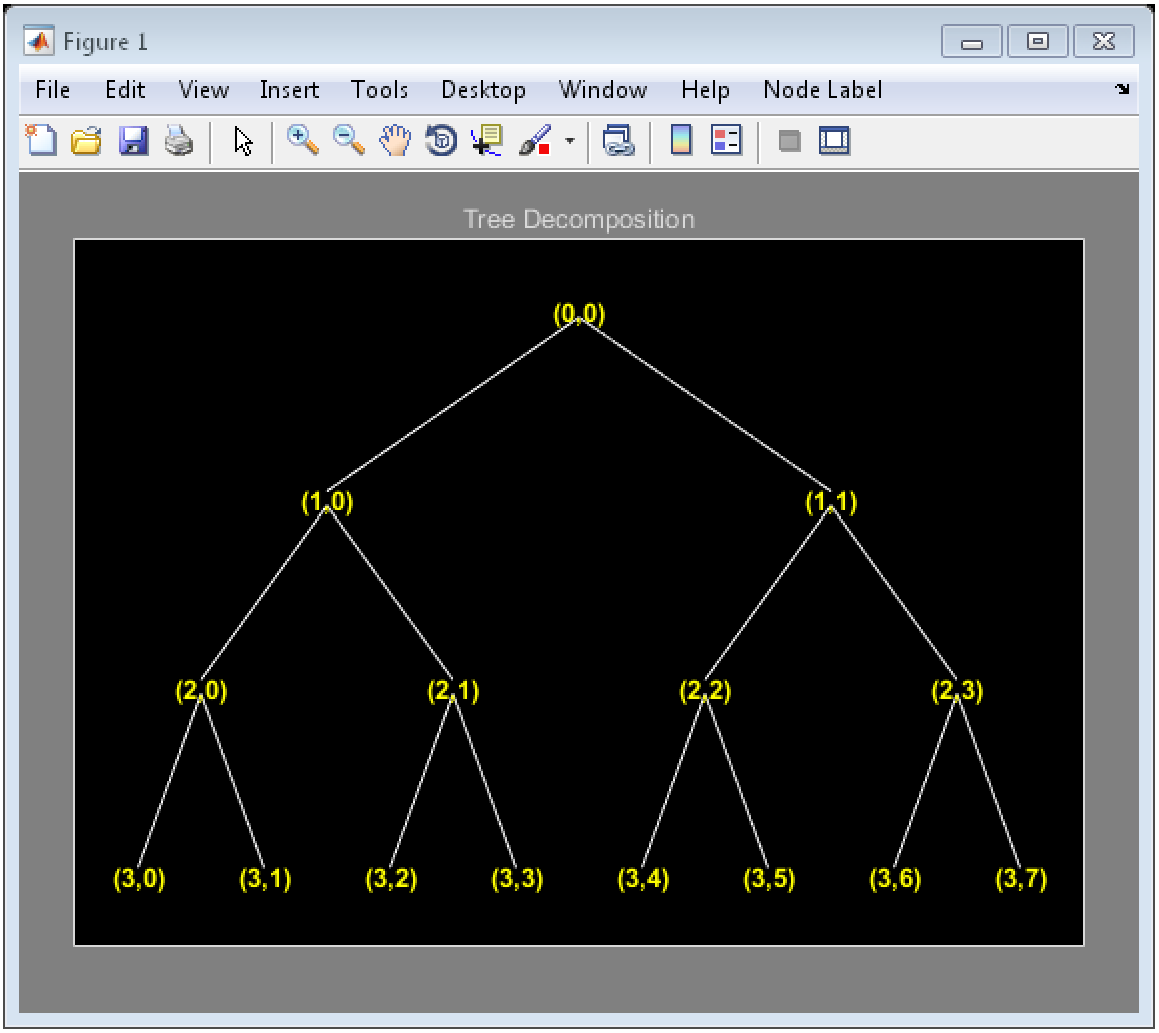
### Depth and Position in Wavelet Packet Tree

Create a binary wavelet packet tree with three levels.

```
Ord = 2;
Lev = 3;
T = ntree(Ord,Lev);
```

Plot the binary wavelet packet tree.

```
plot(T)
```



Obtain the indices of the nodes in linear order.

```
idx = allnodes(T);
```

Conver the indices to depth-position format.

```
[depth,pos] = ind2depo(Ord,idx);
table(depth,pos)
```

```
ans =

    depth    pos
    _____    ___

    0        0
    1        0
    1        1
    2        0
    2        1
    2        2
    2        3
    3        0
    3        1
    3        2
    3        3
    3        4
    3        5
    3        6
    3        7
```

## See Also

`depo2ind`

# intwave

Integrate wavelet function psi (ψ)

## Syntax

```
[INTEG,XVAL] = intwave('wname',PREC)
[INTDEC,XVAL,INTREC] = intwave('wname',PREC)
[INTEG,XVAL] = intwave('wname',PREC)
[INTEG,XVAL] = intwave('wname',PREC,0)
[INTEG,XVAL] = intwave('wname')
[INTEG,XVAL] = intwave('wname',8)
intwave('wname',IN2,IN3), PREC = max(IN2,IN3)
intwave('wname',0)
intwave('wname',8,IN3)
intwave('wname')
intwave('wname',8)
```

## Description

`[INTEG,XVAL] = intwave('wname',PREC)` computes the integral, `INTEG`, of the wavelet function ψ (from −∞ to `XVAL` values): $\int_{-\infty}^{x} \psi(y)dy$ for $x$ in `XVAL`.

The function ψ is approximated on the $2^{\mathrm{PREC}}$ points grid `XVAL` where *`PREC`* is a positive integer. `'wname'` is a string containing the name of the wavelet ψ (see `wfilters` for more information).

Output argument *`INTEG`* is a real or complex vector depending on the wavelet type.

For biorthogonal wavelets,

`[INTDEC,XVAL,INTREC] = intwave('wname',PREC)` computes the integrals, `INTDEC` and `INTREC`, of the wavelet decomposition function $\psi_{\mathrm{dec}}$ and the wavelet reconstruction function $\psi_{\mathrm{rec}}$.

`[INTEG,XVAL] = intwave('wname',PREC)` is equivalent to `[INTEG,XVAL] = intwave('wname',PREC,0)`.

`[INTEG,XVAL] = intwave('`*`wname`*`')` is equivalent to `[INTEG,XVAL] = intwave('`*`wname`*`',8)`.

When used with three arguments `intwave('`*`wname`*`',IN2,IN3), PREC = max(IN2,IN3)` and plots are given.

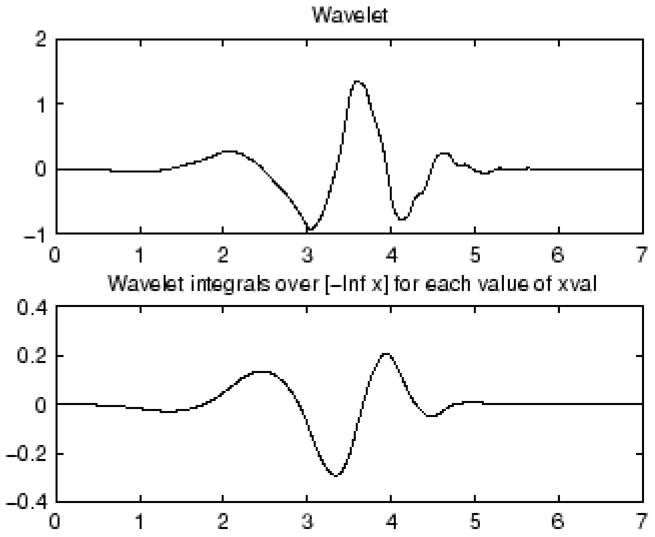
When `IN2` is equal to the special value 0, `intwave('`*`wname`*`',0)` is equivalent to `intwave('`*`wname`*`',8,IN3)`.

`intwave('`*`wname`*`')` is equivalent to `intwave('`*`wname`*`',8)`.

`intwave` is used only for continuous analysis (see `cwt` for more information).

## Examples

```
% Set wavelet name.
wname = 'db4';

% Plot wavelet function.
[phi,psi,xval] = wavefun(wname,7);
subplot(211); plot(xval,psi); title('Wavelet');

% Compute and plot wavelet integrals approximations
% on a dyadic grid.
[integ,xval] = intwave(wname,7);
subplot(212); plot(xval,integ);
title(['Wavelet integrals over [-Inf x] ' ...
       'for each value of xval']);
```



## More About

### Algorithms

First, the wavelet function is approximated on a grid of $2^{\mathrm{PREC}}$ points using `wavefun`. A piecewise constant interpolation is used to compute the integrals using `cumsum`.

## See Also

`wavefun`

## isnode

Existing node test

### Syntax

`R = isnode(T,N)`

### Description

`isnode` is a tree-management utility.

`R = isnode(T,N)` returns 1's for nodes *N*, which exist in the tree *T*, and 0's for others.

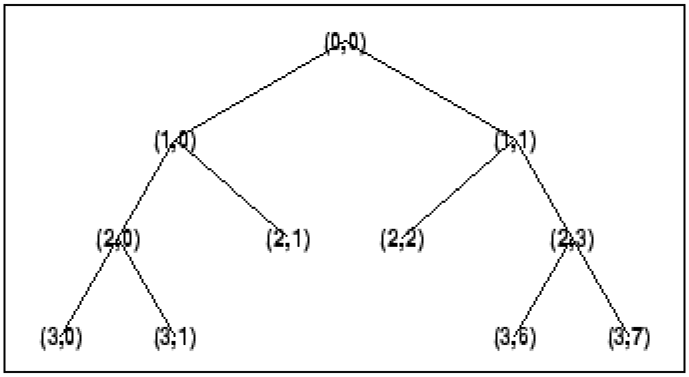
*N* can be a column vector containing the indices of nodes or a matrix, that contains the depths and positions of nodes.

In the last case, `N(i,1)` is the depth of the `i`-th node and `N(i,2)` is the position of the `i`-th node.

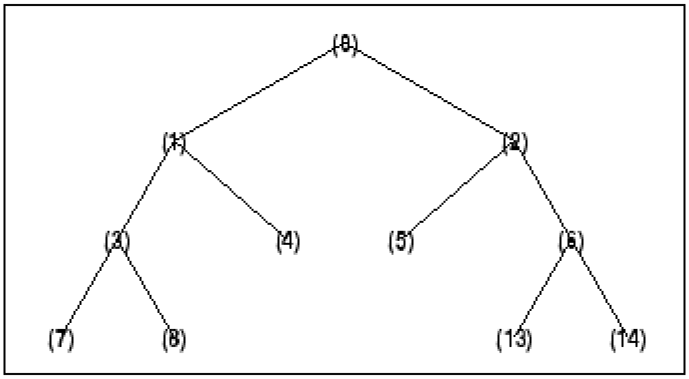
The nodes are numbered from left to right and from top to bottom. The root index is 0.

### Examples

```
% Create initial tree.
ord = 2;
t = ntree(ord,3);    % binary tree of depth 3.
t = nodejoin(t,5);
t = nodejoin(t,4);
plot(t)
```



```
% Change Node Label from Depth_Position to Index
% (see the plot function).
```



```
% Check node index.
isnode(t,[1;3;25])

ans =
     1
     1
     0

% Check node Depth_Position.
isnode(t,[1 0;3 1;4 5])

ans =
     1
     1
     0
```

## See Also

`istnode` | `wtreemgr`

# istnode

Terminal nodes indices test

## Syntax

```
R = istnode(T,N)
```

## Description

`istnode` is a tree-management utility.

`R = istnode(T,N)` returns ranks (in left to right terminal nodes ordering) for terminal nodes *N* belonging to the tree *T*, and 0's for others.

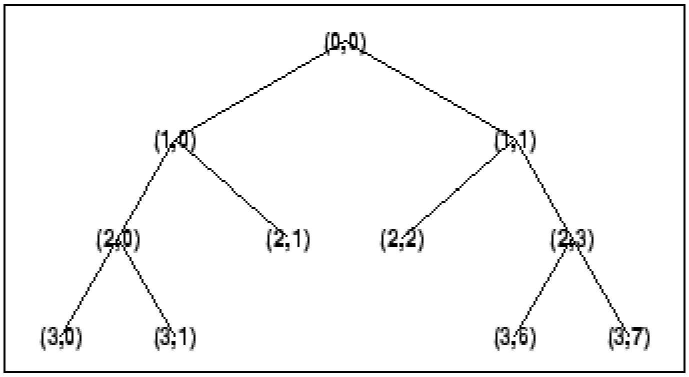
*N* can be a column vector containing the indices of nodes or a matrix that contains the depths and positions of nodes.

In the last case, `N(i,1)` is the depth of the `i`-th node and `N(i,2)` is the position of the `i`-th node.

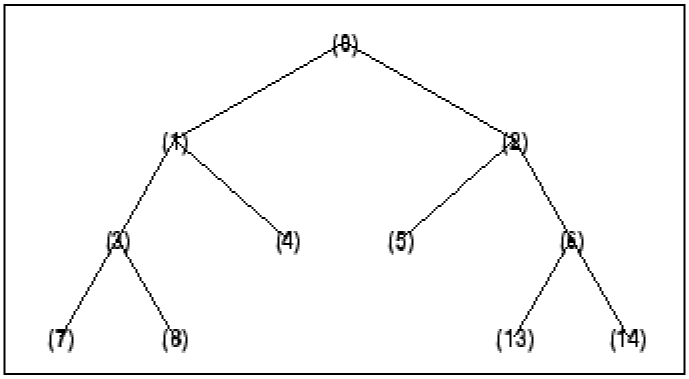
The nodes are numbered from left to right and from top to bottom. The root index is 0.

## Examples

```
% Create initial tree.
ord = 2;
t = ntree(ord,3); % binary tree of depth 3.
t = nodejoin(t,5);
t = nodejoin(t,4);
plot(t)
```



```
% Change Node Label from Depth_Position to Inde
% (see the plot function)x.
```



```
% Find terminal nodes and return indices for terminal
% nodes in the tree.
istnode(t,[14])
ans =
    6

istnode(t,[15])
ans =
    0

istnode(t,[1;7;14;25])
ans =
    0
    1
    6
    0

istnode(t,[1 0;3 1;4 5])
ans =
    0
    2
    0
```

## See Also

`isnode` | `wtreemgr`

# iswt

Inverse discrete stationary wavelet transform 1-D

## Syntax

```
X = iswt(SWC,'wname')
X = iswt(SWA,SWD,'wname')
X = iswt(SWA(end,:),SWD,'wname')
X = iswt(SWC,Lo_R,Hi_R)
X = iswt(SWA,SWD,Lo_R,Hi_R)
X = iswt(SWA(end,:),SWD,Lo_R,Hi_R)
```

## Description

`iswt` performs a multilevel 1-D stationary wavelet reconstruction using either a specific orthogonal wavelet (`'wname'`, see `wfilters` for more information) or specific reconstruction filters (`Lo_R` and `Hi_R`).

`X = iswt(SWC,'wname')` or `X = iswt(SWA,SWD,'wname')` or `X = iswt(SWA(end,:),SWD,'wname')` reconstructs the signal `X` based on the multilevel stationary wavelet decomposition structure `SWC` or `[SWA,SWD]` (see `swt` for more information).

`X = iswt(SWC,Lo_R,Hi_R)` or `X = iswt(SWA,SWD,Lo_R,Hi_R)` or `X = iswt(SWA(end,:),SWD,Lo_R,Hi_R)` reconstruct as above, using filters that you specify.

- `Lo_R` is the reconstruction low-pass filter.
- `Hi_R` is the reconstruction high-pass filter.

`Lo_R` and `Hi_R` must be the same length.

## Examples

```
% Load original 1D signal.
```

```
load noisbloc; s = noisbloc;

% Perform SWT decomposition at level 3 of s using db1.
swc = swt(s,3,'db1');
% Second usage.
[swa,swd] = swt(s,3,'db1');

% Reconstruct s from the stationary wavelet
% decomposition structure swc.
a0 = iswt(swc,'db1');
% Second usage.
a0bis = iswt(swa,swd,'db1');


% Check for perfect reconstruction.
err = norm(s-a0)
err =
  9.6566e-014

errbis = norm(s-a0bis)
errbis =
  9.6566e-014
```

## References

Nason, G.P.; B.W. Silverman (1995), "The stationary wavelet transform and some statistical applications," *Lecture Notes in Statistics*, 103, pp. 281–299.

Coifman, R.R.; Donoho D.L. (1995), "Translation invariant de-noising," *Lecture Notes in Statistics*, 103, pp 125–150.

Pesquet, J.C.; H. Krim, H. Carfatan (1996), "Time-invariant orthonormal wavelet representations," *IEEE Trans. Sign. Proc.*, vol. 44, 8, pp. 1964–1970.

## See Also

`idwt` | `swt` | `waverec`

# iswt2

Inverse discrete stationary wavelet transform 2-D

## Syntax

```
X = iswt2(SWC,'wname')
X = iswt2(A,H,V,D,wname)
X = iswt2(A(:,:,end),H,V,D,'wname')
X = iswt2(SWC,Lo_R,Hi_R)
X = iswt2(A,H,V,D,Lo_R,Hi_R)
X = iswt2(A(:,:,end),H,V,D,Lo_R,Hi_R)
```

## Description

`iswt2` performs a multilevel 2-D stationary wavelet reconstruction using either a specific orthogonal wavelet (`'wname'` see `wfilters` for more information) or specific reconstruction filters (`Lo_R` and `Hi_R`).

`X = iswt2(SWC,'wname')` or `X = iswt2(A,H,V,D,wname)` or `X = iswt2(A(:,:,end),H,V,D,'wname')` reconstructs the signal `X`, based on the multilevel stationary wavelet decomposition structure `SWC` or `[A,H,V,D]` (see `swt2`).

`X = iswt2(SWC,Lo_R,Hi_R)` or `X = iswt2(A,H,V,D,Lo_R,Hi_R)` or `X = iswt2(A(:,:,end),H,V,D,Lo_R,Hi_R)` reconstructs as in the previous syntax, using filters that you specify:

- `Lo_R` is the reconstruction low-pass filter.
- `Hi_R` is the reconstruction high-pass filter.

`Lo_R` and `Hi_R` must be the same length.

## Examples

```
% Load original image.
load nbarb1;
```

```
% Perform SWT decomposition
% of X at level 3 using sym4.
swc = swt2(X,3,'sym4');
% Second usage.
[ca,chd,cvd,cdd] = swt2(X,3,'sym4');

% Reconstruct s from the stationary wavelet
% decomposition structure swc.
a0 = iswt2(swc,'sym4');
% Second usage.
a0 = iswt2(ca,chd,cvd,cdd,'sym4');
% Check for perfect reconstruction.
err = max(max(abs(X-a0)))
ans =
  2.3482e-010

errbis = max(max(abs(X-a0bis)))
ans =
  2.3482e-010
```

## More About

### Tips

If SWC or (cA,cH,cV,cD) are obtained from an indexed image analysis or a truecolor image analysis, then X is an `m`-by-`n` matrix or an `m`-by-`n`-by-3 array, respectively.

For more information on image formats, see the `image` and `imfinfo` reference pages.

## References

Nason, G.P.; B.W. Silverman (1995), “The stationary wavelet transform and some statistical applications,” *Lecture Notes in Statistics*, 103, pp. 281–299.

Coifman, R.R.; Donoho D.L. (1995), “Translation invariant de-noising,” *Lecture Notes in Statistics*, 103, pp. 125–150.

Pesquet, J.C.; H. Krim, H. Carfatan (1996), “Time-invariant orthonormal wavelet representations,” *IEEE Trans. Sign. Proc.*, vol. 44, 8, pp. 1964–1970.

## See Also

`idwt2` | `swt2` | `waverec2`

# laurmat

Laurent matrices constructor

## Syntax

```
M = laurmat(V)
```

## Description

`M = laurmat(V)` returns the Laurent matrix object `M` associated with *V* which can be a cell array (at most two dimensional) of Laurent polynomials (see `laurpoly`) or an ordinary matrix.

## Examples

```
% Define Laurent matrices.
M1 = laurmat(eye(2,2))

      | 1     0  |
      |          |
 M1 = |          |
      |          |
      | 0     1  |

Z  = laurpoly(1,1);
M2 = laurmat({1 Z;0 1})

      | 1     z^(+1)  |
      |               |
 M2 = |               |
      |               |
      | 0       1     |

% Calculus on Laurent polynomials.
P = M1 * M2

     | 1     z^(+1)  |
       |                 |
 P =   |                 |
       |                 |
       | 0        1      |
```

```
d = det(P)
```

```
d(z) = 1
```

## References

Strang, G.; T. Nguyen (1996), *Wavelets and filter banks*, Wellesley-Cambridge Press.

Sweldens, W. (1998), “The Lifting Scheme: a Construction of Second Generation of Wavelets,” *SIAM J. Math. Anal.*, 29 (2), pp. 511–546.

### See Also

`laurpoly`

# laurpoly

Laurent polynomials constructor

## Syntax

```
P = laurpoly(C,d)
P = laurpoly(C,'dmin',d)
P = laurpoly(C,'dmax',d)
P = laurpoly(C,d)
```

## Description

`P = laurpoly(C,d)` returns a Laurent polynomial object. *C* is a vector whose elements are the coefficients of the polynomial `P` and *d* is the highest degree of the monomials of `P`.

If `m` is the length of the vector *C*, *P* represents the following Laurent polynomial:

```
P(z) = C(1)*z^d + C(2)*z^(d-1) + ... + C(m)*z^(d-m+1)
```

`P = laurpoly(C,'dmin',d)` specifies the lowest degree instead of the highest degree of monomials of *P*. The corresponding output *P* represents the following Laurent polynomial:

```
P(z) = C(1)*z^(d+m-1) + ... + C(m-1)*z^(d+1) + C(m)*z^d
```

`P = laurpoly(C,'dmax',d)` is equivalent to `P = laurpoly(C,d)`.

## Examples

```
% Define Laurent polynomials.
P = laurpoly([1:3],2);
P = laurpoly([1:3],'dmax',2)

P(z) = + z^(+2) + 2*z^(+1) + 3

P = laurpoly([1:3],'dmin',2)
```

```
P(z) = + z^(+4) + 2*z^(+3) + 3*z^(+2)

% Calculus on Laurent polynomials.
Z = laurpoly(1,1)

Z(z) = z^(+1)

Q = Z*P

Q(z) = + z^(+5) + 2*z^(+4) + 3*z^(+3)

R = Z^1 - Z^-1

R(z) = + z^(+1) - z^(-1)
```

## References

Strang, G.; T. Nguyen (1996), *Wavelets and filter banks*, Wellesley-Cambridge Press.

Sweldens, W. (1998), “The Lifting Scheme: a Construction of Second Generation of Wavelets,” *SIAM J. Math. Anal.*, 29 (2), pp. 511–546.

## See Also

`laurmat`

# leaves

Determine terminal nodes

## Syntax

```
N = leaves(T)
[N,K] = leaves(T,'sort')
N = leaves(T,'dp')
[N,K] = leaves(T,'sortdp')
[N,K] = leaves(T,'sdp')
```

## Description

`N = leaves(T)` returns the indices of terminal nodes of the tree *T* where `N` is a column vector.

The nodes are ordered from left to right as in tree *T*.

`[N,K] = leaves(T,'s')` or `[N,K] = leaves(T,'sort')` returns sorted indices. `M = N(K)` are the indices reordered as in tree *T*, from left to right.

`N = leaves(T,'dp')` returns a matrix `N`, which contains the depths and positions of terminal nodes.

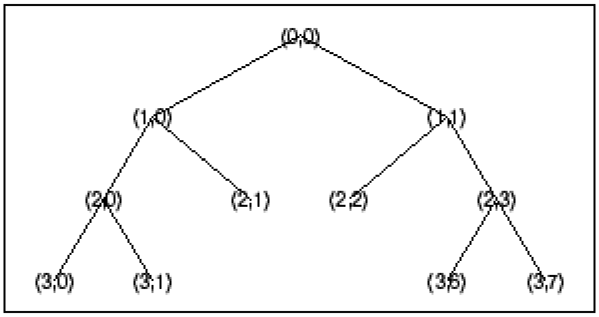
`N(i,1)` is the depth of the `i`-th terminal node, and `N(i,2)` is the position of the `i`-th terminal node.

`[N,K] = leaves(T,'sortdp')` or `[N,K] = leaves(T,'sdp')` returns sorted nodes.

## Examples

```
% Create initial tree.
ord = 2;
t = ntree(ord,3);        % binary tree of depth 3.
t=nodejoin(t,5);
t=nodejoin(t,4);
```

```
plot(t)
```



```
% List terminal nodes (index).
tnodes_ind = leaves(t)
tnodes_ind =
     7
     8
     4
     5
    13
    14

% List terminal nodes (sorted on index).
[tnodes_ind,Ind] = leaves(t,'sort')
tnodes_ind =
     4
     5
     7
     8
    13
    14

Ind =
     3
     4
     1
     2
     5
     6

% List terminal nodes (Depth_Position).
tnodes_depo = leaves(t,'dp')
tnodes_depo =
     3     0
     3     1
     2     1
     2     2
     3     6
     3     7

% List terminal nodes (sorted on Depth_Position).
[tnodes_depo,Ind] = leaves(t,'sortdp')
tnodes_depo =
     2     1
     2     2
     3     0
     3     1
     3     6
     3     7

Ind =
     3
     4
     1
     2
     5
     6
```

## See Also

tnodes | noleaves

## liftfilt

Apply elementary lifting steps on quadruplet of filters

### Syntax

```
[LoDN,HiDN,LoRN,HiRN] = liftfilt(LoD,HiD,LoR,HiR,ELS)
liftfilt(LoD,HiD,LoR,HiR,ELS,TYPE,VALUE)
```

### Description

`[LoDN,HiDN,LoRN,HiRN] = liftfilt(LoD,HiD,LoR,HiR,ELS)` returns the four filters `LoDN`, `HiDN`, `LoRN`, and `HiRN` obtained by an elementary lifting step (`ELS`) starting from the four filters `LoD`, `HiD`, `LoR`, and `HiR`. The four input filters verify the perfect reconstruction condition.

`ELS` is a structure such that

- `TYPE = ELS.type` contains the type of the elementary lifting step. The valid values for `TYPE` are `'p'` (primal) or `'d'` (dual).
- `VALUE = ELS.value` contains the Laurent polynomial `T` associated with the elementary lifting step (see `laurpoly`). If `VALUE` is a vector, the associated Laurent polynomial `T` is equal to `laurpoly(VALUE,0)`.

In addition, `ELS` may be a scaling step. In that case, `TYPE` is equal to `'s'` (scaling) and `VALUE` is a scalar different from zero.

`liftfilt(LoD,HiD,LoR,HiR,ELS,TYPE,VALUE)` gives the same outputs.

**Note** If `TYPE = 'p'` , `HiD` and `LoR` are unchanged.
If `TYPE = 'd'` , `LoD` and `HiR` are unchanged.
If `TYPE = 's'`, the four filters are changed.
If `ELS` is an array of elementary lifting steps, `liftfilt(...,ELS)` performs each step successively.

`liftfilt(...,FLAGPLOT)` plots the successive biorthogonal pairs—scaling function and wavelet.

## Examples

```
% Get Haar filters.
[LoD,HiD,LoR,HiR] = wfilters('haar');

% Lift the Haar filters.
twoels(1) = struct('type','p','value',...
laurpoly([0.125 -0.125],0));
twoels(2) = struct('type','p','value',...
laurpoly([0.125 -0.125],1));
[LoDN,HiDN,LoRN,HiRN] = liftfilt(LoD,HiD,LoR,HiR,twoels);

% The biorthogonal wavelet bior1.3 is obtained up to
% an unsignificant sign.
[LoDB,HiDB,LoRB,HiRB] = wfilters('bior1.3');
samewavelet = ...
isequal([LoDB,HiDB,LoRB,HiRB],[LoDN,-HiDN,LoRN,HiRN])

samewavelet =

     1
```

## See Also

`laurpoly`

# liftwave

Lifting schemes

## Syntax

```
LS = liftwave(WNAME)
LS = liftwave(WNAME,'Int2Int')
```

## Description

`LS = liftwave(WNAME)` returns the lifting scheme associated with the wavelet specified by `WNAME`. `LS` is a structure, not an integer, and used by `lwt`, `ilwt`, `lwt2`, etc.

`LS = liftwave(WNAME,'Int2Int')` performs an integer to integer wavelet transform. Using `'Int2Int'` produces an `LS` such that when you use `[CA,CD] = lwt(X,LS)` or `Y = lwt(X,LS)` and `X` is a vector of integers, the resulting `CA`, `CD`, and `Y` are vectors of integers. If you omit `'Int2Int'` then `lwt` produces vectors of real numbers.

The valid values for `WNAME` are

| WNAME Values | Comments |
|---|---|
| `'lazy'` | A "lazy" wavelet is a second-generation wavelet and is not a true mathematical wavelet. |
| `'haar'` | Same as `'db1'`, `'bior1.1'`, and `'cdf1.1'` |
| `'db1'`, `'db2'`, `'db3'`, `'db4'`, `'db5'`, `'db6'`, `'db7'`, `'db8'` | `'db2'` same as `'sym2'`, `'db3'`, and `'sym4'` |
| `'sym2'`, `'sym3'`, `'sym4'`, `'sym5'`, `'sym6'`, `'sym7'`, `'sym8'` | |
| Cohen-Daubechies-Feauveau wavelets `'cdf1.1'`,`'cdf1.3'`,`'cdf1.5'` `'cdf3.1'`,`'cdf3.3'`,`'cdf3.5'` `'cdf5.1'`,`'cdf5.3'`,`'cdf5.5'` `'cdf2.2'`,`'cdf2.4'`,`'cdf2.6'` | `'cdfX.Y'` same as `'biorX.Y'` except for `bior4.4` and `bior5.5`. |

| WNAME Values | Comments |
|---|---|
| `'cdf4.2'`,`'cdf4.4'`,'cdf4.6'<br>`'cdf6.2'`,`'cdf6.4'`,`'cdf6.6'` | |
| `'biorX.Y'` | See `waveinfo` |
| `'rbioX.Y'` | Reverse of `'biorX.Y'`.<br>See `waveinfo` |
| `'bs3'` | Same as `'cdf4.2'` |
| `'rbs3'` | Reverse of `'bs3'` |
| `'9.7'` | Same as `'bior4.4'` |
| `'r9.7'` | Reverse of `'9.7'` |

For more information about lifting schemes, see `lsinfo`.

## Examples

```
% Start from the db2 wavelet and get the
% corresponding lifting scheme.
lsdb2 = liftwave('db2');

% Visualize the obtained lifting scheme.
displs(lsdb2);

lsdb2 = {...
'd'             [ -1.73205081]              [0]
'p'             [ -0.06698730  0.43301270]  [1]
'd'             [  1.00000000]              [-1]
[  1.93185165]  [  0.51763809]              []
};
```

## See Also

`laurpoly`

# localmax

Identify and chain local maxima

## Syntax

```
[lmaxima,indices] = localmax(inputmatrix)
[lmaxima,indices] = localmax(inputmatrix,initrow)
[lmaxima,indices] = localmax(inputmatrix,initrow,regflag)
```

## Description

`[lmaxima,indices] = localmax(inputmatrix)` identifies and chains the local maxima in the rows of inputmatrix.

`[lmaxima,indices] = localmax(inputmatrix,initrow)` initializes the chaining of local maxima beginning with row initrow. If there are no local maxima in initrow, all rows in lmaxima with indices less than initrow consist of only zeros.

`[lmaxima,indices] = localmax(inputmatrix,initrow,regflag)` replaces initrow of inputmatrix with the level-5 approximation (scaling) coefficients obtained with the `sym4` wavelet.

## Input Arguments

### `inputmatrix`

inputmatrix is a matrix of real or complex numbers. Most often, inputmatrix is a matrix of continuous wavelet transform (CWT) coefficients, and you use `localmax` to identify maxima lines. `localmax` operates on the absolute values of inputmatrix.

### `initrow`

Initialization row for chaining local maxima. The chaining algorithm begins at initrow and decrements the row index by 1 until the first row of the matrix is reached. By specifying initrow, you can exclude rows from the chaining algorithm.

**Default:** `size(inputmatrix,1)`

**`regflag`**

Regularization flag. If you set regflag to `true`, the row of inputmatrix corresponding to initrow is replaced by the level-5 approximation (scaling) coefficients obtained with the `sym4` wavelet.

**Default:** `true`

## Output Arguments

**`lmaxima`**

Matrix with local maxima chains. lmaxima only has nonzero entries at the locations of local maxima in the absolute values of inputmatrix. Denote the row index of lmaxima by `R`. You can determine the value of lmaxima at a local maximum in row `R` as follows:

- If `R>initRow`, the value of lmaxima at a local maximum is 1.
- If `R=initRow`, the value of lmaxima at a local maximum is the column index in row `R`.
- If `R<initRow`, the value of lmaxima at a local maximum in row `R` is the column index of the nearest local maximum in row `R+1`.

  To illustrate this, if inputmatrix is:

  ```
  3    2    5    3
  4    6    3    2
  4    4    7    4
  4    6    2    2
  ```

  lmaxima with `initRow = 4` and `regflag = false` is:

  ```
      0    0    2    0
      0    3    0    0
      0    0    2    0
      0    2    0    0
  ```

  lmaxima with `initRow = 3` and `regflag = false` is:

  ```
      0    0    2    0
      0    3    0    0
  ```

```
     0     0     3     0
     0     1     0     0
```

- If the local maximum in row R lies between two local maxima in row R+1, the value of the local maximum in row R is the higher column index in row R+1.

  To illustrate this, if inputmatrix is:

  ```
     0     0     1     0     0     0
     0     1     0     1     0     0
  ```

  lmaxima with `initRow = 2` and `regflag = false` is:

  ```
     0     0     4     0     0     0
     0     2     0     4     0     0
  ```

  lmaxima with `initRow = 1` and `regflag = false` is:

  ```
     0     0     3     0     0     0
     0     1     0     1     0     0
  ```

**`indices`**

Linear indices of the nonzero values of lmaxima. Use `ind2sub` to convert the linear indices to matrix row and column indices.

## Examples

Construct a 4-by-4 matrix with local maxima at the following row-column indices: (4,2), (3,3), (2,2), and (1,3). Set initrow to 4 and regflag to `false`.

```
inputmatrix = ...
[3     2     5     3
 4     6     3     2
 4     4     7     4
 4     6     2     2];
 [lmaxima,indices] = localmax(inputmatrix,4,false);
 lmaxima
```

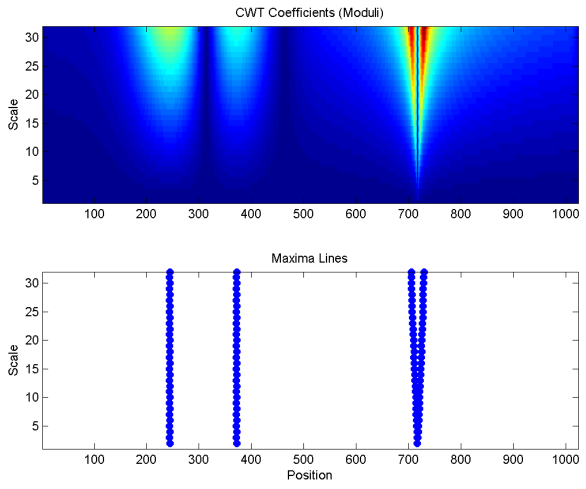
Because `localmax` operates on the absolute values of inputmatrix, setting `inputmatrix(4,2) = -inputmatrix(4,2)` produces an identical lmaxima.

```
 inputmatrix(4,2) = -inputmatrix(4,2);
```

```
[lmaxima1,indices1] = localmax(inputmatrix,4,false);
isequal(lmaxima,lmaxima1)
```

Determine the local maxima from the CWT of the `cuspamax` signal with the Haar wavelet. Plot the CWT coefficient moduli and the maxima lines.

```
load cuspamax;
x = 1:length(cuspamax);
scales = 1:32;
cfs = cwt(cuspamax,scales,'haar');
[lmaxima,indices] = localmax(cfs,[],false);
[iRow,iCol] = find(lmaxima);
subplot(211);
imagesc(abs(cfs)); axis xy;
axis([1 1024 1 32]);
ylabel('Scale'); title('CWT Coefficients (Moduli)');
subplot(212);
plot(x(iCol),scales(iRow),'marker','o','markerfacecolor',[0 0 1],...
    'linestyle','none');
xlabel('Position'); ylabel('Scale'); title('Maxima Lines');
axis([1 1024 1 32]);
```



## ls2filt

Transform lifting scheme to quadruplet of filters

### Syntax

```
[LoD,HiD,LoR,HiR] = ls2filt(LS)
```

### Description

`[LoD,HiD,LoR,HiR] = ls2filt(LS)` returns the four filters `LoD`, `HiD`, `LoR`, and `HiR` associated with the lifting scheme `LS`.

### Examples

```
% Start from the db2 wavelet and get the
% corresponding lifting scheme.
LS = liftwave('db2')

LS =

    'd'         [    -1.7321]    [ 0]
    'p'         [1x2 double]    [ 1]
    'd'         [          1]    [-1]
    [1.9319]    [     0.5176]      []

% Visualize the obtained lifting scheme.

displs(LS);

LS = {...
'd'             [ -1.73205081]               [0]
'p'             [ -0.06698730  0.43301270]  [1]
'd'             [  1.00000000]               [-1]
[  1.93185165]  [  0.51763809]               []
};

% Get the filters from the lifting scheme.
```

```
[LoD,HiD,LoR,HiR] = ls2filt(LS)

LoD =

   -0.1294    0.2241    0.8365    0.4830

HiD =

   -0.4830    0.8365   -0.2241   -0.1294

LoR =

    0.4830    0.8365    0.2241   -0.1294

HiR =

   -0.1294   -0.2241    0.8365   -0.4830

% Get the db2 filters using wfilters.
% You can check the equality.

[LoDref,HiDref,LoRref,HiRref] = wfilters('db2')

LoDref =

   -0.1294    0.2241    0.8365    0.4830

HiDref =

   -0.4830    0.8365   -0.2241   -0.1294

LoRref =

    0.4830    0.8365    0.2241   -0.1294

HiRref =

   -0.1294   -0.2241    0.8365   -0.4830
```

## See Also
`filt2ls` | `lsinfo`

# lsinfo

Lifting schemes information

## Syntax

```
lsinfo
```

## Description

`lsinfo` displays the following information about lifting schemes. A lifting scheme `LS` is a `N` x 3 cell array. The N-1 first rows of the array are elementary lifting steps (`ELS`). The last row gives the normalization of `LS`.

Each `ELS` has this format:

```
{type, coefficients, max_degree}
```

where `type` is `'p'` (primal) or `'d'` (dual), `coefficients` is a vector `C` of real numbers defining the coefficients of a Laurent polynomial `P` described below, and `max_degree` is the highest degree `d` of the monomials of `P`.

The Laurent polynomial `P` is of the form

*P(z) = C(1)*z^d + C(2)*z^(d−1) + ... + C(m)*z^(d−m+1)*

The lifting scheme `LS` is such that for

`k = 1:N-1`, `LS{k,:}` is an `ELS`, where

`LS{k,1}` is the lifting type `'p'` (primal) or `'d'` (dual).

`LS{k,2}` is the corresponding lifting filter.

`LS{k,3}` is the highest degree of the Laurent polynomial corresponding to the filter `LS{k,2}`.

`LS{N,1}` is the primal normalization (real number).

LS{N,2} is the dual normalization (real number).

LS{N,3} is not used.

Usually, the normalizations are such that LS{N,1}*LS{N,2} = 1.

For example, the lifting scheme associated with the wavelet db1 is

```
LS = {...
      'd'         [     -1]    [0]
      'p'         [0.5000]    [0]
      [1.4142]    [0.7071]     []
     }
```

## See Also

displs | laurpoly

# lwt

1-D lifting wavelet transform

## Syntax

```
[CA,CD] = lwt(X,W)
X_InPlace = lwt(X,W)
lwt(X,W,LEVEL)
X_InPlace = lwt(X,W,LEVEL,'typeDEC',typeDEC)
[CA,CD] = lwt(X,W,LEVEL,'typeDEC',typeDEC)
```

## Description

`lwt` performs a 1-D lifting wavelet decomposition with respect to a particular lifted wavelet that you specify.

`[CA,CD] = lwt(X,W)` computes the approximation coefficients vector `CA` and detail coefficients vector `CD`, obtained by a lifting wavelet decomposition, of the vector `X`. `W` is a lifted wavelet name (see `liftwave`).

`X_InPlace = lwt(X,W)` computes the approximation and detail coefficients. These coefficients are stored in place:

`CA = X_InPlace(1:2:end)` and `CD = X_InPlace(2:2:end)`

`lwt(X,W,LEVEL)` computes the lifting wavelet decomposition at level `LEVEL`.

`X_InPlace = lwt(X,W,LEVEL,'typeDEC',typeDEC)` or `[CA,CD] = lwt(X,W,LEVEL,'typeDEC',typeDEC)` with `typeDEC = 'w'` or `'wp'` computes the wavelet or the wavelet packet decomposition using lifting, at level `LEVEL`.

Instead of a lifted wavelet name, you may use the associated lifting scheme `LS`: `lwt(X,LS,...)` instead of `lwt(X,W,...)`.

For more information about lifting schemes, see `lsinfo`.

## Examples

```
% Start from the Haar wavelet and get the
% corresponding lifting scheme.
lshaar = liftwave('haar');

% Add a primal ELS to the lifting scheme.
els = {'p',[-0.125 0.125],0};
lsnew = addlift(lshaar,els);

% Perform LWT at level 1 of a simple signal.
x = 1:8;
[cA,cD] = lwt(x,lsnew)

cA =

    1.9445    4.9497    7.7782   10.6066


cD =

    0.7071    0.7071    0.7071    0.7071

% Perform integer LWT of the same signal.
lshaarInt = liftwave('haar','int2int');
lsnewInt = addlift(lshaarInt,els);
[cAint,cDint] = lwt(x,lsnewInt)

cAint =

     1     3     5     7


cDint =

     1     1     1     1
```

## More About

### Algorithms

This function uses the polyphase algorithm.

`lwt` reduces to `dwt` with zero-padding extension mode and without extra-coefficients.

## References

Strang, G.; T. Nguyen (1996), *Wavelets and filter banks*, Wellesley-Cambridge Press.

Sweldens, W. (1998), “The Lifting Scheme: a Construction of Second Generation of Wavelets,” *SIAM J. Math. Anal.*, 29 (2), pp. 511–546.

## See Also

`ilwt`

# lwt2

2-D lifting wavelet transform

## Syntax

```
[CA,CH,CV,CD] = lwt2(X,W)
X_InPlace = lwt2(X,LS)
lwt2(X,W,LEVEL)
X_InPlace = lwt2(X,W,LEVEL,'typeDEC',typeDEC)
[CA,CH,CV,CD] = LWT2(X,W,LEVEL,'typeDEC',typeDEC)
```

## Description

`lwt2` performs a 2-D lifting wavelet decomposition with respect to a particular lifted wavelet that you specify.

`[CA,CH,CV,CD] = lwt2(X,W)` computes the approximation coefficients matrix `CA` and detail coefficients matrices `CH`, `CV`, and `CD`, obtained by a lifting wavelet decomposition, of the matrix *X*. *W* is a lifted wavelet name (see `liftwave`).

`X_InPlace = lwt2(X,LS)` computes the approximation and detail coefficients. These coefficients are stored in place:

- `CA = X_InPlace(1:2:end,1:2:end)`
- `CH = X_InPlace(2:2:end,1:2:end)`
- `CV = X_InPlace(1:2:end,2:2:end)`
- `CD = X_InPlace(2:2:end,2:2:end)`

`lwt2(X,W,LEVEL)` computes the lifting wavelet decomposition at level `LEVEL`.

`X_InPlace = lwt2(X,W,LEVEL,'typeDEC',typeDEC)` or `[CA,CH,CV,CD] = LWT2(X,W,LEVEL,'typeDEC',typeDEC)` with `typeDEC = 'w'` or `'wp'` computes the wavelet or the wavelet packet decomposition using lifting, at level `LEVEL`.

Instead of a lifted wavelet name, you may use the associated lifting scheme `LS`: `lwt2(X,LS,...)` instead of `LWT2(X,W,...)`.

For more information about lifting schemes, see `lsinfo`.

## Examples

```
% Start from the Haar wavelet and get the
% corresponding lifting scheme.
lshaar = liftwave('haar');

% Add a primal ELS to the lifting scheme.
els = {'p',[-0.125 0.125],0};
lsnew = addlift(lshaar,els);

% Perform LWT at level 1 of a simple image.
x = reshape(1:16,4,4);
[cA,cH,cV,cD] = lwt2(x,lsnew)

cA =

    5.7500   22.7500
   10.0000   27.0000


cH =

    1.0000    1.0000
    1.0000    1.0000


cV =

    4.0000    4.0000
    4.0000    4.0000


cD =

     0     0
     0     0

% Perform integer LWT of the same image.
lshaarInt = liftwave('haar','int2int');
lsnewInt = addlift(lshaarInt,els);
[cAint,cHint,cVint,cDint] = lwt2(x,lsnewInt)

cAint =
     3    11
     5    13


cHint =

     1     1
     1     1


cVint =

     4     4
     4     4


cDint =

     0     0
     0     0
```

## More About

### Tips

When X represents an indexed image, X, as well as the output arrays cA,cH,cV,cD, or `X_InPlace` are `m`-by-`n` matrices. When X represents a truecolor image, it is an `m`-by-`n`-by-3 array, where each `m`-by-`n` matrix represents a red, green, or blue color plane concatenated along the third dimension.

For more information on image formats, see the `image` and `imfinfo` reference pages .

### Algorithms

This function implements the polyphase algorithm.

`lwt` reduces to `dwt` with zero-padding extension mode and without extra-coefficients.

## References

Strang, G.; T. Nguyen (1996), *Wavelets and filter banks*, Wellesley-Cambridge Press.

Sweldens, W. (1998), “The Lifting Scheme: a Construction of Second Generation of Wavelets,” *SIAM J. Math. Anal.*, 29 (2), pp. 511–546.

## See Also

`ilwt2`

# lwtcoef

Extract or reconstruct 1-D LWT wavelet coefficients

## Syntax

```
Y = lwtcoef(TYPE,XDEC,LS,LEVEL,LEVEXT)
Y = lwtcoef(TYPE,XDEC,W,LEVEL,LEVEXT)
```

## Description

`Y = lwtcoef(TYPE,XDEC,LS,LEVEL,LEVEXT)` returns the coefficients or the reconstructed coefficients of level `LEVEXT`, extracted from `XDEC`, the LWT decomposition at level `LEVEL` obtained with the lifting scheme `LS`.

The valid values for `TYPE` are

| TYPE Values | Description |
|---|---|
| `'a'` | Approximations |
| `'d'` | Details |
| `'ca'` | Coefficients of approximations |
| `'cd'` | Coefficients of details |

`Y = lwtcoef(TYPE,XDEC,W,LEVEL,LEVEXT)` returns the same output using `W`, which is the name of a lifted wavelet.

## Examples

```
% Start from the Haar wavelet and get the
% corresponding lifting scheme.
lshaar = liftwave('haar');

% Add a primal ELS to the lifting scheme.
els = {'p',[-0.125 0.125],0};
lsnew = addlift(lshaar,els);

% Perform LWT at level 2 of a simple signal.
```

```
x = 1:8;
xDec = lwt(x,lsnew,2)

xDec =

    4.3438    0.7071    2.1250    0.7071   13.0313    0.7071
    2.0000    0.7071

% Extract approximation coefficients of level 1.
ca1 = lwtcoef('ca',xDec,lsnew,2,1)

ca1 =

    1.9445    4.9497    7.7782   10.6066

% Reconstruct approximations and details.
a1 = lwtcoef('a',xDec,lsnew,2,1)

a1 =

    1.3750    1.3750    3.5000    3.5000    5.5000    5.5000
    7.5000    7.5000

a2 = lwtcoef('a',xDec,lsnew,2,2)

a2 =

    2.1719    2.1719    2.1719    2.1719    6.5156    6.5156
    6.5156    6.5156

d1 = lwtcoef('d',xDec,lsnew,2,1)

d1 =

   -0.3750    0.6250   -0.5000    0.5000   -0.5000    0.5000
   -0.5000    0.5000

d2 = lwtcoef('d',xDec,lsnew,2,2)

d2 =

   -0.7969   -0.7969    1.3281    1.3281   -1.0156   -1.0156
    0.9844    0.9844
```

```
% Check perfect reconstruction.
err = max(abs(x-a2-d2-d1))

err =

  9.9920e-016
```

## See Also

`ilwt` | `lwt`

# lwtcoef2

Extract or reconstruct 2-D LWT wavelet coefficients

## Syntax

```
Y = lwtcoef2(TYPE,XDEC,LS,LEVEL,LEVEXT)
Y = lwtcoef2(TYPE,XDEC,W,LEVEL,LEVEXT)
```

## Description

`Y = lwtcoef2(TYPE,XDEC,LS,LEVEL,LEVEXT)` returns the coefficients or the reconstructed coefficients of level `LEVEXT`, extracted from `XDEC`, the LWT decomposition at level `LEVEL` obtained with the lifting scheme `LS`.

The valid values for `TYPE` are listed in this table.

| TYPE Values | Description |
|---|---|
| `'a'` | Approximations |
| `'h'` | Horizontal details |
| `'v'` | Vertical details |
| `'d'` | Diagonal details |
| `'ca'` | Coefficients of approximations |
| `'ch'` | Coefficients of horizontal details |
| `'cv'` | Coefficients of vertical details |
| `'cd'` | Coefficients of diagonal details |

`Y = lwtcoef2(TYPE,XDEC,W,LEVEL,LEVEXT)` returns the same output using `W`, which is the name of a lifted wavelet.

## Examples

```
% Start from the Haar wavelet and get the
% corresponding lifting scheme.
lshaar = liftwave('haar');
```

```
% Add a primal ELS to the lifting scheme.
els = {'p',[-0.125 0.125],0};
lsnew = addlift(lshaar,els);

% Perform LWT at level 2 of a simple image.
x = reshape(1:16,4,4);
xDec = lwt2(x,lsnew,2)

xDec =

   27.4375    4.0000   17.0000    4.0000
    1.0000         0    1.0000         0
    4.2500    4.0000    0.0000    4.0000
    1.0000         0    1.0000         0

% Extract approximation coefficients of level 1.
ca1 = lwtcoef2('ca',xDec,lsnew,2,1)

ca1 =

    5.7500   22.7500
   10.0000   27.0000

% Reconstruct approximations and details.
a1 = lwtcoef2('a',xDec,lsnew,2,1)

a1 =

    2.8750    2.8750   11.3750   11.3750
    2.8750    2.8750   11.3750   11.3750
    5.0000    5.0000   13.5000   13.5000
    5.0000    5.0000   13.5000   13.5000

a2 = lwtcoef2('a',xDec,lsnew,2,2)

a2 =

    6.8594    6.8594    6.8594    6.8594
    6.8594    6.8594    6.8594    6.8594
    6.8594    6.8594    6.8594    6.8594
    6.8594    6.8594    6.8594    6.8594

h1 = lwtcoef2('h',xDec,lsnew,2,1)
```

```
h1 =

   -0.3750   -0.3750   -0.3750   -0.3750
    0.6250    0.6250    0.6250    0.6250
   -0.5000   -0.5000   -0.5000   -0.5000
    0.5000    0.5000    0.5000    0.5000

v1 = lwtcoef2('v',xDec,lsnew,2,1)

v1 =

   -1.5000    2.5000   -2.0000    2.0000
   -1.5000    2.5000   -2.0000    2.0000
   -1.5000    2.5000   -2.0000    2.0000
   -1.5000    2.5000   -2.0000    2.0000

d1 = lwtcoef2('d',xDec,lsnew,2,1)

d1 =

     0     0     0     0
     0     0     0     0
     0     0     0     0
     0     0     0     0

h2 = lwtcoef2('h',xDec,lsnew,2,2)

h2 =

   -0.7969   -0.7969   -0.7969   -0.7969
   -0.7969   -0.7969   -0.7969   -0.7969
    1.3281    1.3281    1.3281    1.3281
    1.3281    1.3281    1.3281    1.3281

v2 = lwtcoef2('v',xDec,lsnew,2,2)

v2 =

   -3.1875   -3.1875    5.3125    5.3125
   -3.1875   -3.1875    5.3125    5.3125
   -3.1875   -3.1875    5.3125    5.3125
   -3.1875   -3.1875    5.3125    5.3125
```

```
d2 = lwtcoef2('d',xDec,lsnew,2,2)

d2 =

  1.0e-015 *

    0.2498    0.2498   -0.4163   -0.4163
    0.2498    0.2498   -0.4163   -0.4163
   -0.4163   -0.4163    0.6939    0.6939
   -0.4163   -0.4163    0.6939    0.6939

% Check perfect reconstruction.
err = max(max(abs(x-a2-h2-v2-d2-h1-v1-d1)))

err =

  3.5527e-015
```

## More About

### Tips

If XDEC is obtained from an indexed image analysis or a truecolor image analysis, it is an `m`-by-`n` matrix or an `m`-by-`n`-by-3 array, respectively.

For more information on image formats, see the `image` and `imfinfo` reference pages.

## See Also

`ilwt2` | `lwt2`

# mdwtcluster

Multisignals 1-D clustering

## Syntax

```
S = mdwtcluster(X)
S = mdwtcluster(X,'PropName1',PropVal1,'PropName2',PropVal2,...)
```

## Description

`S = mdwtcluster(X)` constructs clusters from a hierarchical cluster tree. The input matrix *X* is decomposed in row direction using the DWT function with the `haar` wavelet and the maximum allowed level.

`S = mdwtcluster(X,'PropName1',PropVal1,'PropName2',PropVal2,...)` allows you to modify some properties. The valid choices for *`PropName`* are:

**Note:** `mdwtcluster` requires the Statistics and Machine Learning Toolbox™

| | |
|---|---|
| `'dirDec'` | `'r'` (row) or `'c'` (column). Default value is `'r'`. |
| `'level'` | Level of the DWT decomposition. Default value is: `level=fix(log2(size(X,d)))` where `d=1` or `d=2`, depending on the `dirDec` value. |
| `'wname'` | Wavelet name used for DWT. Default value is `'haar'`. |
| `'dwtEXTM'` | DWT extension mode (see `dwtmode`). |
| `'pdist'` | See Statistics and Machine Learning Toolbox `pdist` function. Default value is `'euclidean'`. |
| `'linkage'` | See Statistics and Machine Learning Toolbox `linkage` function. Default value is `'ward'`. |
| `'maxclust'` | Number of clusters. Default value is 6. The input variable can be a vector. |

| | |
|---|---|
| `'lst2clu'` | Cell array that contains the list of data to classify.<br><br>If `N` is the level of decomposition, the allowed name values for the cells are:<br><br>• `'s'` — Signal<br>• `'aj'` — Approximation at level `j`<br>• `'dj'` — Detail at level `j`<br>• `'caj'` — Coefficients of approximation at level `j`<br>• `'cdj'` — Coefficients of detail at level `j`<br><br>Default value is `{'s';'ca1';...;'caN'}`. |

The output structure S is such that for each partition `j`:

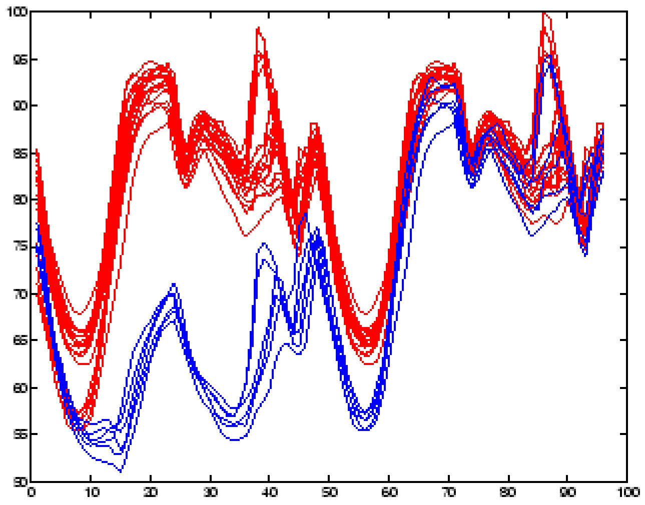
| | |
|---|---|
| `S.Idx(:,j)` | Contains the cluster numbers obtained from the hierarchical cluster tree (see `cluster` in the Statistics and Machine Learning Toolbox software). |
| `S.Incons(:,j)` | Contains the inconsistent values of each non-leaf node in the hierarchical cluster tree (see Statistics and Machine Learning Toolbox software function `inconsistent`). |
| `S.Corr(j)` | Contains the cophenetic correlation coefficients of the partition (see Statistics and Machine Learning Toolbox software function `cophenet`). |

**Note** If `maxclustVal` is a vector, then `IdxCLU` is a multidimensional array such that `IdxCLU(:,j,k)` contains the cluster numbers obtained from the hierarchical cluster tree for `k` clusters.

## Examples

```
load elecsig10
lst2clu = {'s','ca1','ca3','ca6'};

% Compute the structure resulting from multisignal clustering
S = mdwtcluster(signals,'maxclust',4,'lst2clu',lst2clu)
```

```
S = 

    IdxCLU: [70x4 double]
    Incons: [69x4 double]
      Corr: [0.7920 0.7926 0.7947 0.7631]

% Retrieve indices of clusters
IdxCLU = S.IdxCLU;

% Plot the first cluster
plot(signals(IdxCLU(:,1)==1,:)','r');
hold on;

% Plot the third clustering
plot(signals(IdxCLU(:,1)==3,:)','b')
```



```
% Check the equality of partitions
equalPART = isequal(IdxCLU(:,1),IdxCLU(:,3))

equalPART =

     1

% So we can see that we obtain the same partitions using
% coefficents of approximation at level 3 instead of original
% signals. Much less information is then used.
```

## See Also

`mdwtdec` | `wavedec`

# mdwtdec

Multisignal 1-D wavelet decomposition

## Syntax

```
DEC = mdwtdec(DIRDEC,X,LEV,WNAME)
DEC = mdwtdec(DIRDEC,X,LEV,LoD,HiD,LoR,HiR)
DEC = mdwtdec(...,'mode',EXTMODE)
```

## Description

`DEC = mdwtdec(DIRDEC,X,LEV,WNAME)` returns the wavelet decomposition at level `LEV` of each row (if `DIRDEC = 'r'`) or each column (if `DIRDEC = 'c'`) of matrix *X*, using the wavelet `WNAME`.

The output `DEC` is a structure with the following fields:

| | |
|---|---|
| `'dirDec'` | Direction indicator: `'r'` (row) or `'c'` (column) |
| `'level'` | Level of the DWT decomposition |
| `'wname'` | Wavelet name |
| `'dwtFilters'` | Structure with four fields `LoD`, `HiD`, `LoR` and `HiR` |
| `'dwtEXTM'` | DWT extension mode (see `dwtmode`) |
| `'dwtShift'` | DWT shift parameter (0 or 1) |
| `'dataSize'` | Size of `X` |
| `'ca'` | Approximation coefficients at level `LEV` |
| `'cd'` | Cell array of detail coefficients, from level 1 to level `LEV` |

Coefficients `cA` and `cD{k} (for k = 1 to LEV)` are matrices and are stored in rows if `DIRDEC` = `'r'` or in columns if `DIRDEC` = `'c'`.

`DEC = mdwtdec(DIRDEC,X,LEV,LoD,HiD,LoR,HiR)` uses the four filters instead of the wavelet name.

`DEC = mdwtdec(...,'mode',EXTMODE)` computes the wavelet decomposition with the `EXTMODE` extension mode that you specify (see `dwtmode` for the valid extension modes).

## Examples

```
% Load original 1D-multisignal.
load thinker

% Perform a decomposition at level 2 using wavelet db2.
dec = mdwtdec('r',X,2,'db2')
dec =
        dirDec: 'r'
         level: 2
         wname: 'db2'
    dwtFilters: [1x1 struct]
       dwtEXTM: 'sym'
      dwtShift: 0
      dataSize: [192 96]
            ca: [192x26 double]
            cd: {[192x49 double]  [192x26 double]}

% Compute the associated filters of db2 wavelet.
[LoD,HiD,LoR,HiR] = wfilters('db2');

% Perform a decomposition at level 2 using filters.
decBIS = mdwtdec('r',X,2,LoD,HiD,LoR,HiR)

decBIS =
        dirDec: 'r'
         level: 2
         wname: ''
    dwtFilters: [1x1 struct]
       dwtEXTM: 'sym'
      dwtShift: 0
      dataSize: [192 96]
            ca: [192x26 double]
            cd: {[192x49 double]  [192x26 double]}
```

## References

Daubechies, I. , *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics. SIAM Ed., 1992.

Mallat, S., “A theory for multiresolution signal decomposition: the wavelet representation,” *IEEE Pattern Anal. and Machine Intell.*, vol. 11, no. 7, 1989, pp. 674–693.

Meyer, Y. , *Ondelettes et opérateurs*, Tome 1, Hermann Ed. (English translation: *Wavelets and operators*, Cambridge Univ. Press. 1993.)

## See Also

`mdwtdec` | `wavedec`

# mdwtrec

Multisignal 1-D wavelet reconstruction

## Syntax

```
X = mdwtrec(DEC)
X = mdwtrec(DEC,IDXSIG)
Y = mdwtrec(DEC,TYPE,LEV)
A = mdwtrec(DEC,'a')
A = mdwtrec(DEC,'a',LEVDEC)
D = mdwtrec(DEC,'d')
CA = mdwtrec(DEC,'ca')
CA = mdwtrec(DEC,'ca',LEVDEC)
CD = mdwtrec(DEC,'cd',MODE)
CFS = mdwtrec(DEC,'cfs',MODE)
Y = mdwtrec(...,IDXSIG)
```

## Description

`X = mdwtrec(DEC)` returns the original matrix of signals, starting from the wavelet decomposition structure DEC (see `mdwtdec`).

`X = mdwtrec(DEC,IDXSIG)` reconstructs the signals whose indices are given by the vector `IDXSIG`.

`Y = mdwtrec(DEC,TYPE,LEV)` extracts or reconstructs the detail or approximation coefficients at level `LEV` depending on the `TYPE` value. The maximum value for `LEV` is `LEVDEC = DEC.level`.

When TYPE is equal to:

- `'cd'` or `'ca'`, coefficients of level LEV are extracted.
- `'d'` or `'a'`, coefficients of level LEV are reconstructed.
- `'a'` or `'ca'`, LEV must be such that $0 \le$ `LEV` $\le$ `LEVDEC`.
- `'d'` or `'cd'`, LEV must be such that $1 \le$ `LEV` $\le$ `LEVDEC`.

A = mdwtrec(DEC,'a') is equivalent to A = mdwtrec(DEC,'a',LEVDEC).

D = mdwtrec(DEC,'d') returns a matrix containing the sum of all the details, so that X = A + D.

CA = mdwtrec(DEC,'ca') is equivalent to CA = mdwtrec(DEC,'ca',LEVDEC).

CD = mdwtrec(DEC,'cd',MODE) returns a matrix containing all the detail coefficients.

CFS = mdwtrec(DEC,'cfs',MODE) returns a matrix containing all the coefficients.

For MODE = 'descend' the coefficients are concatened from level LEVDEC to level 1 and MODE = 'descend' concatenates from level 1 to level LEVDEC). The default is MODE = 'descend'. The concatenation is made row-wise if DEC.dirDEC = 'r' or column-wise if DEC.dirDEC = 'c'.

Y = mdwtrec(...,IDXSIG) extracts or reconstructs the detail or the approximation coefficients for the signals whose indices are given by the vector IDXSIG.

## Examples

```
% Load original 1D-multisignal.
load thinker

% Perform a decomposition at level 2 using wavelet db2.
dec = mdwtdec('r',X,2,'db2');

% Reconstruct the original matrix of signals, starting from
% the wavelet decomposition structure dec.
XR = mdwtrec(dec);

% Compute the reconstruction error.
errREC = max(max(abs(X-XR)))

errREC =
  2.1026e-010

% Reconstruct the original signal 31, the corresponding
% approximation at level 2, details at levels 1 and 2.
Y = mdwtrec(dec,31);
A2 = mdwtrec(dec,'a',2,31);
D2 = mdwtrec(dec,'d',2,31);
```

```
D1 = mdwtrec(dec,'d',1,31);

% Compute the reconstruction error for signal 31.
errREC = max(abs(Y-A2-D2-D1))

errREC =
  6.8390e-014
```

## References

Daubechies, I., *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics. SIAM Ed., 1992.

Mallat, S., “A theory for multiresolution signal decomposition: the wavelet representation,” *IEEE Pattern Anal. and Machine Intell.*, vol. 11, no. 7, 1989, pp. 674–693.

Meyer, Y., *Ondelettes et opérateurs*, Tome 1, Hermann Ed. (English translation: *Wavelets and operators*, Cambridge Univ. Press. 1993.)

### See Also

`mdwtdec` | `waverec`

## measerr

Approximation quality metrics

### Syntax

```
[PSNR,MSE,MAXERR,L2RAT] = measerr(X,XAPP)
[...] = measerr(...,BPS)
```

### Description

`[PSNR,MSE,MAXERR,L2RAT] = measerr(X,XAPP)` returns the peak signal-to-noise ratio, PSNR, mean square error, MSE, maximum squared error, MAXERR, and ratio of squared norms, L2RAT, for an input signal or image, X, and its approximation, XAPP.

`[...] = measerr(...,BPS)` uses the bits per sample, BPS, to determine the peak signal-to-noise ratio.

### Input Arguments

#### X

X is a real-valued signal or image.

#### XAPP

XAPP is a real-valued signal or image approximation with a size equal to that of the input data, X.

#### BPS

BPS is the number of bits per sample in the data.

**Default:** 8

## Output Arguments

### PSNR

PSNR is the peak signal-to-noise ratio in decibels (dB). The PSNR is only meaningful for data encoded in terms of bits per sample, or bits per pixel. For example, an image with 8 bits per pixel contains integers from 0 to 255.

### MSE

The mean square error (MSE) is the squared norm of the difference between the data and the approximation divided by the number of elements.

### MAXERR

MAXERR is the maximum absolute squared deviation of the data, X, from the approximation, XAPP.

### L2RAT

L2RAT is the ratio of the squared norm of the signal or image approximation, XAPP, to the input signal or image, X.

## Examples

Approximate an image and calculate approximation quality metrics.

```
load woman;
Xapp = X;
Xapp(X<=50) = 1;
[psnr,mse,maxerr,L2rat] = measerr(X,Xapp);
figure; colormap(map);
subplot(1,2,1); image(X);
subplot(1,2,2); image(Xapp);
```

Measure approximation quality in an RGB image.

```
X = imread('africasculpt.jpg');
Xapp = X;
Xapp(X<=100) = 1;
[psnr,mse,maxerr,L2rat] = measerr (X,Xapp)
```

```
figure;
subplot(1,2,1); image(X);
subplot(1,2,2); image(Xapp);
```

## More About

### Peak Signal to Noise Ratio (PSNR)

The following equation defines the PSNR:

$$20\log_{10}(\frac{2^B-1}{\sqrt{MSE}})$$

where $MSE$ represents the mean square error and $B$ represents the bits per sample.

### Mean Square Error (MSE)

The mean square error between a signal or image, $X$, and an approximation, $Y$, is the squared norm of the difference divided by the number of elements in the signal or image:

$$\frac{||X-Y||^2}{N}$$

## References

Huynh-Thu, Q.*Scope of validity of PSNR in image/video quality assessment*, Electronics Letters, 44, 2008, pp. 800–801.

## See Also

`wden` | `wdencmp`

# mexihat

Mexican hat wavelet

## Syntax

```
[PSI,X] = mexihat(LB,UB,N)
```

## Description

`[PSI,X] = mexihat(LB,UB,N)` returns values of the Mexican hat wavelet on an `N` point regular grid, `X`, in the interval `[LB,UB]`.

Output arguments are the wavelet function `PSI` computed on the grid `X`.

This wavelet has [-5 5] as effective support. Although [-5 5] is the correct theoretical effective support, a wider effective support, [-8 8], is used in the computation to provide more accurate results.

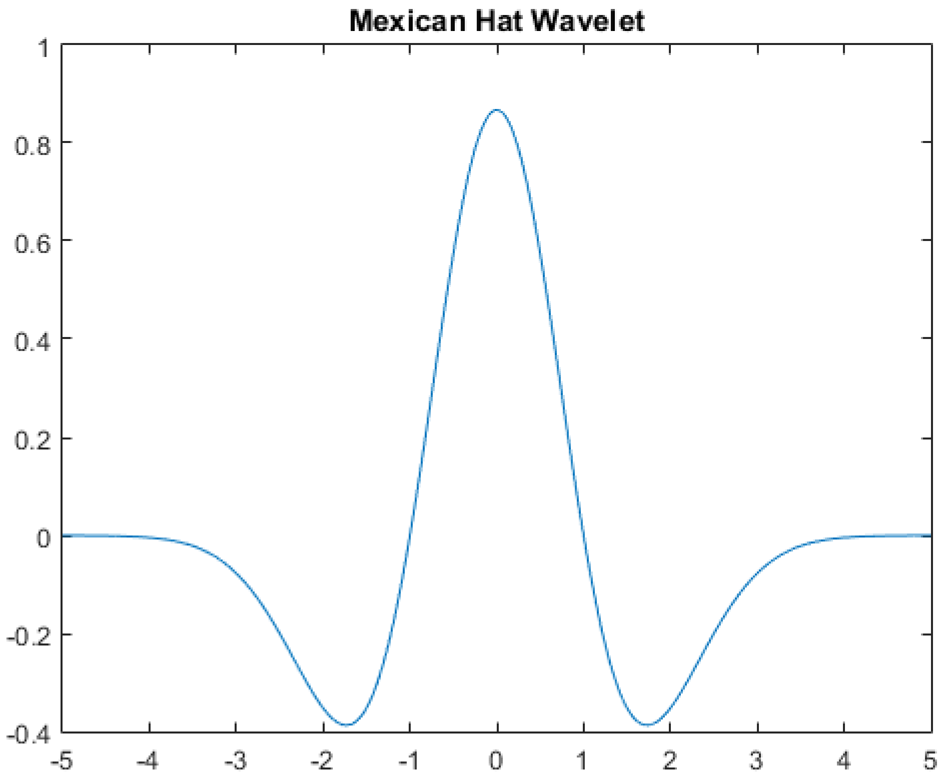
This function is proportional to the second derivative function of the Gaussian probability density function.

## Examples

### Mexican Hat Wavelet

Create a Mexican hat wavelet with support on [-5,5]. Use 1,000 sample points. Plot the result.

```
lb = -5;
ub = 5;
N = 1000;
[psi,xval] = mexihat(lb,ub,N);
plot(xval,psi)
title('Mexican Hat Wavelet');
```



## See Also

`waveinfo`

# meyer

Meyer wavelet

## Syntax

```
[PHI,PSI,T] = meyer(LB,UB,N)
```

## Description

`[PHI,PSI,T] = meyer(LB,UB,N)` returns Meyer scaling and wavelet functions evaluated on an *N* point regular grid in the interval `[LB,UB]`.

*N* must be a power of two.

Output arguments are the scaling function `PHI` and the wavelet function `PSI` computed on the grid *T*. These functions have [-8 8] as effective support.

If only one function is required, a fourth argument is allowed:

```
[PHI,T] = meyer(LB,UB,N,'phi')
[PSI,T] = meyer(LB,UB,N,'psi')
```

When the fourth argument is used, but not equal to `'phi'` or `'psi'`, outputs are the same as in the main option.

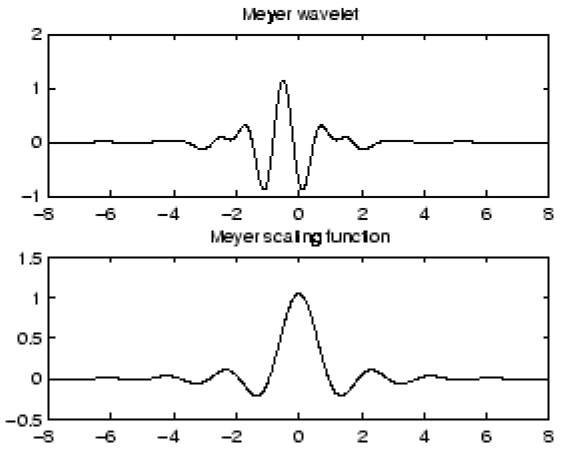
The Meyer wavelet and scaling function are defined in the frequency domain.

By changing the auxiliary function (see `meyeraux` for more information), you get a family of different wavelets.

## Examples

```
% Set effective support and grid parameters.
lb = -8; ub = 8; n = 1024;

% Compute and plot Meyer wavelet and scaling functions.
[phi,psi,x] = meyer(lb,ub,n);
```

```
subplot(211), plot(x,psi)
title('Meyer wavelet')
subplot(212), plot(x,phi)
title('Meyer scaling function')
```



## More About

### Algorithms

Starting from an explicit form of the Fourier transform $\hat{\phi}$ of ϕ, `meyer` computes the values of $\hat{\phi}$ on a regular grid, and then the values of ϕ are computed using `instdfft`, the inverse nonstandard discrete FFT.

The procedure for ψ is along the same lines.

## References

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics, SIAM Ed., pp. 117–119, 137, 152.

### See Also

`meyeraux` | `wavefun` | `waveinfo`

# meyeraux

Meyer wavelet auxiliary function

## Syntax

```
Y = meyeraux(X)
```

## Description

`Y = meyeraux(X)` returns values of the auxiliary function used for Meyer wavelet generation evaluated at the elements of the vector or matrix *X*.

The function is

$$35x^4 - 84x^5 + 70x^6 - 20x^7$$

### See Also

`meyer`

## morlet

Morlet wavelet

### Syntax

```
[PSI,X] = morlet(LB,UB,N)
```

### Description

`[PSI,X] = morlet(LB,UB,N)` returns values of the Morlet wavelet on an `N` point regular grid in the interval `[LB,UB]`.

Output arguments are the wavelet function `PSI` computed on the grid `X`, and the grid `X`.

This wavelet has [-4 4] as effective support. Although [-4 4] is the correct theoretical effective support, a wider effective support, [-8 8], is used in the computation to provide more accurate results.

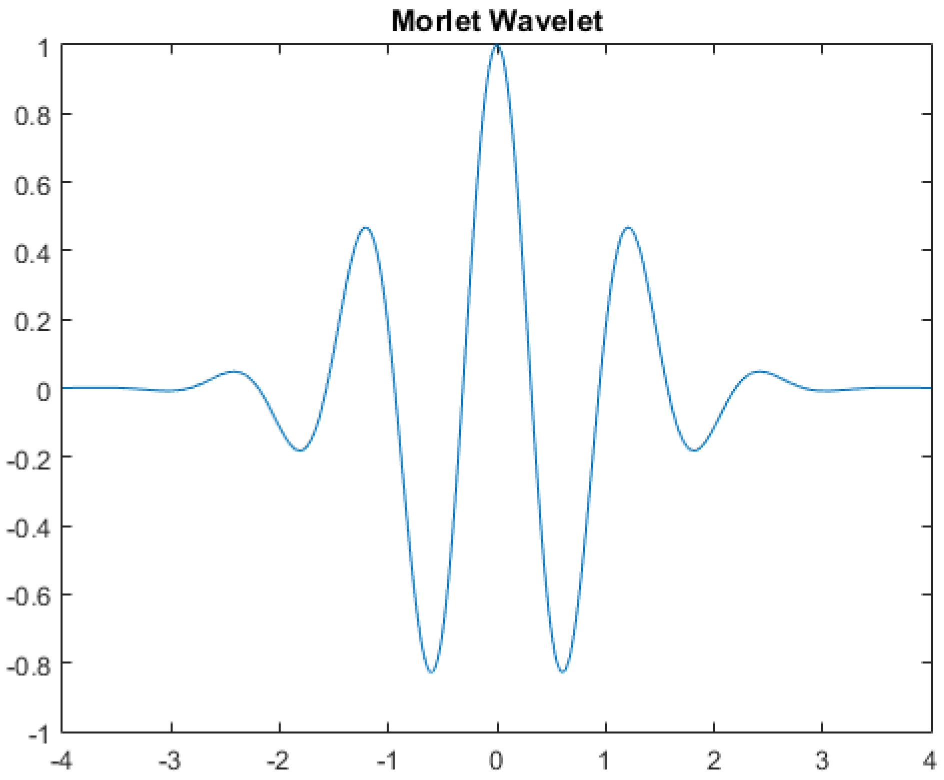
$$\psi(x) = e^{-x^2/2}\cos(5x)$$

### Examples

#### Morlet Wavelet

This example shows how to create a Morlet wavelet. The wavelet has an effective support of [-4,4]. Use 1,000 sample points.

```
lb = -4;
ub = 4;
n = 1000;
[psi,xval] = morlet(lb,ub,n);
plot(xval,psi)
title('Morlet Wavelet');
```



## See Also

`waveinfo`

# mswcmp

Multisignal 1-D compression using wavelets

## Syntax

```
[XC,DECCMP,THRESH] = mswcmp('cmp',DEC,METH)
[XC,DECCMP,THRESH] = mswcmp('cmp',DEC,METH,PARAM)
[XC,THRESH] = mswcmp('cmpsig',...)
[DECCMP,THRESH] = mswcmp('cmpdec',...)
THRESH = mswcmp('thr',...)
[...] = mswcmp(OPTION,DIRDEC,X,WNAME,LEV,METH)
[...] = mswcmp(OPTION,DIRDEC,X,WNAME,LEV,METH,PARAM)
[...] = mswcmp(...,S_OR_H)
[...] = mswcmp(...,S_OR_H,KEEPAPP)
[...] = mswcmp(...,S_OR_H,KEEPAPP,IDXSIG)
```

## Description

`mswcmp` computes thresholds and, depending on the selected option, performs compression of 1-D signals using wavelets.

`[XC,DECCMP,THRESH] = mswcmp('cmp',DEC,METH)` or `[XC,DECCMP,THRESH] = mswcmp('cmp',DEC,METH,PARAM)` returns a compressed (indicated by `'cmp'` input) version `XC` of the original multisignal matrix `X`, whose wavelet decomposition structure is `DEC`. The output `XC` is obtained by thresholding the wavelet coefficients: `DECCMP`, which is the wavelet decomposition associated with `XC` (see `mdwtdec`), and `THRESH` is the matrix of threshold values. The input `METH` is the name of the compression method and `PARAM` is the associated parameter, if required.

Valid compression methods METH are shown in the following tables. For methods that use an associated parameter, the range of allowable `PARAM` values is also shown.

| | |
|---|---|
| `'rem_n0'` | Remove near 0 |
| `'bal_sn'` | Balance sparsity-norm |

| | |
|---|---|
| `'sqrtbal_sn'` | Balance sparsity-norm (`sqrt`) |
| `'scarce'` | Scarce, `PARAM` (any number) |
| `'scarcehi'` | Scarce high, `2.5` ≤ `PARAM` ≤ `10` |
| `'scarceme'` | Scarce medium, `1.5` ≤ `PARAM` ≤ `2.5` |
| `'scarcelo'` | Scarce low, `1` ≤ `PARAM` ≤ `2` |

`PARAM` is a sparsity parameter, and it should be such that: `1` ≤ `PARAM` ≤ `10`. For scarce method no control is done.

| | |
|---|---|
| `'L2_perf'` | Energy ratio |
| `'N0_perf'` | Zero coefficients ratio |

`PARAM` is a real number which represents the required performance:

`0` ≤ `PARAM` ≤ `100`.

| | |
|---|---|
| `'glb_thr'` | Global threshold |

`PARAM` is a real positive number.

| | |
|---|---|
| `'man_thr'` | Manual method |

`PARAM` is an `NbSIG`-by-`NbLEV` matrix or `NbSIG`-by-(`NbLEV+1`) matrix such that:

- - `PARAM(i,j)` is the threshold for the detail coefficients of level `j` for the ith signal (`1` ≤ `j` ≤ `NbLEV`).
- - `PARAM(i,NbLEV+1)` is the threshold for the approximation coefficients for the `ith` signal (if `KEEPAPP` is `0`).

Where `NbSIG` is the number of signals and `NbLEV` the number of levels of decomposition.

`[XC,THRESH] = mswcmp('cmpsig',...)` or
`[DECCMP,THRESH] = mswcmp('cmpdec',...)` or
`THRESH = mswcmp('thr',...)` Instead of the `'cmp'` input `OPTION`, you can use `'cmpsig'`, `'cmpdec'` or `'thr'` to select other output arguments. `'thr'` returns the computed thresholds, but compression is not performed.

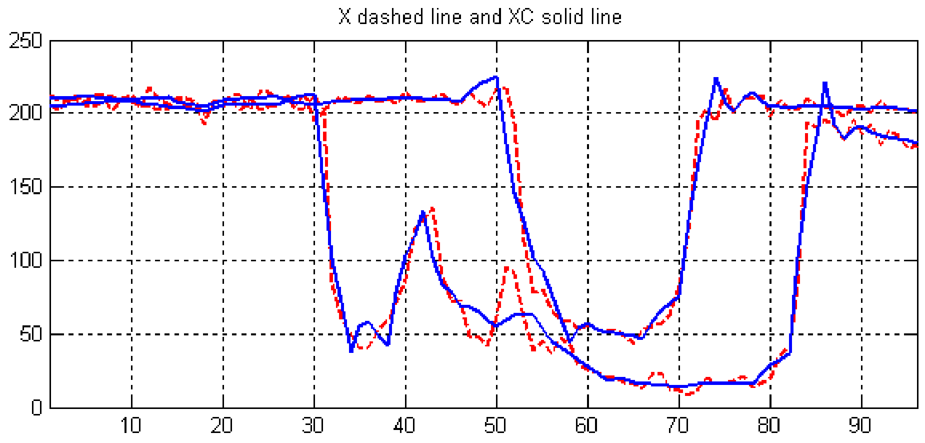
`[...] = mswcmp(OPTION,DIRDEC,X,WNAME,LEV,METH)`
`[...] = mswcmp(OPTION,DIRDEC,X,WNAME,LEV,METH,PARAM)` The decomposition structure input argument `DEC` can be replaced by four arguments: `DIRDEC`, `X`, `WNAME`, and `LEV`. Before performing a compression or computing thresholds, the multisignal matrix `X` is decomposed at level `LEV` using the wavelet `WNAME`, in the direction `DIRDEC`.

`[...] = mswcmp(...,S_OR_H)`
`[...] = mswcmp(...,S_OR_H,KEEPAPP)`
`[...] = mswcmp(...,S_OR_H,KEEPAPP,IDXSIG)` Three more optional inputs may be used:

- `S_OR_H ('s' or 'h')` stands for soft or hard thresholding (see `mswthresh` for more details). Default is `'h'`.
- `KEEPAPP (true or false)` indicates whether to keep approximation coefficients (`true`) or not (`false`). Default is `false`.
- `IDXSIG` is a vector which contains the indices of the initial signals, or the string `'all'`. Default is `'all'`.

## Examples

```
% Load original 1D-multisignal.
load thinker

% Perform a decomposition at level 2 using wavelet db2.
dec = mdwtdec('r',X,2,'db2');

% Compress the signals to obtain a percentage of zeros
% near 95% for the wavelet coefficients.
[XC,decCMP,THRESH] = mswcmp('cmp',dec,'N0_perf',95);
[Ecmp,PECcmp,PECFScmp] = wdecenergy(decCMP);

% Plot the original signals 1 and 31, and
% the corresponding compressed signals.
figure;
plot(X([1 31],:)','r--','linewidth',2);   hold on
plot(XC([1 31],:)','b','linewidth',2);
grid; set(gca,'Xlim',[1,96])
title('X dashed line and XC solid line')
```



## References

Birgé L.; P. Massart (1997), "From Model Selection to Adaptive Estimation," in D. Pollard (ed), *Festchrift for L. Le Cam*, Springer, pp. 55–88.

DeVore, R.A.; B. Jawerth, B.J. Lucier (1992), "Image Compression Through Wavelet Transform Coding," *IEEE Trans. on Inf. Theory*, vol. 38, No 2, pp. 719–746.

Donoho, D.L. (1993), "Progress in Wavelet Analysis and WVD: a Ten Minute Tour," in Progress in Wavelet Analysis and Applications, Y. Meyer, S. Roques, pp. 109–128. Frontières Ed.

Donoho, D.L.; I.M. Johnstone(1994), "Ideal Spatial Adaptation by Wavelet Shrinkage," *Biometrika*, vol. 81, pp. 425–455.

Donoho, D.L.; I.M. Johnstone, G. Kerkyacharian, D. Picard (1995), "Wavelet Shrinkage: Asymptopia," *Jour. Roy. Stat. Soc.*, *series B*, vol. 57 no. 2, pp. 301–369.

Donoho, D.L.; I.M. Johnstone, "Ideal De-noising in an Orthonormal Basis Chosen from a Library of Bases," C.R.A.S. Paris, t. 319, Ser. I, pp. 1317–1322.

Donoho, D.L. (1995), "De-noising by Soft-thresholding," *IEEE Trans. on Inf. Theory*, 41, 3, pp. 613–627.

## See Also

`mdwtdec` | `mdwtrec` | `mswthresh` | `wthresh`

## mswcmpscr

Multisignal 1-D wavelet compression scores

### Syntax

```
[THR,L2SCR,NOSCR,IDXSORT] = mswcmpscr(DEC)
```

### Description

`[THR,L2SCR,NOSCR,IDXSORT] = mswcmpscr(DEC)` computes four matrices: thresholds `THR`, compression scores `L2SCR` and `NOSCR`, and indices `IDXSORT`. The decomposition `DEC` corresponds to a matrix of wavelet coefficients `CFS` obtained by concatenation of detail and (optionally) approximation coefficients, where

`CFS = [cd{DEC.level}, ... , cd{1}]` or `CFS = [ca, cd{DEC.level}, ... , cd{1}]`

The concatenation is made rowwise if `DEC.dirDec` is equal to `'r'` or columnwise if `DEC.dirDec` is equal to `'c'` .

If `NbSIG` is the number of original signals and `NbCFS` the number of coefficients for each signal (all or only the detail coefficients), then `CFS` is an `NbSIG`-by-`NbCFS` matrix. Therefore,

- `THR`, `L2SCR`, `NOSCR` are `NbSIG`-by-(`NbCFS+1`) matrices
- `IDXSORT` is an `NbSIG`-by-`NbCFS` matrix
- `THR(:,2:end)` is equal to `CFS` sorted by row in ascending order with respect to the absolute value.
- For each row, `IDXSORT` contains the order of coefficients and `THR(:,1)=0`.

For the ith signal:

- `L2SCR(i,j)` is the percentage of preserved energy (L2-norm), corresponding to a threshold equal to `CFS(i,j-1)` ($2 \leq j \leq$ `NbCFS`), and `L2SCR(:,1)=100`.
- `NOSCR(i,j)` is the percentage of zeros corresponding to a threshold equal to `CFS(i,j-1)` ($2 \leq j \leq$ `NbCFS`), and `NOSCR(:,1)=0`.

Three more optional inputs may be used:

```
[...] = mswcmpscr(...,S_OR_H,KEEPAPP,IDXSIG)
```

- `S_OR_H ('s' or 'h')` stands for soft or hard thresholding (see `mswthresh` for more details).
- `KEEPAPP (true or false)` indicates whether to keep approximation coefficients (`true`) or not (`false`).
- `IDXSIG` is a vector that contains the indices of the initial signals, or the string `'all'`.

The defaults are, respectively, `'h'`, false and `'all'`.

## Examples

```
% Load original 1D-multisignal.
load thinker

% Perform a decomposition at level 2 using wavelet db2.
dec = mdwtdec('r',X,2,'db2');

% Compute compression performances for soft an hard thresholding.
[THR_S,L2SCR_S,NOSCR_S] = mswcmpscr(dec,'s');
[THR_H,L2SCR_H,NOSCR_H] = mswcmpscr(dec,'h');
```

## References

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics. SIAM Ed.

Mallat, S. (1989), “A theory for multiresolution signal decomposition: the wavelet representation,” *IEEE Pattern Anal. and Machine Intell.*, vol. 11, no. 7, pp. 674–693.

Meyer, Y. (1990), *Ondelettes et opérateurs*, Tome 1, Hermann Ed. (English translation: *Wavelets and operators*, Cambridge Univ. Press. 1993.)

### See Also

`mdwtdec` | `mdwtrec` | `ddencmp` | `wdencmp`

## mswcmptp

Multisignal 1-D compression thresholds and performances

### Syntax

```
[THR_VAL,L2_Perf,NO_Perf] = mswcmptp(DEC,METH)
[THR_VAL,L2_Perf,NO_Perf] = mswcmptp(DEC,METH,PARAM)
```

### Description

`[THR_VAL,L2_Perf,NO_Perf] = mswcmptp(DEC,METH)` or `[THR_VAL,L2_Perf,NO_Perf] = mswcmptp(DEC,METH,PARAM)` computes the vectors `THR_VAL`, `L2_Perf` and `NO_Perf` obtained after a compression using the `METH` method and, if required, the `PARAM` parameter (see `mswcmp` for more information on `METH` and `PARAM`).

For the ith signal:

- `THR_VAL(i)` is the threshold applied to the wavelet coefficients. For a level dependent method, `THR_VAL(i,j)` is the threshold applied to the detail coefficients at level `j`
- `L2_Perf(i)` is the percentage of energy (L2_norm) preserved after compression.
- `NO_Perf(i)` is the percentage of zeros obtained after compression.

You can use three more optional inputs:

```
[...] = mswcmptp(...,S_OR_H,KEEPAPP,IDXSIG)
```

- `S_OR_H ('s' or 'h')` stands for soft or hard thresholding (see `mswthresh` for more details).
- `KEEPAPP (true or false)` indicates whether to keep approximation coefficients (`true`) or not (`false`)
- `IDXSIG` is a vector which contains the indices of the initial signals, or the string `'all'`.

The defaults are, respectively, `'h'`, false and `'all'`.

## Examples

```
% Load original 1D-multisignal.
load thinker

% Perform a decomposition at level 2 using wavelet db2.
dec = mdwtdec('r',X,2,'db2');

% Compute compression thresholds and exact performances
% obtained after a compression using the method 'NO_perf' and
% requiring a percentage of zeros near 95% for the wavelet
% coefficients.
[THR_VAL,L2_Perf,NO_Perf] = mswcmptp(dec,'NO_perf',95);
```

## References

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics. SIAM Ed.

Mallat, S. (1989), “A theory for multiresolution signal decomposition: the wavelet representation,” *IEEE Pattern Anal. and Machine Intell.*, vol. 11, no. 7, pp. 674–693.

Meyer, Y. (1990), *Ondelettes et opérateurs*, Tome 1, Hermann Ed. (English translation: *Wavelets and operators*, Cambridge Univ. Press. 1993.)

### See Also

`mdwtdec` | `mdwtrec` | `ddencmp` | `wdencmp`

# mswden

Multisignal 1-D denoising using wavelets

## Syntax

```
[XD,DECDEN,THRESH] = mswden('den',...)
[XD,THRESH] = mswden('densig',...)
[DECDEN,THRESH] = mswden('dendec',...)
THRESH = mswden('thr',...)
[...] = mswden(OPTION,DIRDEC,X,WNAME,LEV,METH,PARAM)
[...] = mswden(...,S_OR_H)
[...] = mswden(...,S_OR_H,KEEPAPP)
[...] = mswden(...,S_OR_H,KEEPAPP,IDXSIG)
```

## Description

`mswden` computes thresholds and, depending on the selected option, performs denoising of 1-D signals using wavelets.

`[XD,DECDEN,THRESH] = mswden('den',...)` returns a denoised version `XD` of the original multisignal matrix `X`, whose wavelet decomposition structure is `DEC`. The output `XD` is obtained by thresholding the wavelet coefficients, `DECDEN` is the wavelet decomposition associated to `XD` (see `mdwtdec`), and `THRESH` is the matrix of threshold values. The input `METH` is the name of the denoising method and `PARAM` is the associated parameter, if required.

Valid denoising methods `METH` and associated parameters `PARAM` are:

| | |
|---|---|
| `'rigrsure'` | Principle of Stein's Unbiased Risk |
| `'heursure'` | Heuristic variant of the first option |
| `'sqtwolog'` | Universal threshold `sqrt(2*log(.))` |
| `'minimaxi'` | Minimax thresholding (see `thselect`) |

For these methods `PARAM` defines the multiplicative threshold rescaling:

| | |
|---|---|
| `'one'` | No rescaling |
| `'sln'` | Rescaling using a single estimation of level noise based on first level coefficients |
| `'mln'` | Rescaling using a level dependent estimation of level noise |

## Penalization methods

| | |
|---|---|
| `'penal'` | Penal |
| `'penalhi'` | Penal high, 2.5 ℜ≤ PARAM ℜ≤ 10 |
| `'penalme'` | Penal medium, 1.5 ℜ≤ PARAM ℜ≤ 2.5 |
| `'penallo'` | Penal low, 1 ℜ≤ PARAM ℜ≤ 2 |

`PARAM` is a sparsity parameter, and it should be such that: 1 ≤ `PARAM` ≤ 10. For `penal` method, no control is done.

## Manual method

| | |
|---|---|
| `'man_thr'` | Manual method |

`PARAM` is an `NbSIG`-by-`NbLEV` matrix or `NbSIG`-by-(`NbLEV+1`) matrix such that:

- `PARAM(i,j)` is the threshold for the detail coefficients of level `j` for the ith signal (1 ≤ `j` ≤ `NbLEV`).
- `PARAM(i,NbLEV+1)` is the threshold for the approximation coefficients for the `i`th signal (if `KEEPAPP` is `0`).

where `NbSIG` is the number of signals and `NbLEV` the number of levels of decomposition.

Instead of the `'den'` input `OPTION`, you can use `'densig'`, `'dendec'` or `'thr'` `OPTION` to select output arguments:

`[XD,THRESH] = mswden('densig',...)` or `[DECDEN,THRESH] = mswden('dendec',...)`

`THRESH = mswden('thr',...)` returns the computed thresholds, but denoising is not performed.

The decomposition structure input argument `DEC` can be replaced by four arguments: `DIRDEC`, `X`, `WNAME` and `LEV`.

`[...] = mswden(OPTION,DIRDEC,X,WNAME,LEV,METH,PARAM)` before performing a denoising or computing thresholds, the multisignal matrix `X` is decomposed at level `LEV` using the wavelet `WNAME`, in the direction `DIRDEC`.

You can use three more optional inputs:

`[...] = mswden(...,S_OR_H)` or
`[...] = mswden(...,S_OR_H,KEEPAPP)` or
`[...] = mswden(...,S_OR_H,KEEPAPP,IDXSIG)`

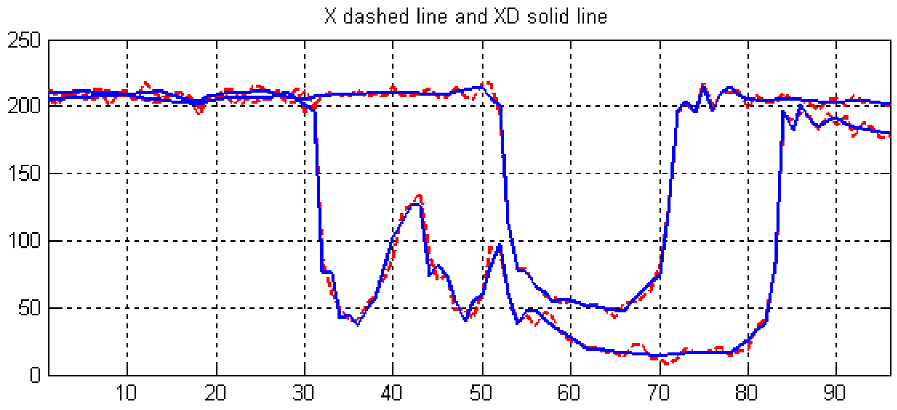
- `S_OR_H ('s' or 'h')` stands for soft or hard thresholding (see `mswthresh` for more details).
- `KEEPAPP (true or false)` indicates whether to keep approximation coefficients (`true`) or not (`false`).
- `IDXSIG` is a vector that contains the indices of the initial signals, or the string `'all'`.

The defaults are, respectively, `'h'`, `false` and `'all'`.

## Examples

```
% Load original 1D-multisignal.
load thinker

% Perform a decomposition at level 2 using the wavelet db2.
dec = mdwtdec('r',X,2,'db2');

% Denoise signals using the universal method
% of thresholding (sqtwolog) and the 'sln'
% threshold rescaling (with a single estimation
% of level noise, based on first level coefficients).
[XD,decDEN,THRESH] = mswden('den',dec,'sqtwolog','sln');

% Plot the original signals 1 and 31, and the
% corresponding denoised signals.
figure;
plot(X([1 31],:)','r--','linewidth',2);   hold on
plot(XD([1 31],:)','b','linewidth',2);
```

```
grid; set(gca,'Xlim',[1,96])
title('X dashed line and XD solid line')
```



## References

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## See Also

`mdwtdec` | `mdwtrec` | `mswthresh` | `wthresh`

# mswthresh

Perform multisignal 1-D thresholding

## Syntax

```
Y = mswthresh(X,SORH,T)
Y = mswthresh(X,SORH,T,'c')
Y = mswthresh(X,'s',T)
Y = mswthresh(X,'h',T)
```

## Description

`Y = mswthresh(X,SORH,T)` returns soft (if `SORH='s'`) or hard (if `SORH='h'`) `T`-thresholding of the input matrix `X`. `T` can be a single value, a matrix of the same size as `X` or a vector. In this last case, thresholding is performed rowwise and `LT = length(T)` must be such that `size(X,1)` ≤ `LT`.

`Y = mswthresh(X,SORH,T,'c')` performs a columnwise thresholding and `size(X,2)` ≤ `LT`.

`Y = mswthresh(X,'s',T)` returns `Y = SIGN(X).(|X|-T)+`, soft thresholding is shrinkage.

`Y = mswthresh(X,'h',T)` returns `Y = X.1_(|X|>T)`, hard thresholding is cruder.

### See Also

`mswden` | `mswcmp` | `wthresh` | `wden` | `wdencmp` | `wpdencmp`

# nodeasc

Node ascendants

## Syntax

```
A = nodeasc(T,N)
```

## Description

`nodeasc` is a tree-management utility.

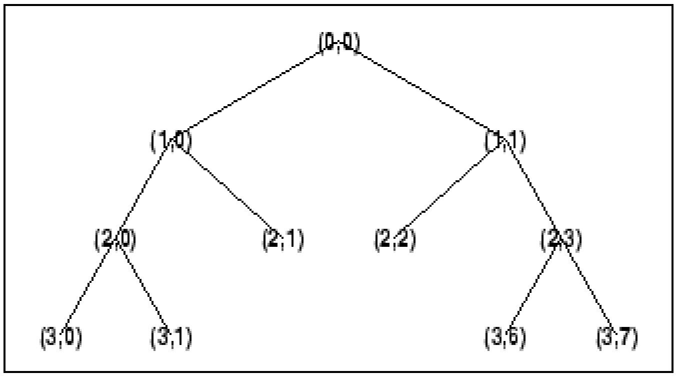
`A = nodeasc(T,N)` returns the indices of all the ascendants of the node *N* in the tree *T* where N can be the index node or the depth and position of the node. A is a column vector with A(1) = index of node *N*.

`A = nodeasc(T,N,'deppos')` is a matrix, which contains the depths and positions of all ascendants. `A(i,1)` is the depth of the `i`-th ascendant and `A(i,2)` is the position of the `i`-th ascendant.

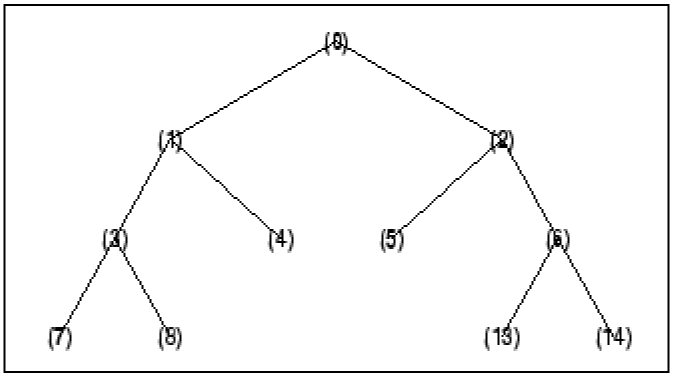
The nodes are numbered from left to right and from top to bottom. The root index is 0.

## Examples

```
% Create binary tree of depth 3.
t = ntree(2,3);
t = nodejoin(t,5);
t = nodejoin(t,4);
plot(t)
```



```
% Change Node Label from Depth_Position to Index
% (see the plot function).
```



```
nodeasc(t,[2 2])
ans =
    5
    2
    0

nodeasc(t,[2 2],'deppos')
ans =
    2     2
    1     1
    0     0
```

## See Also

`nodedesc` | `nodepar` | `wtreemgr`

# nodedesc

Node descendants

## Syntax

```
D = nodedesc(T,N)
D = nodedesc(T,N,'deppos')
```

## Description

`nodedesc` is a tree-management utility.

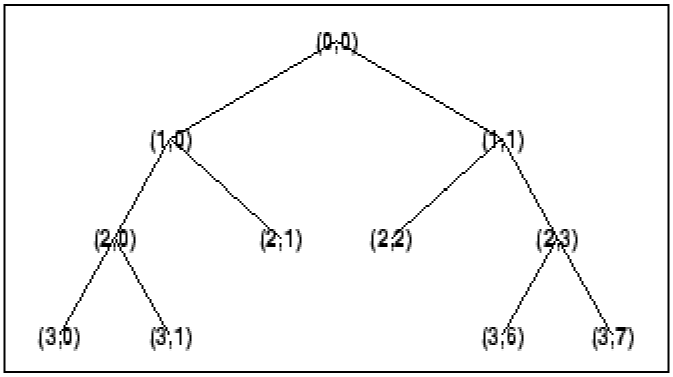
`D = nodedesc(T,N)` returns the indices of all the descendants of the node *N* in the tree *T* where *N* can be the index node or the depth and position of node. `D` is a column vector with `D(1)` = index of node *N*.

`D = nodedesc(T,N,'deppos')` is a matrix that contains the depths and positions of all descendants. `D(i,1)` is the depth of the `i`-th descendant and `D(i,2)` is the position of the `i`-th descendant.

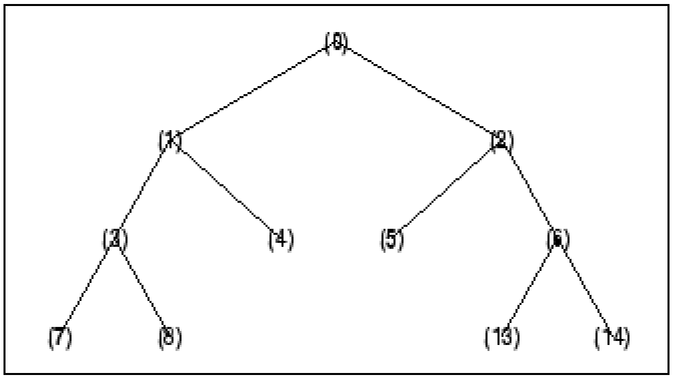
The nodes are numbered from left to right and from top to bottom. The root index is 0.

## Examples

```
% Create binary tree of depth 3.
t = ntree(2,3);
t = nodejoin(t,5);
t = nodejoin(t,4);
plot(t)
```



```
% Change Node Label from Depth_Position to Index
% (see the plot function).
```



```
% Node descendants.
nodedesc(t,2)
ans =
     2
     5
     6
    13
    14

nodedesc(t,2,'deppos')
ans =
     1     1
     2     2
    2     3
    3     6
    3     7

nodedesc(t,[1 1],'deppos')
ans =
    1     1
    2     2
    2     3
    3     6
    3     7

nodedesc(t,[1 1])
ans =
    2
    5
    6
   13
   14
```

## See Also

nodeasc | nodepar | wtreemgr

# nodejoin

Recompose node

## Syntax

```
T = nodejoin(T,N)
T = nodejoin(T)
T = nodejoin(T,0)
```

## Description

`nodejoin` is a tree-management utility.

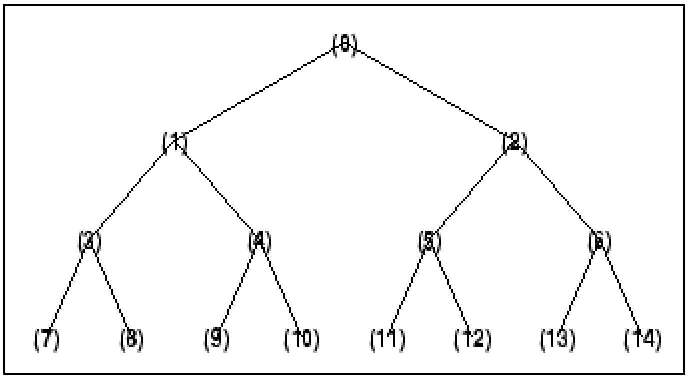
`T = nodejoin(T,N)` returns the modified tree *T* corresponding to a recomposition of the node *N*.

The nodes are numbered from left to right and from top to bottom. The root index is 0.

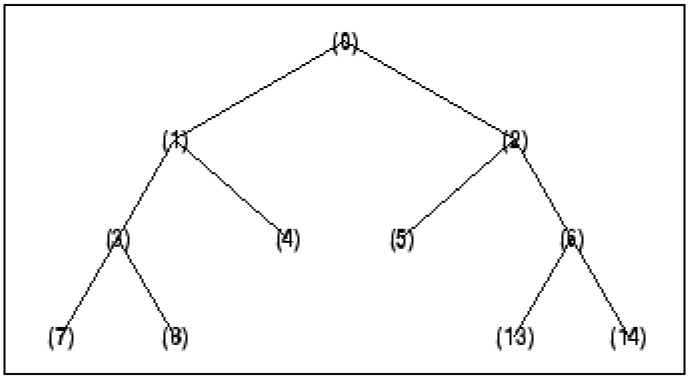
`T = nodejoin(T)` is equivalent to `T = nodejoin(T,0)`.

## Examples

```
% Create binary tree of depth 3.
t = ntree(2,3);

% Plot tree t.
plot(t)

% Change Node Label from Depth_Position to Index
% (see the plot function).
```



```
% Merge nodes of indices 4 and 5.
t = nodejoin(t,5);
t = nodejoin(t,4);
% Plot new tree t.
plot(t)

% Change Node Label from Depth_Position to Index
% (see the plot function).
```



## See Also

`nodesplt`

# nodepar

Node parent

## Syntax

```
F = nodepar(T,N)
F = nodepar(T,N,'deppos')
```

## Description

`nodepar` is a tree-management utility.

`F = nodepar(T,N)` returns the indices of the Äúparent(s)Äù of the nodes *N* in the tree *T* where *N* can be a column vector containing the indices of nodes or a matrix that contains the depths and positions of nodes. In the last case, `N(i,1)` is the depth of the `i`-th node and `N(i,2)` is the position of the `i`-th node.

`F = nodepar(T,N,'deppos')` is a matrix that contains the depths and positions of returned nodes. `F(i,1)` is the depth of the `i`-th node and `F(i,2)` is the position of the `i`-th node.

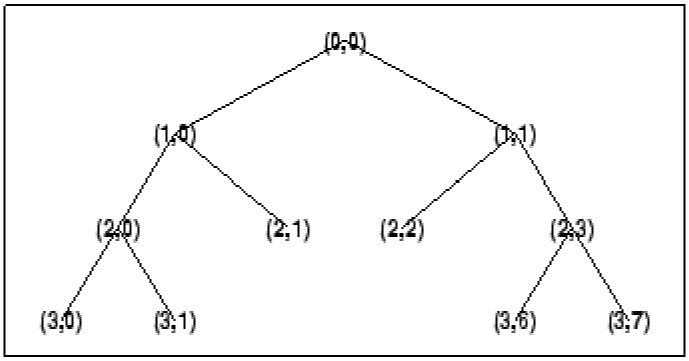
`nodepar(T,0)` or `nodepar(T,[0,0])` returns `-1`.

`nodepar(T,0,'deppos')` or `nodepar(T,[0,0],'deppos')` returns `[-1,0]`.

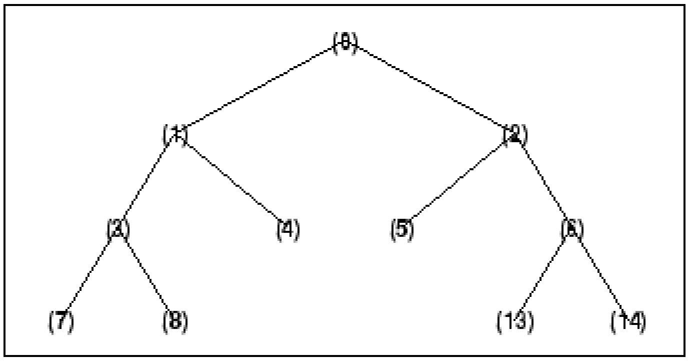
The nodes are numbered from left to right and from top to bottom. The root index is 0.

## Examples

```
% Create binary tree of depth 3.
t = ntree(2,3);
t = nodejoin(t,5);
t = nodejoin(t,4);
plot(t)
```



```
% Change Node Label from Depth_Position to Index
% (see the plot function).
```



```
% Nodes parent.
nodepar(t,[2 2],'deppos')

ans =
    1     1

nodepar(t,[1;7;14])

ans =
    0
    3
    6
```

## See Also

nodeasc | nodedesc | wtreemgr

## nodesplt

Split (decompose) node

### Syntax

```
T = nodesplt(T,N)
```

### Description

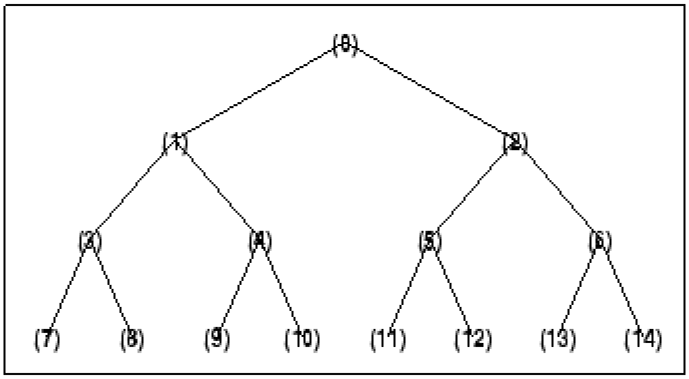
`nodesplt` is a tree-management utility.

`T = nodesplt(T,N)` returns the modified tree *T* corresponding to the decomposition of the node *N*.

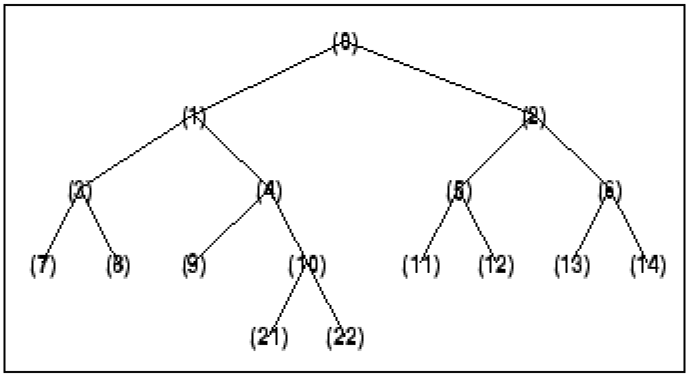
The nodes are numbered from left to right and from top to bottom. The root index is 0.

### Examples

```
% Create binary tree (tree of order 2) of depth 3.
t = ntree(2,3);

% Plot tree t.
plot(t)

% Change Node Label from Depth_Position to Index
% (see the plot function).
```



```
% Split node of index 10.
t = nodesplt(t,10);

% Plot new tree t.
plot(t)
% Change Node Label from Depth_Position to Index
% (see the plot function).
```



## See Also

`nodejoin`

## noleaves

Determine nonterminal nodes

### Syntax

```
N = noleaves(T)
N = noleaves(T,'dp')
```

### Description

`N = noleaves(T)` returns the indices of nonterminal nodes of the tree *T* (i.e., nodes that are not leaves). `N` is a column vector.

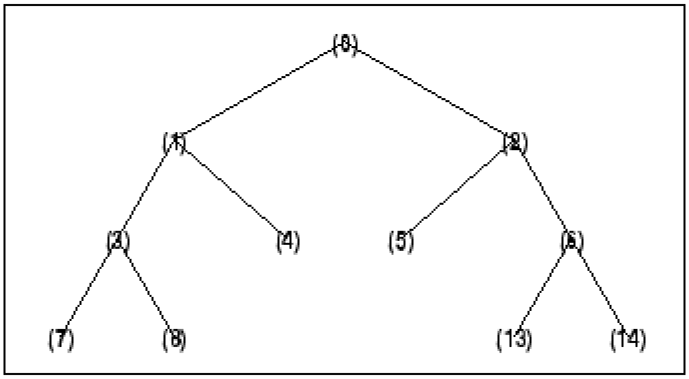
The nodes are ordered from left to right as in tree *T*.

`N = noleaves(T,'dp')` returns a matrix `N`, which contains the depths and positions of nonterminal nodes.

`N(i,1)` is the depth of the `i`-th nonterminal node and
`N(i,2)` is the position of the `i`-th nonterminal node.

### Examples

```
% Create initial tree.
ord = 2;
t = ntree(ord,3);        % binary tree of depth 3.
t=nodejoin(t,5);
t=nodejoin(t,4);
plot(t)

% Change Node Label from Depth_Position to Index
% (see the plot function).
```



```
% List nonterminal nodes (index).
ntnodes_ind = noleaves(t)

ntnodes_ind =
     0
     1
     2
     3
     6

% List nonterminal nodes (Depth_Position).
ntnodes_depo = noleaves(t,'dp')

ntnodes_depo =
     0     0
     1     0
     1     1
     2     0
     2     3
```

## See Also

`leaves`

# ntnode

Number of terminal nodes

## Syntax

```
NB = ntnode(T)
```

## Description

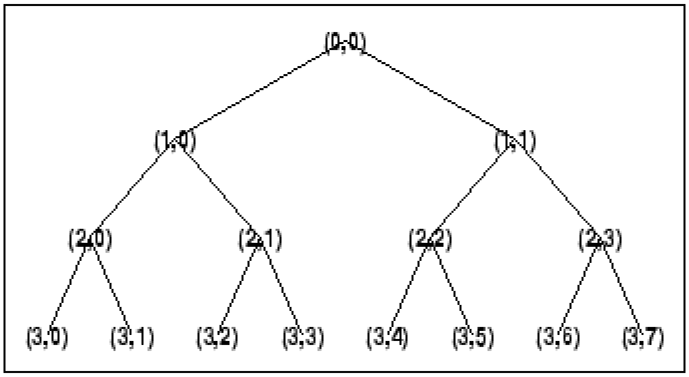
`ntnode` is a tree-management utility.

`NB = ntnode(T)` returns the number of terminal nodes in the tree `T`.

The nodes are numbered from left to right and from top to bottom. The root index is 0.

## Examples

```
% Create binary tree (tree of order 2) of depth 3.
t = ntree(2,3);

% Plot tree t.
plot(t)
```



```
% Number of terminal nodes.
ntnode(t)

ans =
    8
```

## See Also

`wtreemgr`

## ntree

NTREE constructor

### Syntax

```
T = ntree(ORD,D)
T = ntree
T = ntree(2,0)
T = ntree(ORD)
T = ntree(ORD,0)
T = ntree(ORD,D,S)
T = ntree(ORD,D,S,U)
```

### Description

`T = ntree(ORD,D)` returns an NTREE object, which is a complete tree of order `ORD` and depth `D`.

`T = ntree` is equivalent to `T = ntree(2,0)`.

`T = ntree(ORD)` is equivalent to `T = ntree(ORD,0)`.

With `T = ntree(ORD,D,S)` you can set a “split scheme” for nodes. The split scheme field `S` is a logical array of size `ORD` by 1.

The root of the tree can be split and it has `ORD` children. You can split the `j`-th child if `S(j) = 1`.

Each node that you can split has the same property as the root node.

With `T = ntree(ORD,D,S,U)` you can, in addition, set a userdata field.

Inputs can be given in another way:

`T = ntree('order',ORD,'depth',D,'spsch',S,'ud',U)`. For “missing” inputs the defaults are `ORD = 2` and `D = 0 , S = ones([1:ORD]) , U = {}`.

`[T,NB] = ntree( ... )` returns also the number of terminal nodes (leaves) of `T`.

For more information on object fields, type `help ntree/get`.

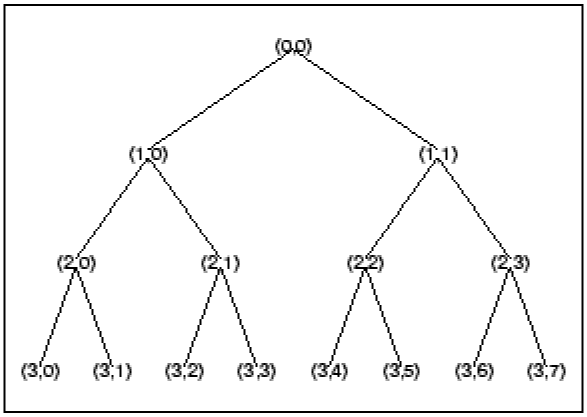
Class NTREE (Parent class: WTBO)

## Fields

| | |
|---|---|
| `wtbo` | Parent object |
| `order` | Tree order |
| `depth` | Tree depth |
| `spsch` | Split scheme for nodes |
| `tn` | Column vector with terminal node indices |

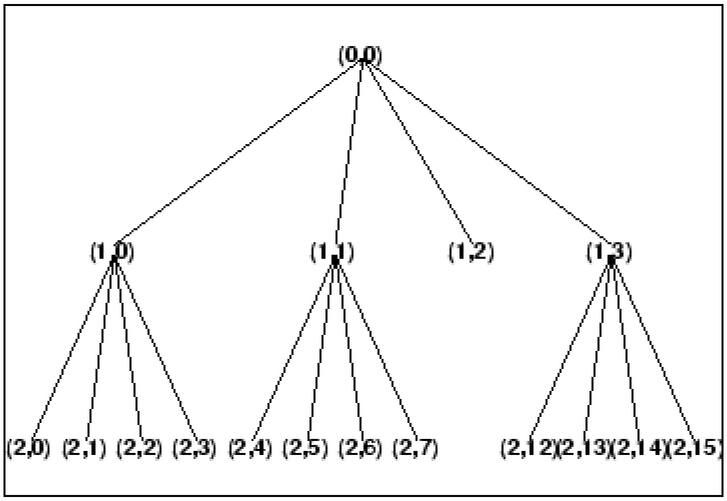
## Examples

```
% Create binary tree (tree of order 2) of depth 3.
t2 = ntree(2,3);

% Plot tree t2.
plot(t2)
```

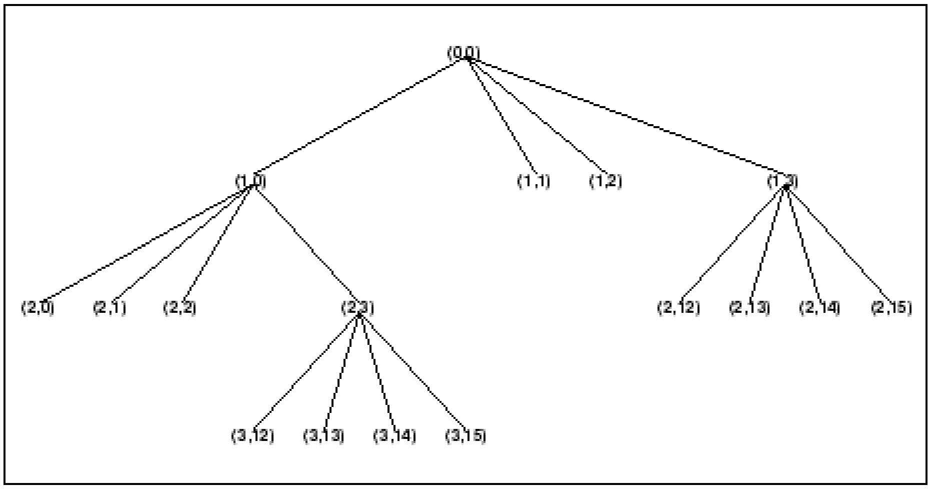


```
% Create a quadtree (tree of order 4) of depth 2.
```

```
t4 = ntree(4,2,[1 1 0 1]);

% Plot tree t4.
plot(t4)
```



```
% Split and merge some nodes using the gui
% generated by plot (see the plot function).
% The figure becomes:
```



## See Also

`wtbo`

# orthfilt

Orthogonal wavelet filter set

## Syntax

```
[Lo_D,Hi_D,Lo_R,Hi_R] = orthfilt(W)
```

## Description

`[Lo_D,Hi_D,Lo_R,Hi_R] = orthfilt(W)` computes the four filters associated with the scaling filter *W* corresponding to a wavelet:

| | |
|---|---|
| `Lo_D` | Decomposition low-pass filter |
| `Hi_D` | Decomposition high-pass filter |
| `Lo_R` | Reconstruction low-pass filter |
| `Hi_R` | Reconstruction high-pass filter |

For an orthogonal wavelet, in the multiresolution framework, we start with the scaling function $\phi$ and the wavelet function $\psi$. One of the fundamental relations is the twin-scale relation:

$$\frac{1}{2}\phi\left(\frac{x}{2}\right) = \sum_{n \in Z} w_n \phi(x-n)$$

All the filters used in `dwt` and `idwt` are intimately related to the sequence $(w_n)_{n \in Z}$. Clearly if $\phi$ is compactly supported, the sequence $(w_n)$ is finite and can be viewed as a FIR filter. The scaling filter *W* is

- A low-pass FIR filter
- Of length $2N$
- Of sum 1

- Of norm $\frac{1}{\sqrt{2}}$

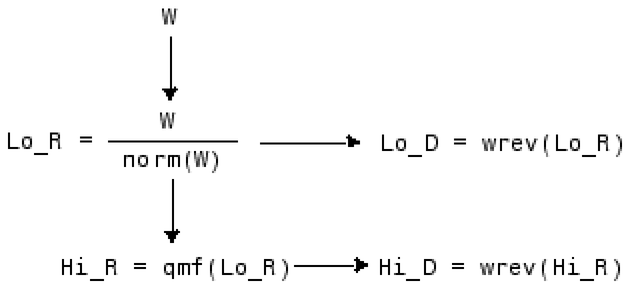
For example, for the `db3` scaling filter,

```
load db3
db3
db3 =
    0.2352 0.5706 0.3252 -0.0955 -0.0604 0.0249

sum(db3)
ans =
    1.000
    norm(db3)
ans =
    0.7071
```

From filter `W`, we define four FIR filters, of length 2N and norm 1, organized as follows:

| Filters | Low-Pass | High-Pass |
|---|---|---|
| Decomposition | `Lo_D` | `Hi_D` |
| Reconstruction | `Lo_R` | `Hi_R` |

The four filters are computed using the following scheme:

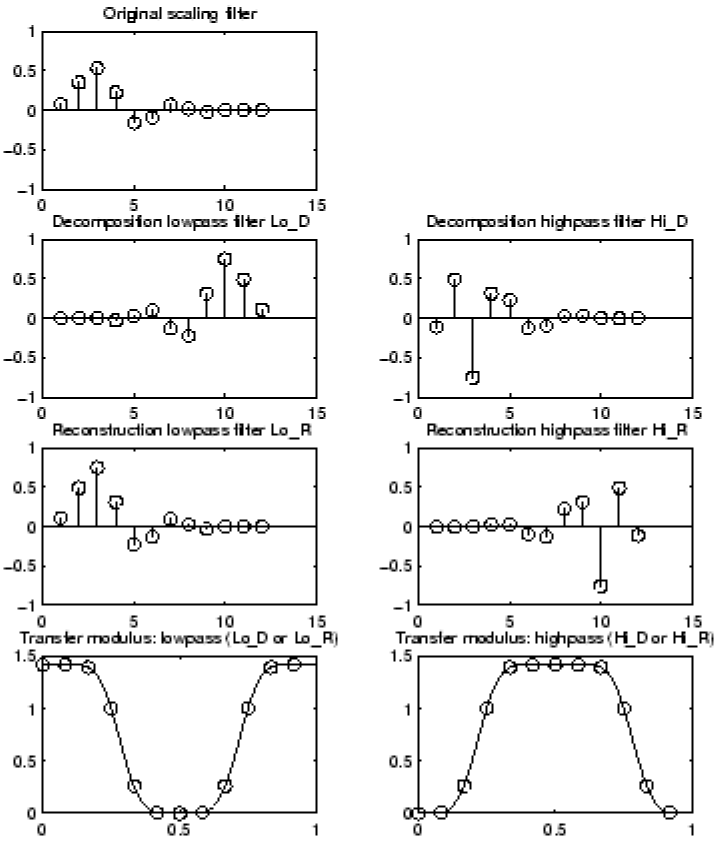


where `qmf` is such that `Hi_R` and `Lo_R` are quadrature mirror filters (i.e., `Hi_R(k) =` `(-1)`$^{k}$`Lo_R(2N + 1 - k)`, for `k = 1, 2, Ä, 2N`), and where `wrev` flips the filter coefficients. So `Hi_D` and `Lo_D` are also quadrature mirror filters. The computation of these filters is performed using `orthfilt`.

## Examples

```
% Load scaling filter.
load db8; w = db8;
subplot(421); stem(w);
title('Original scaling filter');

% Compute the four filters.
[Lo_D,Hi_D,Lo_R,Hi_R] = orthfilt(w);
subplot(423); stem(Lo_D);
title('Decomposition low-pass filter');
subplot(424); stem(Hi_D);
title('Decomposition high-pass filter');
subplot(425); stem(Lo_R);
title('Reconstruction low-pass filter');
subplot(426); stem(Hi_R);
title('Reconstruction high-pass filter');

% Check for orthonormality.
df = [Lo_D;Hi_D];
rf = [Lo_R;Hi_R];
id = df*df'

id =
    1.0000         0
         0    1.0000

id = rf*rf'

id =
    1.0000         0
         0    1.0000

% Check for orthogonality by dyadic translation, for example:
df = [Lo_D 0 0;Hi_D 0 0];
dft = [0 0 Lo_D; 0 0 Hi_D];
zer = df*dft'

zer =

    1.0e-12 *
    -0.1883 0.0000
    -0.0000 -0.1883
```

```
% High- and low-frequency illustration.
fftld = fft(Lo_D); ffthd = fft(Hi_D);
freq = [1:length(Lo_D)]/length(Lo_D);
subplot(427); plot(freq,abs(fftld));
title('Transfer modulus: low-pass');
subplot(428); plot(freq,abs(ffthd));
title('Transfer modulus: high-pass')
% Editing some graphical properties,
% the following figure is generated.
```



## References

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics, SIAM Ed. pp. 117–119, 137, 152.

### See Also

`biorfilt` | `qmf` | `wfilters`

# otnodes

Order terminal nodes of binary wavelet packet tree

## Syntax

```
[Tn_Pal,Tn_Seq] = otnodes(WPT)
[Tn_Pal,Tn_Seq,I,J] = otnodes(WPT)
[DP_Pal,DP_Seq] = otnodes(WPT,'dp')
```

## Description

`[Tn_Pal,Tn_Seq] = otnodes(WPT)` returns the terminal nodes of the binary wavelet packet tree, WPT, in Paley (natural) ordering, Tn_Pal, and sequency (frequency) ordering, Tn_Seq. Tn_Pal and Tn_Seq are *N*-by-1 column vectors where *N* is the number of terminal nodes.

`[Tn_Pal,Tn_Seq,I,J] = otnodes(WPT)` returns the permutations of the terminal node indices such that `Tn_Seq = Tn_Pal(I)` and `Tn_Pal = Tn_Seq(J)`.

`[DP_Pal,DP_Seq] = otnodes(WPT,'dp')` returns the Paley and frequency-ordered terminal nodes in node depth-position format. DP_Pal and DP_Seq are *N*-by-2 matrices. The first column contains the depth index, and the second column contains the position index.

## Input Arguments

### WPT

Binary wavelet packet tree. You can use `treeord` to determine the order of your wavelet packet tree.

### dp

String variable indicating that the Paley-ordered or sequency-ordered nodes are returned in depth-position format.

## Output Arguments

**`Tn_Pal`**

Terminal nodes in Paley (natural) ordering

**`Tn_Seq`**

Terminal nodes in sequency ordering

**`DP_Pal`**

Paley-ordered terminal nodes in depth-position format. This output argument only applies when you use the `'dp'` input argument.

**`DP_Seq`**

Sequency-ordered terminal nodes in depth-position format. This output argument only applies when you use the `'dp'` input argument.

## Examples

Order terminal nodes with Paley and frequency ordering:

```
x = randn(8,1);
wpt = wpdec(x,2,'haar');
[Tn_Pal,Tn_Seq] = otnodes(wpt);
% Tn_Pal is [3 4 5 6]
% Tn_Seq is [3 4 6 5]
```

Return permutations for Paley and frequency ordering:

```
load noisdopp;
wpt = wpdec(noisdopp,6,'sym4');
[Tn_Pal,Tn_Seq,I,J] = otnodes(wpt);
isequal(Tn_Seq(J),Tn_Pal)
isequal(Tn_Seq,Tn_Pal(I))
```

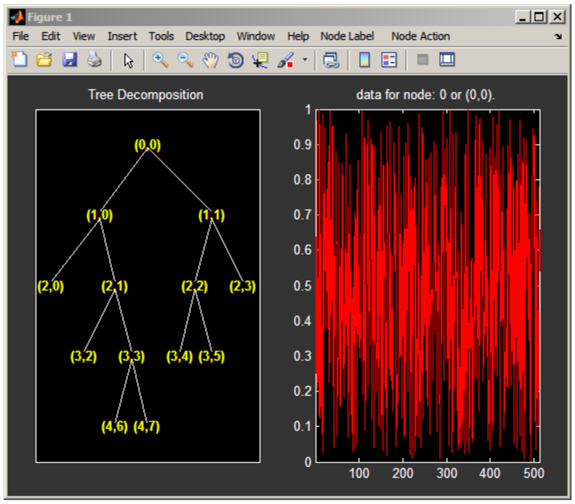
Order terminal nodes by depth and position:

```
x = randn(8,1);
```

```
wpt = wpdec(x,2,'haar');
[DP_Pal,DP_Seq] = otnodes(wpt,'dp');
```

Order terminal nodes from a modified wavelet packet tree:

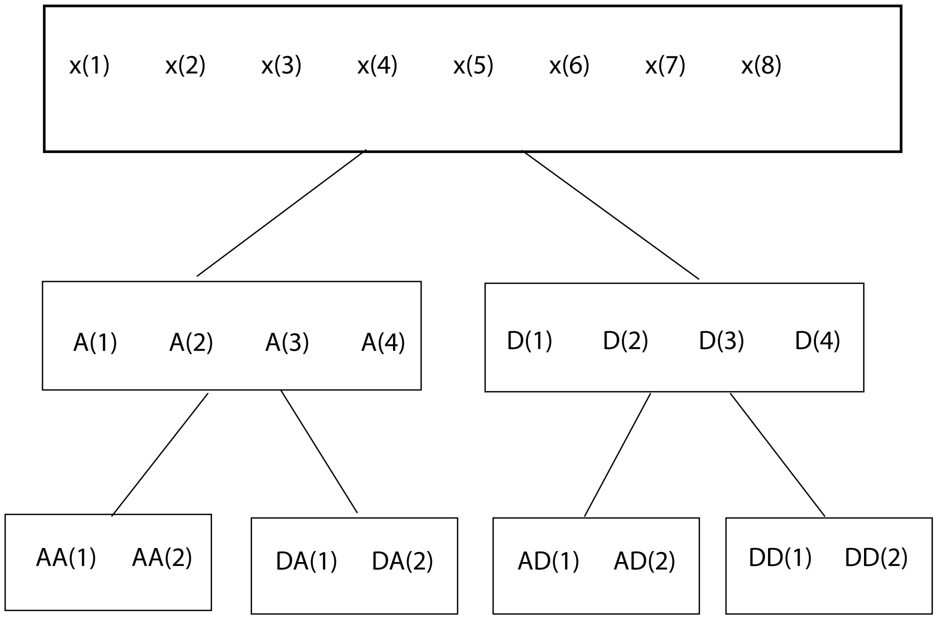
```
 t = wptree(2,2,rand(1,512),'haar');
 t = wpsplt(t,4);
 t = wpsplt(t,5);
 t = wpsplt(t,10);
 plot(t);
 [tn_Pal,tn_Seq,I,J] = otnodes(t);
```



## More About

### Paley (Natural) and Sequency (Frequency) Ordering

The discrete wavelet packet transform iterates on both approximation and detail coefficients at each level. In this transform, *A* denotes the lowpass (approximation) filter followed by downsampling. *D* denotes the highpass (detail) filter followed by downsampling. The following figure represents a wavelet packet transform in Paley ordering acting on a time series of length 8. The transform has a depth of two.



Because of aliasing introduced by downsampling, the frequency content extracted by the operator *AD* is higher than the frequency content extracted by the *DD* operator. Therefore, the terminal nodes in frequency (sequency) order are: *AA,DA,DD,AD*. The terminal nodes in Paley order have the following indices: 3,4,5,6. The frequency order has the indices: 3,4,6,5.

## References

Wickerhauser, M.V. *Lectures on Wavelet Packet Algorithms*, Technical Report, Washington University, Department of Mathematics, 1992.

### See Also

`leaves` | `treeord`

# pat2cwav

Build wavelet from pattern

## Syntax

```
[PSI,XVAL,NC] = pat2cwav(YPAT,METHOD,POLDEGREE,REGULARITY)
```

## Description

`[PSI,XVAL,NC] = pat2cwav(YPAT,METHOD,POLDEGREE,REGULARITY)` computes an admissible wavelet for CWT (given by `XVAL` and `PSI`) adapted to the pattern defined by the vector `YPAT`, and of norm equal to 1.

The underlying x-values pattern is set to

```
xpat = linspace(0,1,length(YPAT))
```

The constant `NC` is such that `NC*PSI` approximates `YPAT` on the interval `[0,1]` by least squares fitting using

- a polynomial of degree `POLDEGREE` when `METHOD` is equal to `'polynomial'`
- a projection on the space of functions orthogonal to constants when `METHOD` is equal to `'orthconst'`

The `REGULARITY` parameter defines the boundary constraints at the points 0 and 1. Allowable values are `'continuous'`, `'differentiable'`, and `'none'`.

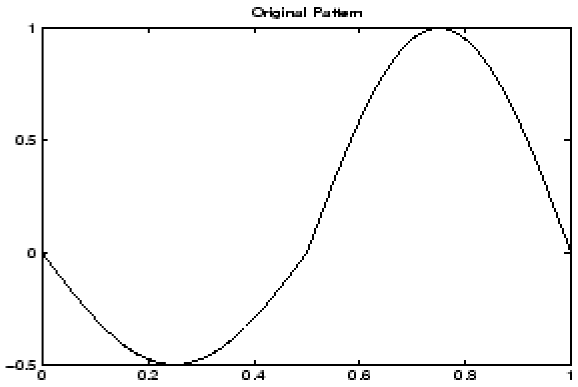
When `METHOD` is equal to `'polynomial'`

- if `REGULARITY` is equal to `'continuous'`, `POLDEGREE` must be greater than or equal to 3.
- if `REGULARITY` is equal to `'differentiable'`, `POLDEGREE` must be greater than or equal to 5.

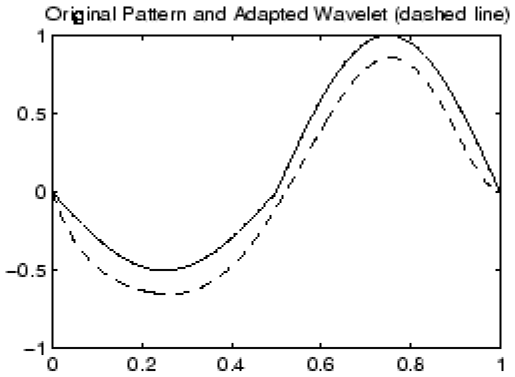
## Examples

The principle for designing a new wavelet for CWT is to approximate a given pattern using least squares optimization under constraints leading to an admissible wavelet well suited for the pattern detection using the continuous wavelet transform (see Misiti et al.).

```
load ptpssin1;
plot(X,Y), title('Original Pattern')
```



```
[psi,xval,nc] = pat2cwav(Y, 'polynomial',6, 'continuous') ;
plot(X,Y,'-',xval,nc*psi,'--'),
title('Original Pattern and Adapted Wavelet (dashed line)')
```



You can check that `psi` satisfies the definition of a wavelet by noting that it integrates to zero and that its $L_2$ norm is equal to 1.

```
dx = xval(2)-xval(1);
Mu = sum(psi*dx)
L2norm = sum(abs(psi).^2*dx)
```

## References

Misiti, M., Y. Misiti, G. Oppenheim, J.-M. Poggi (2003), “Les ondelettes et leurs applications,” Hermes.

# plot

Plot tree GUI

## Syntax

```
plot(T)
plot(T,FIG)
```

## Description

`plot` is a graphical tree-management utility.

`plot(T)` plots the tree *T*.

The figure that contains the tree is a GUI tool. It lets you change the **Node Label** to **Depth_Position** or **Index**, and **Node Action** to **Split-Merge** or **Visualize**.

The default values are **Depth_Position** and **Visualize**.

You can click the nodes to execute the current **Node Action**.

`plot(T,FIG)` plots the tree *T* in the figure whose handle is `FIG`. This figure was already used to plot a tree, for example using the command

```
FIG = plot(T)
```

After some split or merge actions, you can get the new tree using its parent figure handle. The following syntax lets you perform this functionality:

```
NEWT = plot(T,'read',FIG)
```

In fact, the first argument is dummy. The most general syntax is

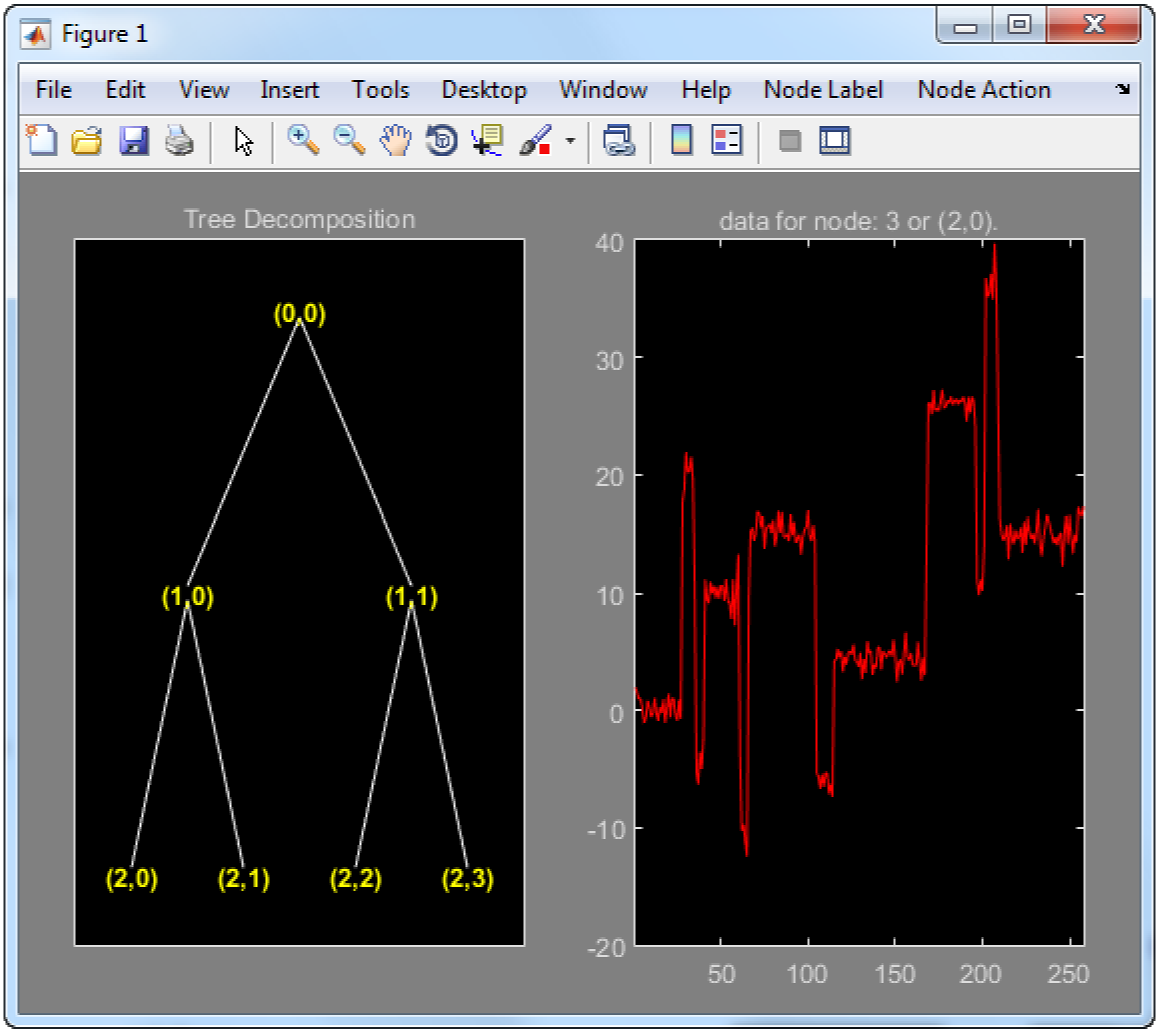
```
NEWT = plot(DUMMY,'read',FIG)
```

where `DUMMY` is any object parented by an NTREE object. More generally, `DUMMY` can be any object constructor name returning an NTREE parented object. For example:

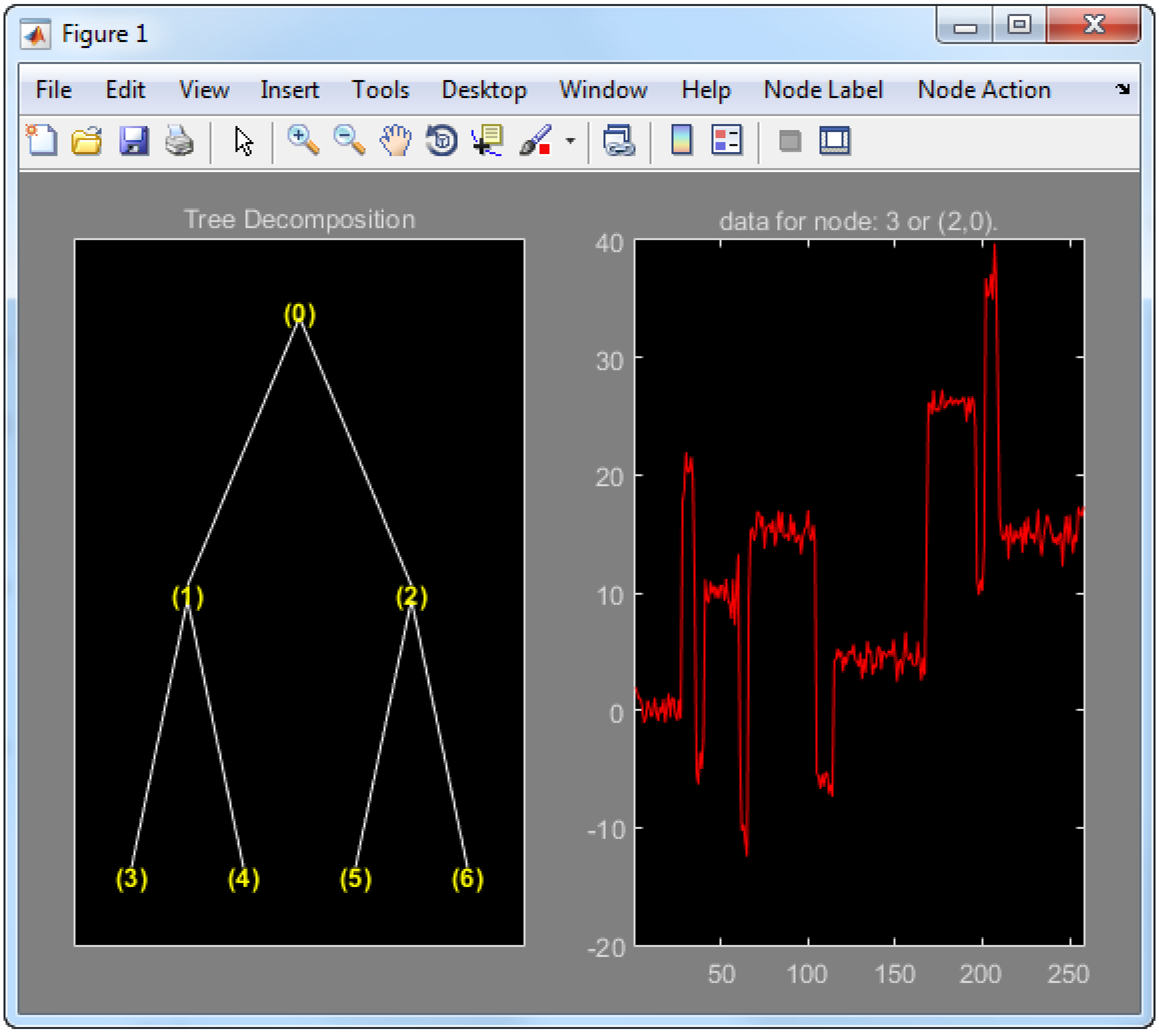
```
NEWT = plot(ntree,'read',FIG)
NEWT = plot(dtree,'read',FIG)
NEWT = plot(wptree,'read',FIG)
```

## Examples

```
% Create a wavelet packets tree (1-D)
load noisbloc
x = noisbloc;
t = wpdec(x,2,'db2');

% Plot tree t.
plot(t)
```



```
% Change Node Label from Depth_Position to Index.

% Click the node (3). You get the following figure.
```



Now set the **Node Label** back to **Depth_Position**. Change **Node Action** to **Split-Merge**. Click on the `(1,1)` node.

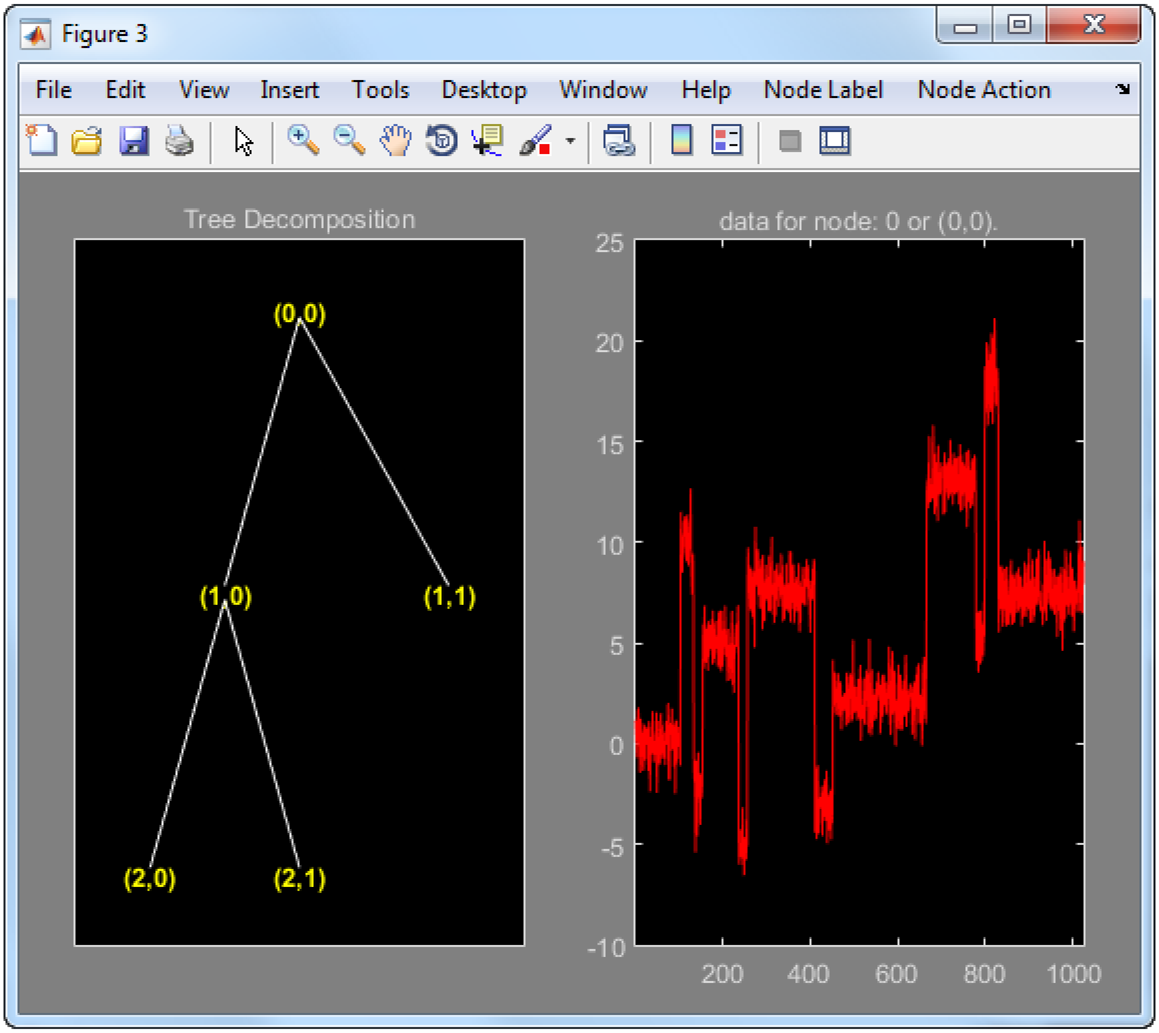
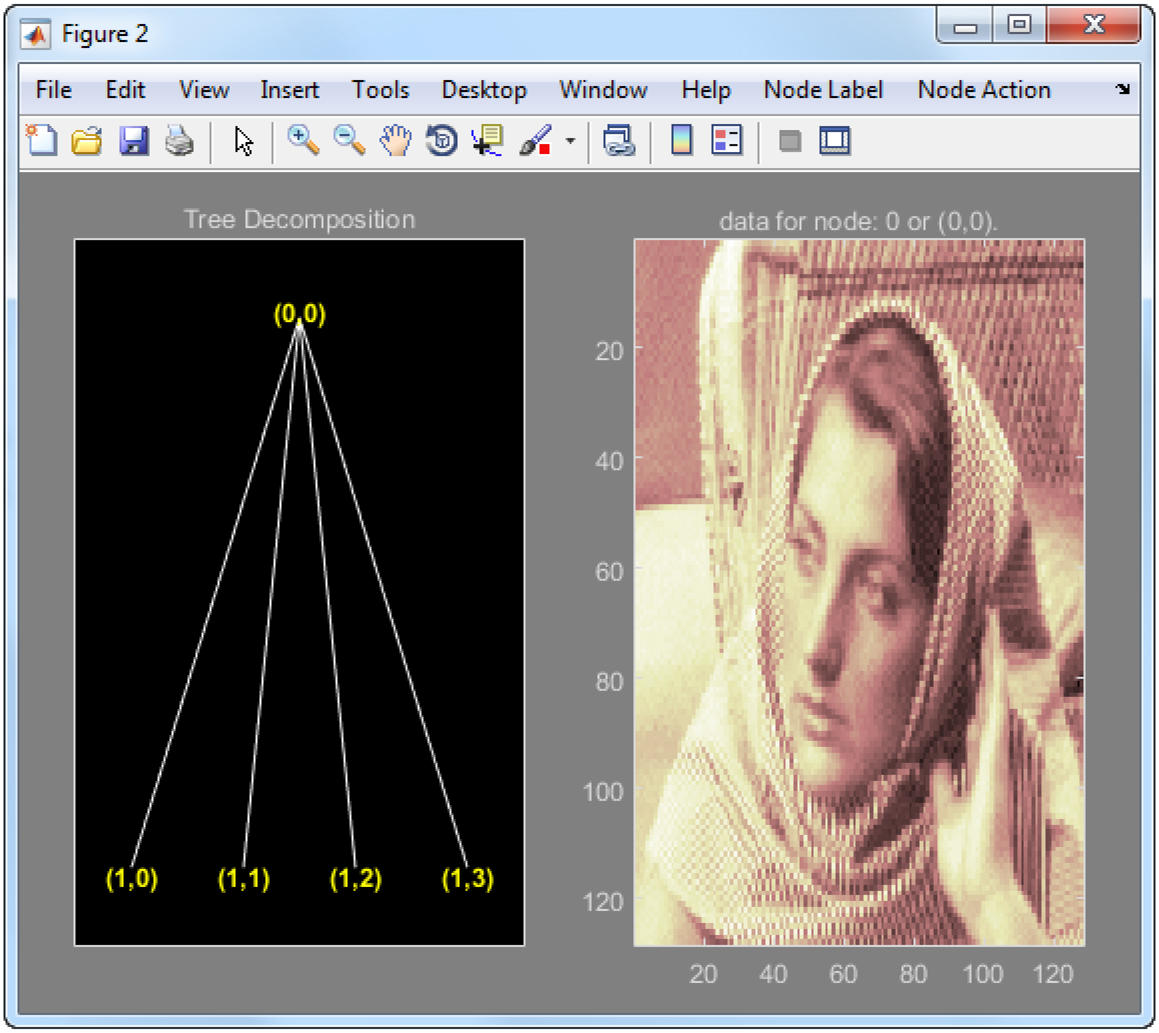


Figure 3
File Edit View Insert Tools Desktop Window Help Node Label Node Action
Tree Decomposition
(0,0)
(1,0)
(1,1)
(2,0)
(2,1)
data for node: 0 or (0,0).
25
20
15
10
5
0
-5
-10
200
400
600
800
1000

The above figure now shows the discrete wavelet transform down to level 2.

```
% Create a wavelet packets tree (2-D)
load woman2
t = wpdec2(X,1,'sym4');

% Plot tree t.
plot(t)
```

```
% Change Node Label from Depth_Position to Index.
% Click the node (1). You get the following figure.
```



## plotdt

Plot dual-tree or double-density wavelet transform

### Syntax

```
plotdt(wt)
```

### Description

`plotdt(wt)` plots the coefficients of the 1-D or 2-D wavelet filter bank decomposition, wt.

### Examples

#### Plot Complex Dual-Tree Wavelet Transform of 1-D Signal

Plot the complex dual-tree wavelet transform of the noisy Doppler signal.

Load the noisy Doppler signal. Obtain the complex dual-tree wavelet transform down to level 4.

```
load noisdopp;
wt = dddtree('cplxdt',noisdopp,4,'dtf1');
```

Plot the coefficients.

```
plotdt(wt)
```

#### Plot Complex Oriented Dual-Tree Wavelet Transform of 2-D Image

Plot the complex oriented dual-tree wavelet transform of an image.

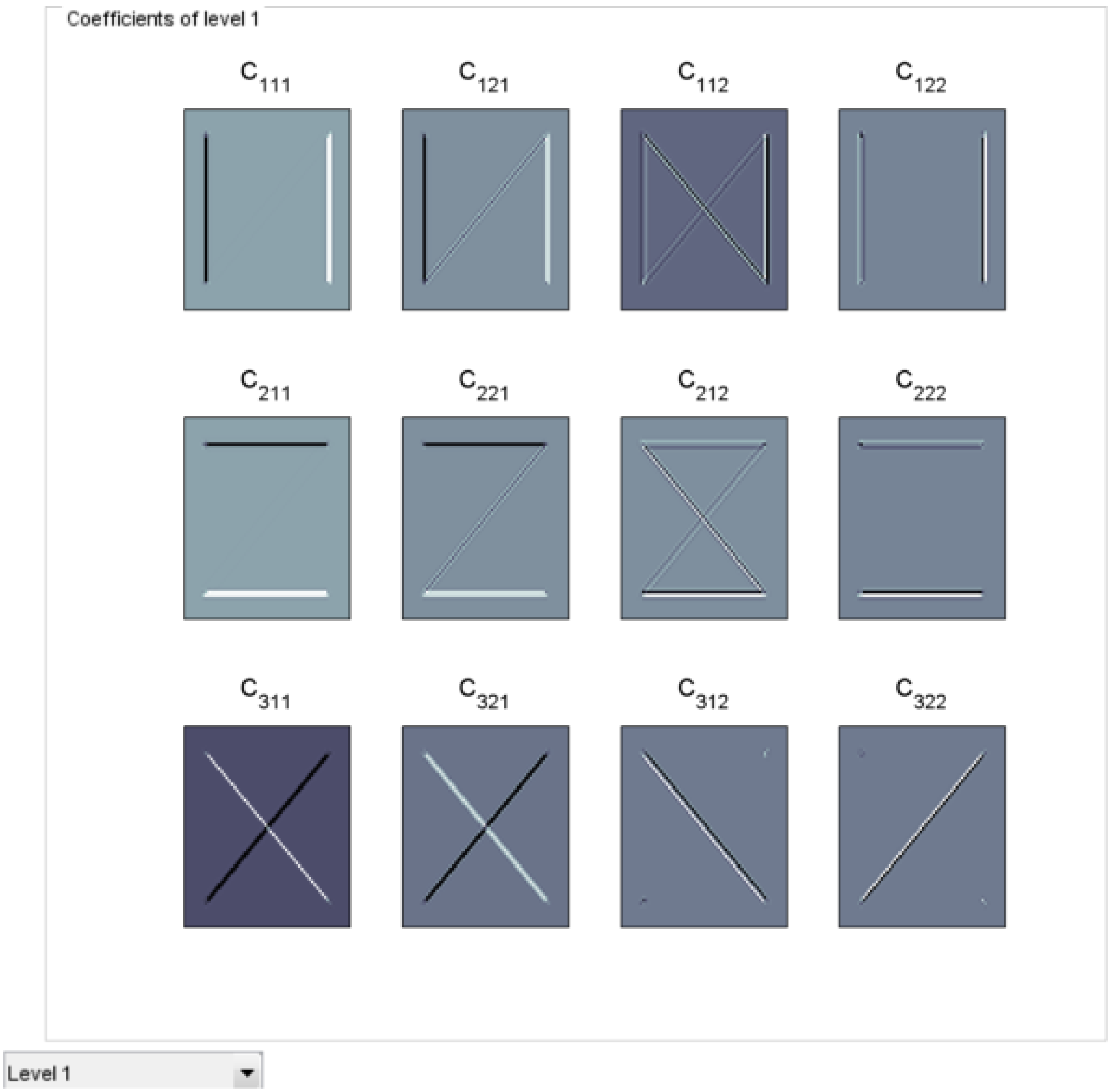
Load the “xbox” image. Obtain the complex oriented dual-tree wavelet transform down to level 3.

```
load xbox;
wt = dddtree2('cplxdt',xbox,3,'dtf1');
```

Plot the coefficients.

```
plotdt(wt)
```

Select the level-one detail coefficients from the drop-down list in the lower left corner.



- “Analytic Wavelets Using the Dual-Tree Wavelet Transform”

## Input Arguments

### `wt` — Wavelet transform

structure

Wavelet transform, returned as a structure from `dddtree` or `dddtree2` with these fields:

#### `type` — Type of wavelet decomposition (filter bank)

`'dwt'` | `'ddt'` | `'realdt'` | `'cplxdt'` | `'realdddt'` | `'cplxdddt'`

Type of wavelet decomposition (filter bank), specified as one of `'dwt'`, `'ddt'`, `'realdt'`, `'cplxdt'`,, `'realdddt'`, or `'cplxdddt'`. `'realdt'` and `'realdddt'` are only valid for the 2-D wavelet transform. The type, `'dwt'`, is a critically sampled (nonredundant) discrete wavelet transform for 1-D data or 2-D images. The other decomposition types are oversampled wavelet transforms. For details about transform types see `dddtree` for 1-D wavelet transforms and `dddtree2` for 2-D wavelet transforms.

#### `level` — Level of the wavelet decomposition

positive integer

Level of the wavelet decomposition, specified as a positive integer.

#### `filters` — Decomposition (analysis) and reconstruction (synthesis) filters

structure

Decomposition (analysis) and reconstruction (synthesis) filters, specified as a structure with these fields:

##### `Fdf` — First-stage analysis filters

matrix | cell array

First level decomposition filters specified as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage analysis filters for the corresponding tree.

##### `Df` — Analysis filters for levels > 1

matrix | cell array

Analysis filters for levels > 1, specified as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the analysis filters for the corresponding tree.

### `Frf` — First-level reconstruction filters

matrix | cell array

First-level reconstruction filters, specified as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage synthesis filters for the corresponding tree.

### `Rf` — Reconstruction filters for levels > 1

matrix | cell array

Reconstruction filters for levels > 1, specified as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage synthesis filters for the corresponding tree.

### `cfs` — Wavelet transform coefficients

cell array of matrices

Wavelet transform coefficients, specified as a 1-by-(level+1) cell array of matrices. The size and structure of the matrix elements of the cell array depend on the type of wavelet transform and whether the decomposition is 1-D or 2-D. For a 1-D wavelet transform, the coefficients are organized by transform type as follows:

- `'dwt'` — `cfs{j}`

    - j = 1,2,...level is the level.
    - `cfs{level+1}` are the lowpass, or scaling, coefficients.
- `'ddt'` — `cfs{j}(:,:,k)`
    - j = 1,2,... level is the level.
    - k = 1,2 is the wavelet filter.
    - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'cplxdt'` — `cfs{j}(:,:,m)`
    - j = 1,2,... level is the level.
    - m = 1,2 are the real and imaginary parts.
    - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'realdddt'` — `cfs{j}(:,:,d,k)`
    - j = 1,2,... level is the level.
    - d = 1,2,3 is the orientation.
    - k = 1,2 is the wavelet transform tree.
    - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'cplxdddt'` — `cfs{j}(:,:,d,k,m)`
    - j = 1,2,... level is the level.
    - k = 1,2 is the wavelet transform tree.
    - m = 1,2 are the real and imaginary parts.
    - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.

For a 2-D wavelet transform, the coefficients are organized by transform type as follows:

- `'dwt'` — `cfs{j}(:,:,d)`
    - j = 1,2,... level is the level.
    - d = 1,2,3 is the orientation.
    - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'ddt'` — `cfs{j}(:,:,d)`
    - j = 1,2,... level is the level.

  - d = 1,2,3,4,5,6,7,8 is the orientation.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'realddt'` — `cfs{j}(:,:,d,k)`
  - j = 1,2,... level is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'cplxdt'` — `cfs{j}(:,:,d,k,m)`
  - j = 1,2,... level is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - m = 1,2 are the real and imaginary parts.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'realdddt'` — `cfs{j}(:,:,d,k)`
  - j = 1,2,... level is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'cplxdddt'` — `cfs{j}(:,:,d,k,m)`
  - j = 1,2,... level is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - m = 1,2 are the real and imaginary parts.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.

## More About

- “Critically Sampled and Oversampled Wavelet Filter Banks”

## See Also

`dddtree` | `dddtree2` | `dddtreecfs`

## qmf

Scaling and Wavelet Filter

### Syntax

```
Y = qmf(X,P)
Y = qmf(X)
Y = qmf(X,0)
```

### Description

`Y = qmf(X,P)` changes the signs of the even index entries of the reversed vector filter coefficients `X` if `P` is even. If `P` is odd the same holds for odd index entries. `Y = qmf(X)` is equivalent to `Y = qmf(X,0)`.

Let `x` be a finite energy signal. Two filters $F_0$ and $F_1$ are quadrature mirror filters (QMF) if, for any $x$,

$$\|y_0\|^2 + \|y_1\|^2 = \|x\|^2$$

where $y_0$ is a decimated version of the signal $x$ filtered with $F_0$ so $y_0$ defined by $x_0 = F_0(x)$ and $y_0(n) = x_0(2n)$, and similarly, $y_1$ is defined by $x_1 = F_1(x)$ and $y_1(n) = x_1(2n)$. This property ensures a perfect reconstruction of the associated two-channel filter banks scheme (see Strang-Nguyen p. 103).

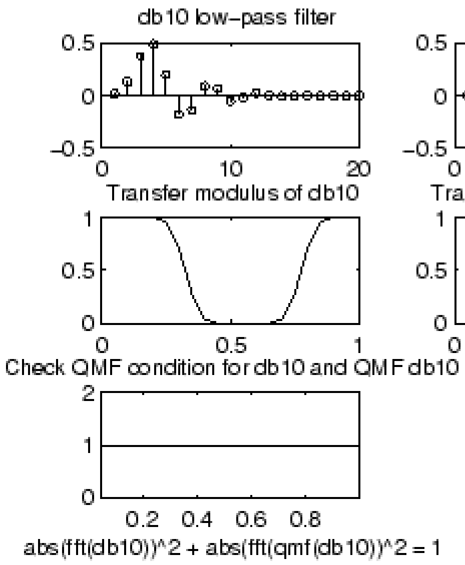
For example, if $F_0$ is a Daubechies scaling filter and $F_1$ = `qmf`($F_0$), then the transfer functions $F_0(z)$ and $F_1(z)$ of the filters $F_0$ and $F_1$ satisfy the condition (see the example for *db*10):

$$|F_0(z)|^2 + |F_1(z)|^2 = 1$$

### Examples

```
% Load scaling filter associated with an orthogonal wavelet.
load db10;
```

```
subplot(321); stem(db10); title('db10 low-pass filter');

% Compute the quadrature mirror filter.
qmfdb10 = qmf(db10);
subplot(322); stem(qmfdb10); title('QMF db10 filter');

% Check for frequency condition (necessary for orthogonality):
% abs(fft(filter))^2 + abs(fft(qmf(filter))^2 = 1 at each
% frequency.
m = fft(db10);
mt = fft(qmfdb10);
freq = [1:length(db10)]/length(db10);
subplot(323); plot(freq,abs(m));
title('Transfer modulus of db10')
subplot(324); plot(freq,abs(mt));
title('Transfer modulus of QMF db10')
subplot(325); plot(freq,abs(m).^2 + abs(mt).^2);
title('Check QMF condition for db10 and QMF db10')
xlabel(' abs(fft(db10))^2 + abs(fft(qmf(db10))^2 = 1')

% Editing some graphical properties,
% the following figure is generated.
```



```
% Check for orthonormality.
```

```
df = [db10;qmfdb10]*sqrt(2);
id = df*df'

id =
    1.0000 0.0000
    0.0000 1.0000
```

## References

Strang, G.; T. Nguyen (1996), *Wavelets and Filter Banks*, Wellesley-Cambridge Press.

# rbiowavf

Reverse biorthogonal spline wavelet filters

## Syntax

```
[RF,DF] = rbiowavf(W)
```

## Description

`[RF,DF] = rbiowavf(W)` returns the two scaling filters associated with the biorthogonal wavelet specified by the string *W*.

`W = 'rbioNd.Nr'` where possible values for `Nd` and `Nr` are

| | |
|---|---|
| `Nd = 1` | `Nr = 1 , 3 or 5` |
| `Nd = 2` | `Nr = 2 , 4 , 6 or 8` |
| `Nd = 3` | `Nr = 1 , 3 , 5 , 7 or 9` |
| `Nd = 4` | `Nr = 4` |
| `Nd = 5` | `Nr = 5` |
| `Nd = 6` | `Nr = 8` |

The output arguments are filters.

- `RF` is the reconstruction filter.
- `DF` is the decomposition filter.

## Reverse Biorthgonal Scaling Filter

Obtain the reverse biorthogonal reconstruction and decomposition scaling filters for the `'rbio3.1'` wavelet. The `'rbio3.1'` wavelet has 3 vanishing moments for the decomposition (analysis) wavelet and 1 vanishing moment for the reconstruction (synthesis) wavelet.

```
[RF,DF] = rbiowavf('rbio3.1');
```

The reconstruction scaling filter, RF, and the decomposition filter, DF, are equal to the filters returned by wfilters scaled by $\sqrt{2}$.

```
[LoD,HiD,LoR,HiR] = wfilters('rbio3.1');
max(abs(sqrt(2)*DF-LoD))
max(abs(sqrt(2)*RF-LoR))


ans =

     0


ans =

     0
```

## See Also

biorfilt | waveinfo

# read

Read values of WPTREE

## Syntax

```
VARARGOUT = read(T,VARARGIN)
```

## Description

`VARARGOUT = read(T,VARARGIN)` is the most general syntax to read one or more property values from the fields of a WPTREE object .

The different ways to call the `read` function are

```
PropValue = read(T,'PropName') or
PropValue = read(T,'PropName','PropParam')
```

or any combination of the previous syntaxes:

```
[PropValue1,PropValue2, ] =
read(T,'PropName1','PropParam1','PropName2','PropParam2', )
```

where '*PropParam*' is optional.

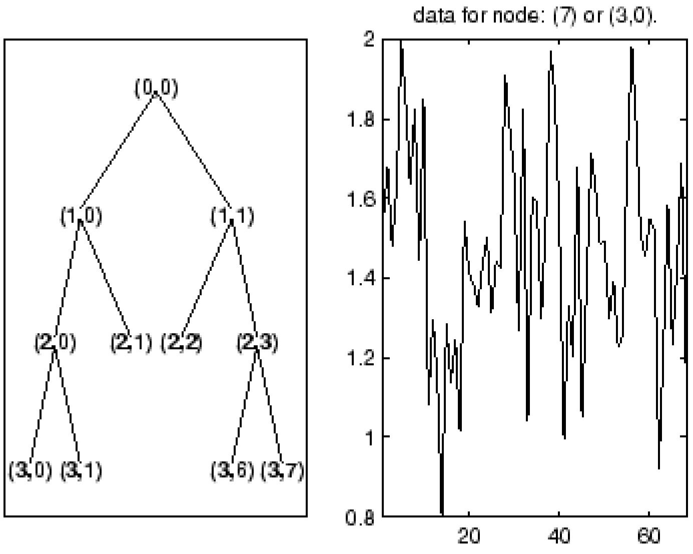
The valid choices for '*PropName*' and '*PropParam*' are listed in this table.

| ***PropName*** | ***PropParam*** |
|---|---|
| `'ent'`, `'ento'` or `'sizes'` (see `wptree`) | Without '*PropParam*' or with '*PropParam*' = Vector of node indices, `PropValue` contains the entropy (or optimal entropy, or size) of the tree nodes in ascending node index order. |
| `'cfs'` | With '*PropParam*' = One terminal node index. `cfs = read(T,'cfs',NODE)` is equivalent to cfs = read(T,'data',NODE) and returns the coefficients of the terminal node `NODE`. |

| ***PropName*** | ***PropParam*** |
|---|---|
| `'entName'`, `'entPar'`, `'wavName'` (see `wptree`) or `'allcfs'` | Without `'`*`PropParam`*`'`. `cfs = read(T,'allcfs')` is equivalent to `cfs = read(T,'data')`. `PropValue` contains the desired information in ascending node index order of the tree nodes. |
| `'wfilters'` (see `wfilters`) | Without `'`*`PropParam`*`'` or with `'`*`PropParam`*`' = 'd','r','l','h'`. |
| `'data'` | Without `'`*`PropParam`*`'` or with `'`*`PropParam`*`'` = One terminal node index or `'`*`PropParam`*`'` = Column vector of terminal node indices.In this last case, `PropValue` is a cell array. Without `'`*`PropParam`*`'`, `PropValue` contains the coefficients of the tree nodes in ascending node index order. |

## Examples

```
% Create a wavelet packet tree.
x = rand(1,512);
t = wpdec(x,3,'db3');
t = wpjoin(t,[4;5]);
plot(t);

% Click the node (3,0), (see the plot function).
l% Read values.

sAll = read(t,'sizes');
sNod = read(t,'sizes',[0,4,5]);
eAll = read(t,'ent');
eNod = read(t,'ent',[0,4,5]);
dAll = read(t,'data');
dNod = read(t,'data',[4;5]);
[lo_D,hi_D,lo_R,hi_R] = read(t,'wfilters');
[lo_D,lo_R,hi_D,hi_R] = read(t,'wfilters','l','wfilters','h');
[ent,ento,cfs4,cfs5]  = read(t,'ent','ento','cfs',4,'cfs',5);
```



## See Also

`disp` | `get` | `set` | `wptree` | `write`

## readtree

Read wavelet packet decomposition tree from figure

### Syntax

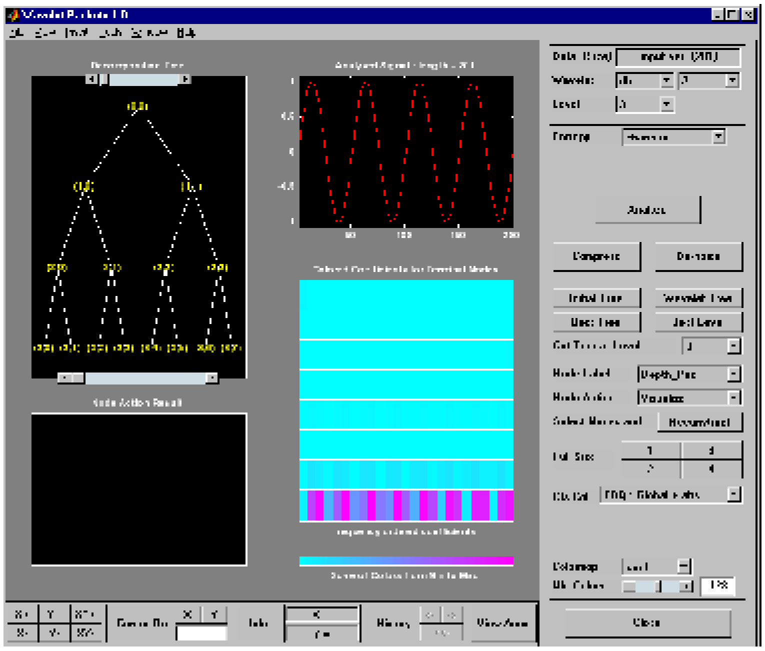
```
T = readtree(F)
```

### Description

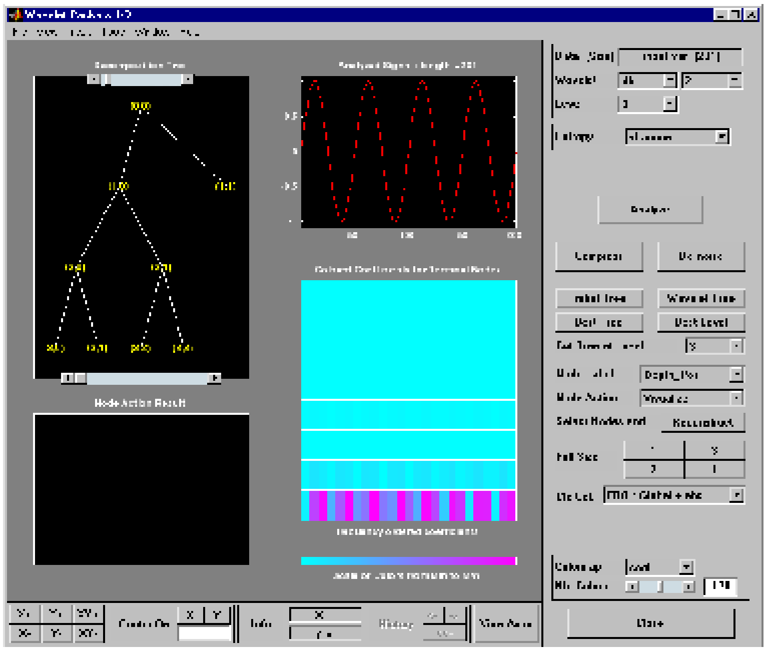
`T = readtree(F)` reads the wavelet packet decomposition tree from the figure whose handle is *F*.

### Examples

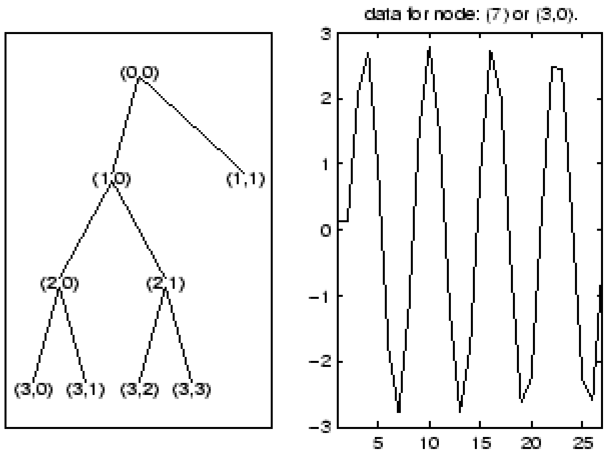
```
% Create a wavelet packet tree.
x   = sin(8*pi*[0:0.005:1]);
t   = wpdec(x,3,'db2');

% Display the generated tree in a Wavelet Packet 1-D GUI window.
fig = drawtree(t);
```



```
%-------------------------------------
% Use the GUI to split or merge Nodes.
%-------------------------------------
```



```
t = readtree(fig);
plot(t)

% Click the node (3,0), (see the plot function).
```



## See Also

`drawtree`

## scal2frq

Scale to frequency

### Syntax

```
F = scal2frq(A,'wname',DELTA)
scal2frq(A,'wname')
scal2frq(A,'wname',1)
```

### Description

`F = scal2frq(A,'wname',DELTA)` returns the pseudo-frequencies corresponding to the scales given by `A` and the wavelet function `'wname'` (see `wavefun` for more information) and the sampling period `DELTA`.

`scal2frq(A,'wname')` is equivalent to `scal2frq(A,'wname',1)`.

There is only an approximate answer for the relationship between scale and frequency.

In wavelet analysis, the way to relate scale to frequency is to determine the center frequency of the wavelet, $F_c$, and use the following relationship.

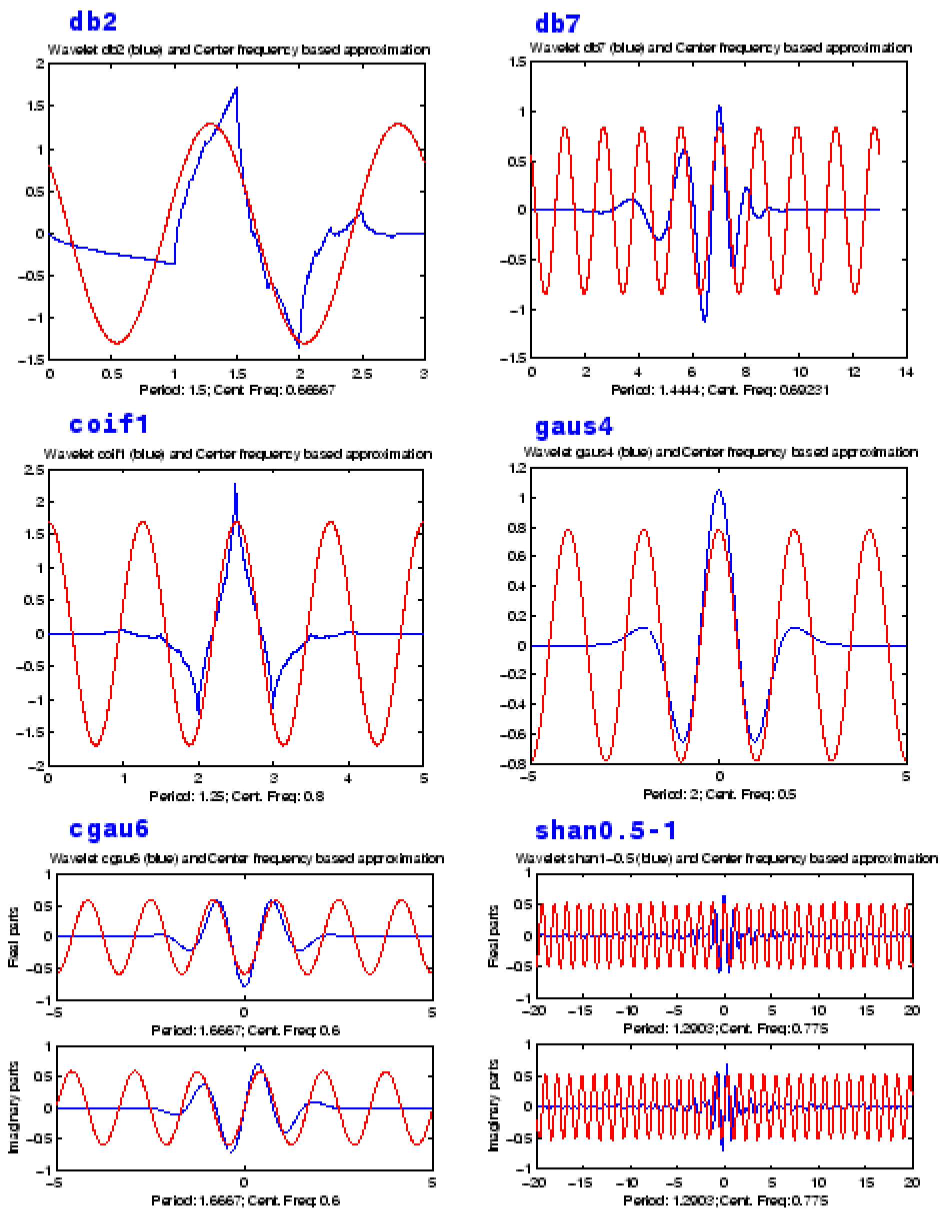
$$F_a = \frac{F_c}{a \cdot \Delta}$$

where

- $a$ is a scale.
- $\Delta$ is the sampling period.
- $F_c$ is the center frequency of a wavelet in Hz.
- $F_a$ is the pseudo-frequency corresponding to the scale $a$, in Hz.

The idea is to associate with a given wavelet a purely periodic signal of frequency $F_c$. The frequency maximizing the Fourier transform of the wavelet modulus is $F_c$. `centfrq` computes the center frequency for a specified wavelet.

Some examples of the correspondence between the center frequency and the wavelet are shown in the following figure.



**Center Frequencies for Real and Complex Wavelets**

As you can see, the center frequency-based approximation captures the main wavelet oscillations. Therefore, the center frequency is a convenient and simple characterization of the dominant frequency of the wavelet.

Dilating the wavelet by $a$, changes the center frequency to $F_c$ / $a$. If the underlying sampling period is $\Delta$, the scale $a$ corresponds to the frequency

$$F_a = \frac{F_c}{a \cdot \Delta}$$

`scal2frq` computes this correspondence.

## Examples

### Scales To Frequencies

Construct a vector of scales with 10 voices per octave over five octaves. Assume the data are sampled at 10 kHz.

```
voicesperoctave = 10;
numoctaves = 5;
a0 = 2^(1/voicesperoctave);
Fs = 1e4;
scales = ...
    a0.^(voicesperoctave:1/voicesperoctave:numoctaves*voicesperoctave);
```

Convert the scales to approximate frequencies in hertz for the Morlet wavelet.

```
Frq = scal2frq(scales,'morl',1/Fs);
```

Determine the corresponding periods. Construct a table with the scales, the corresponding frequencies, and periods. Display the smallest 20 scales along with their corresponding frequencies and periods.

```
Frq = Frq(:);
scales = scales(:);
T = [scales.*(1/Fs) Frq 1./Frq];
T = array2table(T,'VariableNames',{'Scale','Frequency','Period'});
T(1:20,:)
```

```
ans = 

      Scale        Frequency       Period   
    __________    _________    __________

        0.0002    4062.5       0.00024615
    0.00020139    4034.4       0.00024787
    0.00020279    4006.6       0.00024959
     0.0002042    3978.9       0.00025133
    0.00020562    3951.4       0.00025307
    0.00020705    3924.1       0.00025483
    0.00020849      3897       0.00025661
    0.00020994    3870.1       0.00025839
     0.0002114    3843.4       0.00026019
    0.00021287    3816.8         0.000262
    0.00021435    3790.4       0.00026382
    0.00021585    3764.3       0.00026566
    0.00021735    3738.3        0.0002675
    0.00021886    3712.4       0.00026936
    0.00022038    3686.8       0.00027124
    0.00022191    3661.3       0.00027312
    0.00022346      3636       0.00027502
    0.00022501    3610.9       0.00027694
    0.00022658      3586       0.00027886
    0.00022815    3561.2        0.0002808
```

## Plot CWT with Frequencies instead of Scales

The example shows how to plot the CWT using approximate frequencies in Hz instead of scales. This creates a time-frequency plot instead of a time-scale plot.

Create a signal consisting of two sine waves with disjoint support in additive noise. Assume the signal is sampled at 1 kHz.

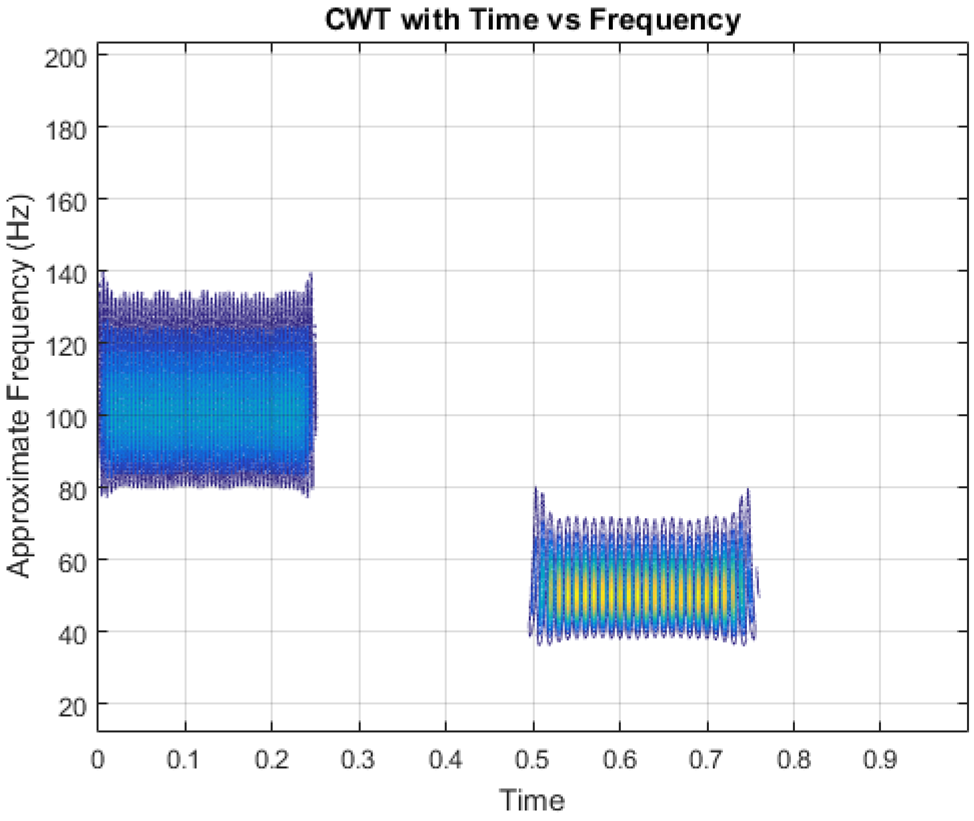
```
Fs = 1000;
t = 0:1/Fs:1-1/Fs;
x = 1.5*cos(2*pi*100*t).*(t<0.25)+1.5*cos(2*pi*50*t).*(t>0.5 & t<=0.75);
x = x+0.05*randn(size(t));
```

Obtain the CWT of the input signal with the Morlet wavelet. Set the number of voices per octave to 32. Create a scale vector to cover 6 octaves. The initial scale is $4\Delta t$ where $\Delta t$ is the sampling interval.

```
numvoices = 32;
a0 = 2^(1/numvoices);
numoctaves = 6;
scales = a0.^(2*numvoices:1/numvoices:numoctaves*numvoices);
cfs = cwt(x,scales,'morl');
```

Convert the scales to approximate frequencies and plot the result.

```
pfreq = scal2frq(scales,'morl',1/Fs);
contour(t,pfreq,abs(cfs).^2);
axis tight;
grid on;
xlabel('Time');
ylabel('Approximate Frequency (Hz)');
title('CWT with Time vs Frequency');
```



## References

Abry, P. (1997), *Ondelettes et turbulence. Multirésolutions, algorithmes de décomposition, invariance d'échelles*, Diderot Editeur, Paris.

## See Also

`centfrq`

# set

WPTREE field contents

## Syntax

```
T = set(T,'FieldName1',FieldValue1,'FieldName2',FieldValue2, ...)
```

## Description

`T = set(T,'FieldName1',FieldValue1,'FieldName2',FieldValue2, ...)` sets the content of the specified fields for the WPTREE object `T`.

For the fields that are objects or structures, you can set the subfield contents, giving the name of these subfields as `'FieldName'` values.

The valid choices for `'FieldName'` are

| | |
|---|---|
| `'dtree'` | DTREE parent object |
| `'wavInfo'` | Structure (wavelet information) |

The fields of the wavelet information structure, `'wavInfo'`, are also valid for `'FieldName'`:

| | |
|---|---|
| `'wavName'` | Wavelet name |
| `'Lo_D'` | Low Decomposition filter |
| `'Hi_D'` | High Decomposition filter |
| `'Lo_R'` | Low Reconstruction filter |
| `'Hi_R'` | High Reconstruction filter |

| | |
|---|---|
| `'entInfo'` | Structure (entropy information) |

The fields of the entropy information structure, `'entInfo'`, are also valid for `'FieldName'`:

| | |
|---|---|
| `'entName'` | Entropy name |
| `'entPar'` | Entropy parameter |

Or fields of DTREE parent object:

| | |
|---|---|
| `'ntree'` | NTREE parent object |
| `'allNI'` | All nodes information |
| `'terNI'` | Terminal nodes information |

Or fields of NTREE parent object:

| | |
|---|---|
| `'wtbo'` | WTBO parent object |
| `'order'` | Order of the tree |
| `'depth'` | Depth of the tree |
| `'spsch'` | Split scheme for nodes |
| `'tn'` | Array of terminal nodes of the tree |

Or fields of WTBO parent object:

| | |
|---|---|
| `'wtboInfo'` | Object information |
| `'ud'` | Userdata field |

**Caution** The `set` function should only be used to set the field `'ud'`.

## See Also

`disp` | `get` | `read` | `write`

# shanwavf

Complex Shannon wavelet

## Syntax

```
[PSI,X] = shanwavf(LB,UB,N,FB,FC)
```

## Description

`[PSI,X] = shanwavf(LB,UB,N,FB,FC)` returns values of the complex Shannon wavelet. The complex Shannon wavelet is defined by a bandwidth parameter `FB`, a wavelet center frequency `FC`, and the expression

```
PSI(X) = (FB^0.5)*(sinc(FB*X).*exp(2*i*pi*FC*X))
```

on an `N` point regular grid in the interval `[LB,UB]`.

`FB` and `FC` must be such that `FC > 0` and `FB > 0`.

Output arguments are the wavelet function `PSI` computed on the grid `X`.

## Examples

### Complex Shannon Wavelet

Obtain and plot a complex Shannon wavelet. Set the bandwidth and center frequency parameters.

```
fb = 1; fc = 1.5;
```

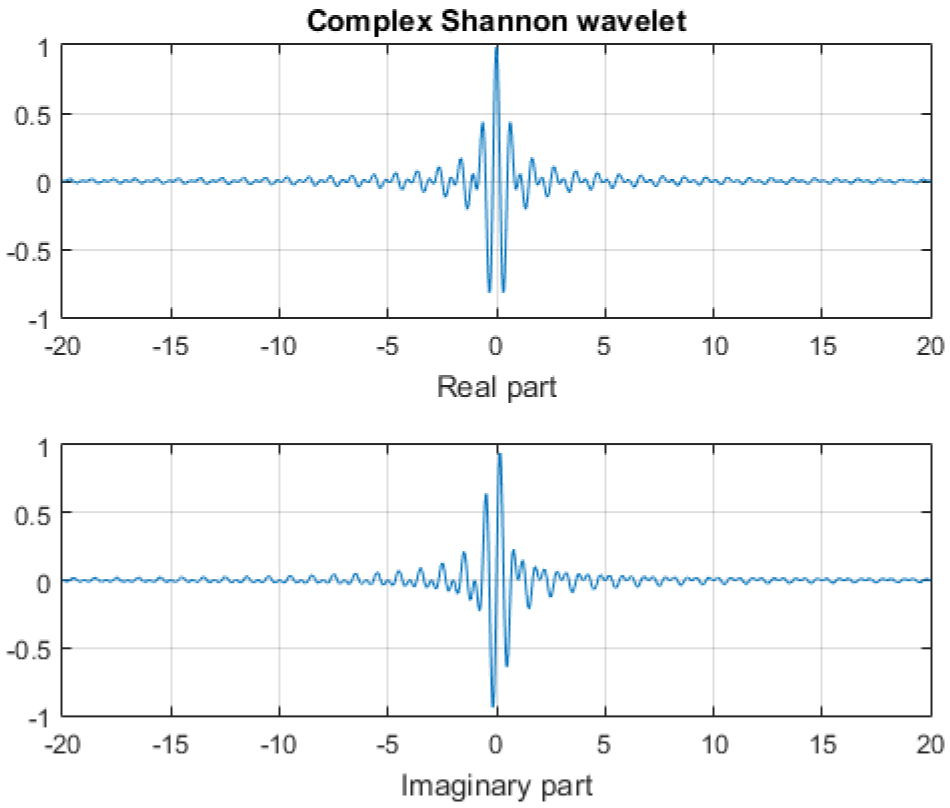
Set the effective support and number of sample points.

```
lb = -20;
ub = 20;
n = 1000;
```

Obtain the complex-valued Shannon wavelet and plot the real and imaginary parts.

```
[psi,x] = shanwavf(lb,ub,n,fb,fc);
```

```
subplot(211)
plot(x,real(psi))
title('Complex Shannon wavelet')
xlabel('Real part');
grid on;
subplot(212)
plot(x,imag(psi))
xlabel('Imaginary part')
grid on;
```



## References

Teolis, A. (1998), *Computational signal processing with wavelets*, Birkäuser, p. 62.

## See Also

`waveinfo`

# swt

Discrete stationary wavelet transform 1-D

## Syntax

```
SWC = swt(X,N,'wname')
SWC = swt(X,N,Lo_D,Hi_D)
```

## Description

`swt` performs a multilevel 1-D stationary wavelet decomposition using either a specific orthogonal wavelet (`'wname'`, see `wfilters` for more information) or specific orthogonal wavelet decomposition filters.

`SWC = swt(X,N,'wname')` computes the stationary wavelet decomposition of the signal `X` at level `N`, using `'wname'`.

`N` must be a strictly positive integer (see `wmaxlev` for more information) and `length(X)` must be a multiple of $2^N$ .

`SWC = swt(X,N,Lo_D,Hi_D)` computes the stationary wavelet decomposition as above, given these filters as input:

- `Lo_D` is the decomposition low-pass filter.
- `Hi_D` is the decomposition high-pass filter.

`Lo_D` and `Hi_D` must be the same length.

The output matrix `SWC` contains the vectors of coefficients stored row-wise:

For $1 \leq i \leq N$, the output matrix `SWC(i,:)` contains the detail coefficients of level `i` and `SWC(N+1,:)` contains the approximation coefficients of level `N`.

`[SWA,SWD] = swt( )` computes approximations, `SWA`, and details, `SWD`, stationary wavelet coefficients.

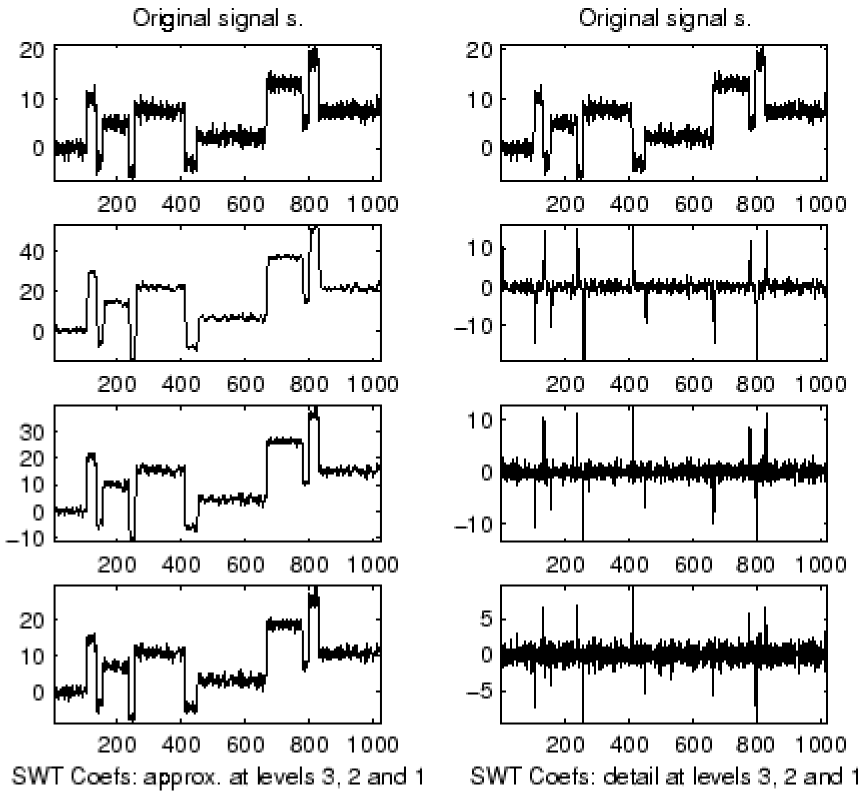
The vectors of coefficients are stored row-wise:

For `1` $\leq$ `i` $\leq$ `N`, the output matrix `SWA(i,:)` contains the approximation coefficients of level `i` and the output matrix `SWD(i,:)` contains the detail coefficients of level `i`.

**Note** `swt` is defined using `dwt` with periodic extension.

## Examples

```
% Load original 1D signal.
load noisbloc; s = noisbloc;

% Perform SWT decomposition at level 3 of s using db1.
[swa,swd] = swt(s,3,'db1');

% Plots of SWT coefficients of approximations and details
% at levels 3 to 1.

% Using some plotting commands,
% the following figure is generated.
```

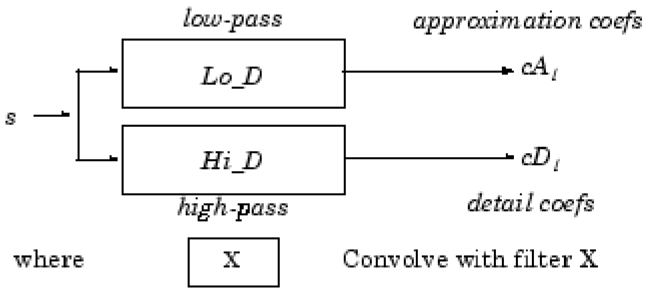


## More About

### Algorithms

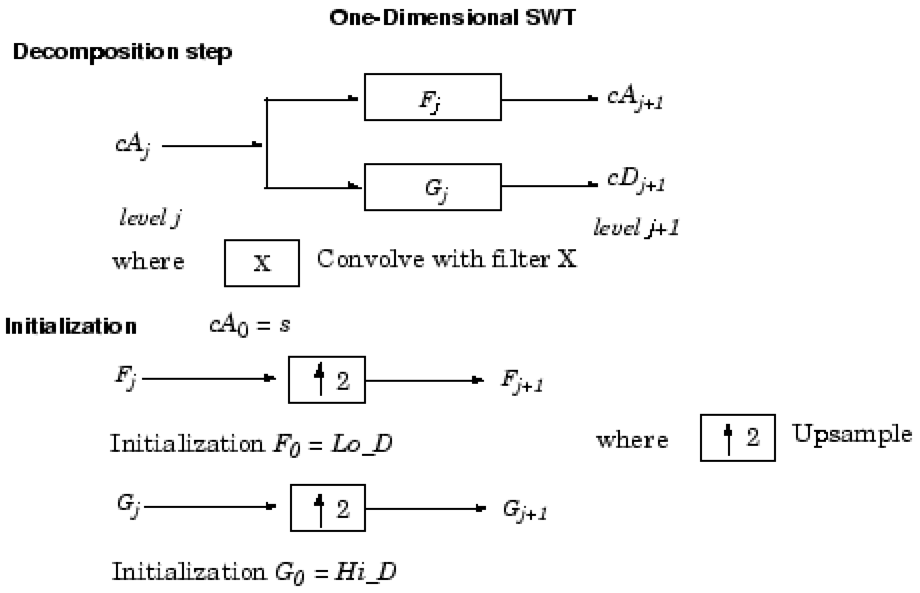
Given a signal $s$ of length $N$, the first step of the SWT produces, starting from $s$, two sets of coefficients: approximation coefficients $cA_1$ and detail coefficients $cD_1$. These vectors are obtained by convolving $s$ with the low-pass filter `Lo_D` for approximation, and with the high-pass filter `Hi_D` for detail.

More precisely, the first step is



**Note** $cA_1$ and $cD_1$ are of length `N` instead of `N/2` as in the DWT case.

The next step splits the approximation coefficients $cA_1$ in two parts using the same scheme, but with modified filters obtained by upsampling the filters used for the previous step and replacing $s$ by $cA_1$. Then, the SWT produces $cA_2$ and $cD_2$. More generally,



## References

Nason, G.P.; B.W. Silverman (1995), “The stationary wavelet transform and some statistical applications,” *Lecture Notes in Statistics*, 103, pp. 281–299.

Coifman, R.R.; Donoho, D.L. (1995), “Translation invariant de-noising,” *Lecture Notes in Statistics*, 103, pp. 125–150.

Pesquet, J.C.; H. Krim, H. Carfatan (1996), “Time-invariant orthonormal wavelet representations,” *IEEE Trans. Sign. Proc.*, vol. 44, 8, pp. 1964–1970.

## See Also

`dwt` | `wavedec`

## swt2

Discrete stationary wavelet transform 2-D

### Syntax

```
SWC = swt2(X,N,'wname')
[A,H,V,D] = swt2(X,N,'wname')
SWC = swt2(X,N,Lo_D,Hi_D)
[A,H,V,D] = swt2(X,N,Lo_D,Hi_D)
```

### Description

`swt2` performs a multilevel 2-D stationary wavelet decomposition using either a specific orthogonal wavelet (`'wname'`— see `wfilters` for more information) or specific orthogonal wavelet decomposition filters.

`SWC = swt2(X,N,'wname')` or `[A,H,V,D] = swt2(X,N,'wname')` compute the stationary wavelet decomposition of the matrix `X` at level `N`, using `'wname'`.

`N` must be a strictly positive integer (see `wmaxlev` for more information), and $2^N$ must divide `size(X,1)` and `size(X,2)`.

Outputs `[A,H,V,D]` are 3-D arrays, which contain the coefficients:

- For `1` $\leq$ `i` $\leq$ `N`, the output matrix `A(:,:,i)` contains the coefficients of approximation of level `i`.
- The output matrices `H(:,:,i)`, `V(:,:,i)` and `D(:,:,i)` contain the coefficients of details of level `i` (horizontal, vertical, and diagonal):

  ```
  SWC = [H(:,:,1:N) ; V(:,:,1:N) ; D(:,:,1:N) ; A(:,:,N)]
  ```

`SWC = swt2(X,N,Lo_D,Hi_D)` or `[A,H,V,D] = swt2(X,N,Lo_D,Hi_D)`, computes the stationary wavelet decomposition as in the previous syntax, given these filters as input:

- `Lo_D` is the decomposition low-pass filter.

- Hi_D is the decomposition high-pass filter.

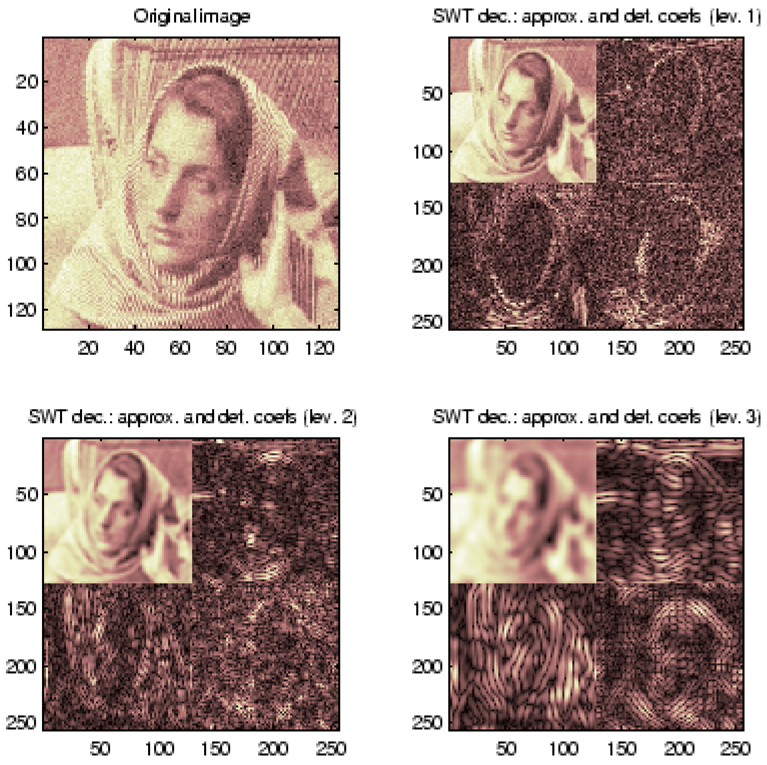
Lo_D and Hi_D must be the same length.

**Note** swt2 is defined using dwt with periodic extension.

## Examples

```
% Load original image.
load nbarb1;

% Image coding.
nbcol = size(map,1);
cod_X = wcodemat(X,nbcol);

% Visualize the original image.
subplot(221)
image(cod_X)
title('Original image');
colormap(map)

% Perform SWT decomposition
% of X at level 3 using sym4.
[ca,chd,cvd,cdd] = swt2(X,3,'sym4');

% Visualize the decomposition.

for k = 1:3
    % Images coding for level k.
    cod_ca  = wcodemat(ca(:,:,k),nbcol);
    cod_chd = wcodemat(chd(:,:,k),nbcol);
    cod_cvd = wcodemat(cvd(:,:,k),nbcol);
    cod_cdd = wcodemat(cdd(:,:,k),nbcol);
    decl = [cod_ca,cod_chd;cod_cvd,cod_cdd];

    % Visualize the coefficients of the decomposition
    % at level k.
    subplot(2,2,k+1)
    image(decl)

    title(['SWT dec.: approx. ', ...
   'and det. coefs (lev. ',num2str(k),')']);
```

```
    colormap(map)
end
% Editing some graphical properties,
% the following figure is generated.
```



## More About

### Tips

When X represents an indexed image, X is an `m`-by-`n` matrix and the output arrays SWC or cA,cH,cV, and cD are `m`-by-`n`-by-`p` arrays.

When X represents a truecolor image, it becomes an `m`-by-`n`-by-3 array. This array is an `m`-by-`n`-by-3 array, where each `m`-by-`n` matrix represents a red, green, or blue color plane concatenated along the third dimension. The output arrays SWC or cA,cH,cV, and cD are `m`-by-`n`-by-`p`-by-3 arrays.

For more information on image formats, see the `image` and `imfinfo` reference pages.

## Algorithms

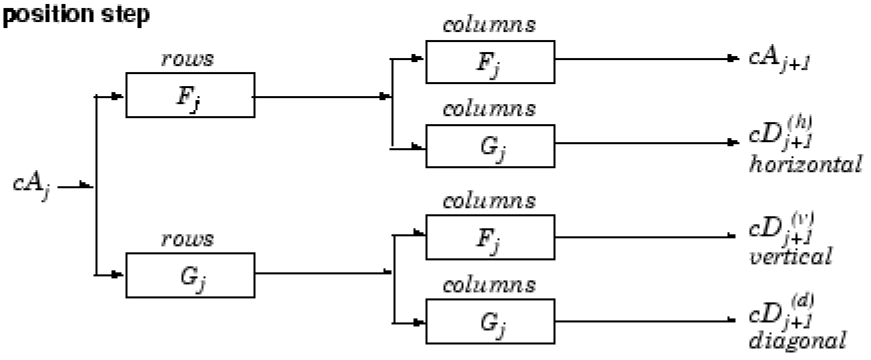
For images, an algorithm similar to the one-dimensional case is possible for two-dimensional wavelets and scaling functions obtained from one-dimensional ones by tensor product.

This kind of two-dimensional SWT leads to a decomposition of approximation coefficients at level*j* in four components: the approximation at level *j*+1, and the details in three orientations (horizontal, vertical, and diagonal).

The following chart describes the basic decomposition step for images:

## Two-Dimensional SWT

**Decomposition step**



where

$\overset{rows}{\boxed{X}}$ Convolve with filter X the rows of the entry

$\overset{columns}{\boxed{X}}$ Convolve with filter X the columns of the entry

**Initialization** $cA_0 = s$ for the decomposition initialization

$$\begin{tikzcd} F_j \arrow[r] & \boxed{\uparrow 2} \arrow[r] & F_{j+1} \end{tikzcd}$$

Initialization $F_0 = Lo\_D$

where $\boxed{\uparrow 2}$ Upsample

$$\begin{tikzcd} G_j \arrow[r] & \boxed{\uparrow 2} \arrow[r] & G_{j+1} \end{tikzcd}$$

Initialization $G_0 = Hi\_D$

**Note** $size(cA_j) = size(cD_j^{(h)}) = size(cD_j^{(v)}) = size(cD_j^{(d)}) = s$

where $s = size\ of\ the\ analyzed\ image$

## References

Nason, G.P.; B.W. Silverman (1995), “The stationary wavelet transform and some statistical applications,” *Lecture Notes in Statistics*, 103, pp. 281–299.

Coifman, R.R.; Donoho, D.L. (1995), “Translation invariant de-noising,” *Lecture Notes in Statistics*, 103, pp. 125–150.

Pesquet, J.C.; H. Krim, H. Carfatan (1996), “Time-invariant orthonormal wavelet representations,” *IEEE Trans. Sign. Proc.*, vol. 44, 8, pp. 1964–1970.

## See Also

`dwt2` | `iswt2` | `wavedec2`

## symaux

Symlet wavelet filter computation

### Syntax

```
W = SYMAUX(N,SUMW)
W = SYMAUX(N)
W = SYMAUX(N,1)
W = SYMAUX(N,0)
W = SYMAUX(N,1)
```

### Description

Symlets are the Äúleast asymmetricÄù Daubechies wavelets.

`W = SYMAUX(N,SUMW)` is the order `N` Symlet scaling filter such that `SUM(W) = SUMW`. Possible values for `N` are 1, 2, 3, ...

**Note** Instability may occur when `N` is too large.

`W = SYMAUX(N)` is equivalent to `W = SYMAUX(N,1)`.

`W = SYMAUX(N,0)` is equivalent to `W = SYMAUX(N,1)`.

### Examples

```
% Generate wdb4 the order 4 Daubechies scaling filter.
wdb4 = dbaux(4)

wdb4 =

  Columns 1 through 7

    0.1629    0.5055    0.4461   -0.0198   -0.1323    0.0218    0.0233
  Column 8

   -0.0075


% wdb4 is a solution of the equation: P = conv(wrev(w),w)*2,
% where P is the "Lagrange  trous" filter for N=4.
% wdb4 is a minimum phase solution of the previous equation,
% based on the roots of P (see dbaux).
P = conv(wrev(wdb4),wdb4)*2;

% Generate wsym4 the order 4 symlet scaling filter.
% The Symlets are the "least asymmetric" Daubechies'
% wavelets obtained from another choice between the roots of P.
wsym4 = symaux(4)

wsym4 =

  Columns 1 through 7

    0.0228   -0.0089   -0.0702    0.2106    0.5683    0.3519   -0.0210

  Column 8

   -0.0536


% Compute conv(wrev(wsym4),wsym4) * 2 and check that wsym4
% is another solution of the equation P = conv(wrev(w),w)*2.
Psym = conv(wrev(wsym4),wsym4)*2;
err = norm(P-Psym)

err =

  7.4988e-016
```

## See Also

symwavf | wfilters

## symwavf

Symlet wavelet filter

### Syntax

```
F = symwavf(W)
```

### Description

`F = symwavf(W)` returns the scaling filter associated with the symlet wavelet specified by the string *W* where `W = 'symN'`. Possible values for `N` are 2, 3, ..., 45.

### Examples

```
% Compute the scaling filter corresponding to wavelet sym4.
w = symwavf('sym4')

w =
 Columns 1 through 7
    0.0228 -0.0089 -0.0702 0.2106 0.5683 0.3519 -0.0210
 Column 8
    -0.0536
```

### See Also

`symaux` | `waveinfo`

# thselect

Threshold selection for de-noising

## Syntax

```
THR = thselect(X,TPTR)
```

## Description

`thselect` is a one-dimensional de-noising oriented function.

`THR = thselect(X,TPTR)` returns threshold `X`-adapted value using selection rule defined by string `TPTR`.

Available selection rules are

- `TPTR = 'rigrsure'`, adaptive threshold selection using principle of Stein's Unbiased Risk Estimate.
- `TPTR = 'heursure'`, heuristic variant of the first option.
- `TPTR = 'sqtwolog'`, threshold is `sqrt(2*log(length(X)))`.
- `TPTR = 'minimaxi'`, minimax thresholding.

Threshold selection rules are based on the underlying model $y = f(t) + e$ where $e$ is a white noise $N(0,1)$. Dealing with unscaled or nonwhite noise can be handled using rescaling output threshold `THR` (see `SCAL` parameter in `wden` for more information).

Available options are

- `tptr = 'rigrsure'` uses for the soft threshold estimator, a threshold selection rule based on SteinÄôs Unbiased Estimate of Risk (quadratic loss function). One gets an estimate of the risk for a particular threshold value ($t$). Minimizing the risks in ($t$) gives a selection of the threshold value.
- `tptr = 'sqtwolog'` uses a fixed-form threshold yielding minimax performance multiplied by a small factor proportional to `log(length(X))`.
- `tptr = 'heursure'` is a mixture of the two previous options. As a result, if the signal to noise ratio is very small, the SURE estimate is very noisy. If such a situation is detected, the fixed form threshold is used.

- `tptr = 'minimaxi'` uses a fixed threshold chosen to yield minimax performance for mean square error against an ideal procedure. The minimax principle is used in statistics in order to design estimators. Since the de-noised signal can be assimilated to the estimator of the unknown regression function, the minimax estimator is the one that realizes the minimum of the maximum mean square error obtained for the worst function in a given set.

## Examples

```
% The current extension mode is zero-padding (see dwtmode).
% Generate Gaussian white noise.
x = randn(1,1000);

% Find threshold for each selection rule.
% Adaptive threshold using SURE.
thr = thselect(x,'rigrsure')
% Fixed form threshold.
thr = thselect(x,'sqtwolog')
% Heuristic variant of the first option.
thr = thselect(x,'heursure')
% Minimax threshold.
thr = thselect(x,'minimaxi')
```

## References

Donoho, D.L. (1993), “Progress in wavelet analysis and WVD: a ten minute tour,” in *Progress in wavelet analysis and applications*, Y. Meyer, S. Roques, pp. 109–128. Frontières Ed.

Donoho, D.L., I.M. Johnstone (1994), “Ideal spatial adaptation by wavelet shrinkage,” *Biometrika*, vol 81, pp. 425–455.

Donoho, D.L. (1995), “De-noising by soft-thresholding,” *IEEE Trans. on Inf. Theory*, 41, 3, pp. 613–627.

### See Also

`wden`

# tnodes

Determine terminal nodes

## Syntax

```
N = tnodes(T)
N = tnodes(T,'deppos')
[N,K] = tnodes(T)
[N,K] = tnodes(T,'deppos'), M = N(K)
```

## Description

`tnodes` is a tree-management utility.

`N = tnodes(T)` returns the indices of terminal nodes of the tree *T*. `N` is a column vector.

The nodes are numbered from left to right and from top to bottom. The root index is 0.

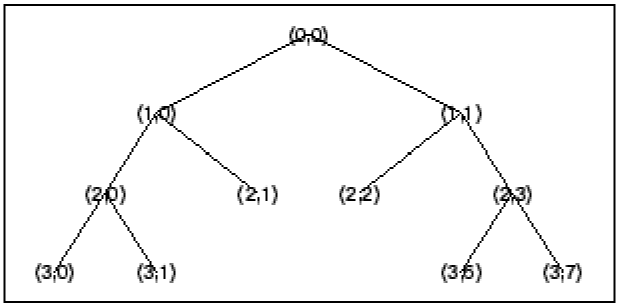
`N = tnodes(T,'deppos')` returns a matrix `N`, which contains the depths and positions of terminal nodes.

`N(i,1)` is the depth of the `i`-th terminal node. `N(i,2)` is the position of the `i`-th terminal node.

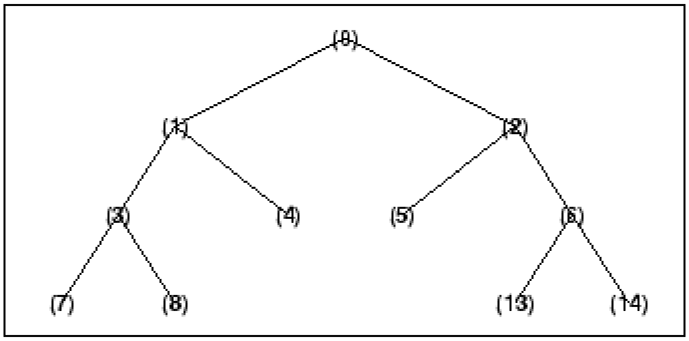
For `[N,K] = tnodes(T)` or `[N,K] = tnodes(T,'deppos'), M = N(K)` are the indices reordered as in tree *T*, from left to right.

## Examples

```
% Create initial tree.
ord = 2;
t = ntree(ord,3);      % Binary tree of depth 3.
t = nodejoin(t,5);
t = nodejoin(t,4);
plot(t)
```



```
% Change Node Label from Depth_Position to Index
% (see the plot function).
```



```
% List terminal nodes (index).
tnodes(t)

ans =
    4
    5
    7
    8
    13
    14
% List terminal nodes (Depth_Position).
tnodes(t,'deppos')
ans =
    2   1
    2   2
    3   0
    3   1
```

```
     3     6
     3     7
```

## See Also

leaves | noleaves | wtreemgr

## treedpth

Tree depth

### Syntax

```
D = treedpth(T)
```

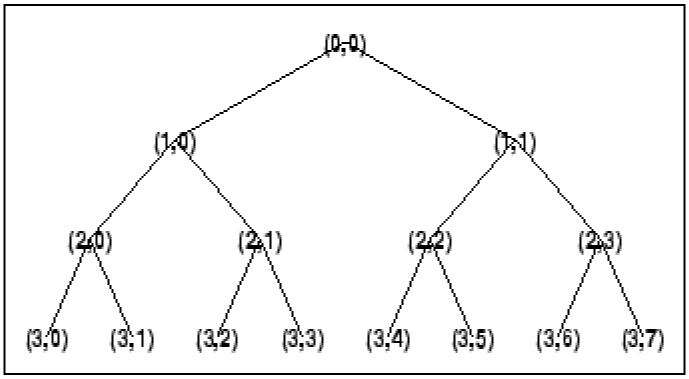
### Description

`treedpth` is a tree-management utility.

`D = treedpth(T)` returns the depth `D` of the tree `T`.

### Examples

```
% Create binary tree (tree of order 2) of depth 3.
t = ntree(2,3);

% Plot tree t.
plot(t)
```



```
% Tree depth.
treedpth(t)
```

```
ans =
    3
```

## See Also

`wtreemgr`

# treeord

Tree order

## Syntax

```
ORD = treeord(T)
```

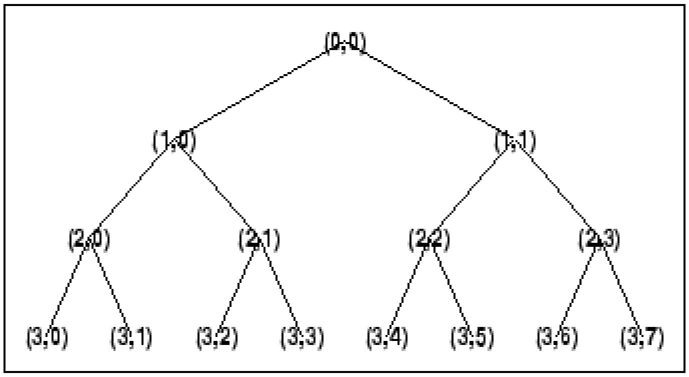
## Description

`treeord` is a tree-management utility.

`ORD = treeord(T)` returns the order `ORD` of the tree *T*.

## Examples

```
% Create binary tree (tree of order 2) of depth 3.
t = ntree(2,3);

% Plot tree t.
plot(t)
```



```
% Tree order.
treeord(t)
```

```
ans =
    2
```

## See Also

`wtreemgr`

## upcoef

Direct reconstruction from 1-D wavelet coefficients

### Syntax

```
Y = upcoef(O,X,'wname',N)
Y = upcoef(O,X,'wname',N,L)
Y = upcoef(O,X,Lo_R,Hi_R,N)
Y = upcoef(O,X,Lo_R,Hi_R,N,L)
Y = upcoef(O,X,'wname'')
Y = upcoef(O,X,'wname'',1)
Y = upcoef(O,X,Lo_R,Hi_R)
Y = upcoef(O,X,Lo_R,Hi_R,1)
```

### Description

`upcoef` is a one-dimensional wavelet analysis function.

`Y = upcoef(O,X,'wname',N)` computes the `N`-step reconstructed coefficients of vector `X`.

`'wname'` is a string containing the wavelet name. See `wfilters` for more information.

`N` must be a strictly positive integer.

If `O` = `'a'`, approximation coefficients are reconstructed.

If `O` = `'d'`, detail coefficients are reconstructed.

`Y = upcoef(O,X,'wname',N,L)` computes the `N`-step reconstructed coefficients of vector `X` and takes the length-`L` central portion of the result.

Instead of giving the wavelet name, you can give the filters.

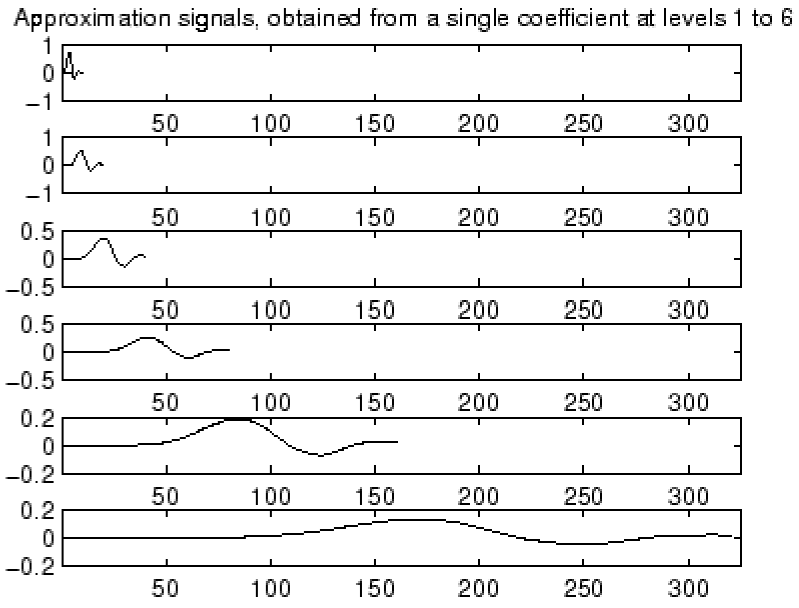
For `Y = upcoef(O,X,Lo_R,Hi_R,N)` or `Y = upcoef(O,X,Lo_R,Hi_R,N,L)`, `Lo_R` is the reconstruction low-pass filter and `Hi_R` is the reconstruction high-pass filter.

`Y = upcoef(O,X,'`*`wname`*`'')` is equivalent to `Y = upcoef(O,X,'`*`wname`*`'',1)`.

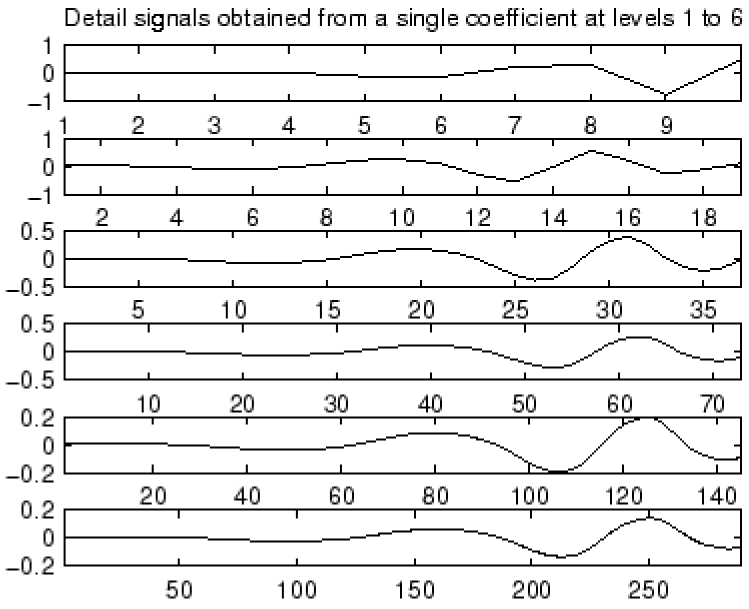
`Y = upcoef(O,X,Lo_R,Hi_R)` is equivalent to `Y = upcoef(O,X,Lo_R,Hi_R,1)`.

## Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Approximation signals, obtained from a single coefficient
% at levels 1 to 6.
cfs = [1];  % Decomposition reduced a single coefficient.
essup = 10; % Essential support of the scaling filter db6.
figure(1)
for i=1:6
    % Reconstruct at the top level an approximation
    % which is equal to zero except at level i where only
    % one coefficient is equal to 1.
    rec = upcoef('a',cfs,'db6',i);

    % essup is the essential support of the
    % reconstructed signal.
    % rec(j) is very small when j is ≥ essup.
    ax = subplot(6,1,i),h = plot(rec(1:essup));
    set(ax,'xlim',[1 325]);
    essup = essup*2;

end
subplot(611)
title(['Approximation signals, obtained from a single ' ...
     'coefficient at levels 1 to 6'])

% Editing some graphical properties,
% the following figure is generated.
```



```
% The same can be done for details.
% Details signals, obtained from a single coefficient
% at levels 1 to 6.

cfs = [1];
mi = 12; ma = 30;   % Essential support of
                    % the wavelet filter db6.
rec = upcoef('d',cfs,'db6',1);
figure(2)
subplot(611), plot(rec(3:12))
for i=2:6
    % Reconstruct at top level a single detail
    % coefficient at level i.
    rec = upcoef('d',cfs,'db6',i);
    subplot(6,1,i), plot(rec(mi*2^(i-2):ma*2^(i-2)))
end
subplot(611)
title(['Detail signals obtained from a single ' ...
    'coefficient at levels 1 to 6'])
% Editing some graphical properties,
% the following figure is generated.
```



## More About

### Algorithms

`upcoef` is equivalent to an `N` time repeated use of the inverse wavelet transform.

## See Also

`idwt`

# upcoef2

Direct reconstruction from 2-D wavelet coefficients

## Syntax

```
Y = upcoef2(O,X,'wname',N,S)
Y = upcoef2(O,X,Lo_R,Hi_R,N,S)
Y = upcoef2(O,X,'wname',N)
Y = upcoef2(O,X,Lo_R,Hi_R,N)
Y = upcoef2(O,X,'wname')
Y = upcoef2(O,X,'wname',1)
Y = upcoef2(O,X,Lo_R,Hi_R)
Y = upcoef2(O,X,Lo_R,Hi_R,1)
```

## Description

`upcoef2` is a two-dimensional wavelet analysis function.

`Y = upcoef2(O,X,'wname',N,S)` computes the `N`-step reconstructed coefficients of matrix `X` and takes the central part of size `S`. `'wname'` is a string containing the name of the wavelet. See `wfilters` for more information.

If `O = 'a'`, approximation coefficients are reconstructed; otherwise if `O = 'h'` (`'v'` or `'d'`, respectively), horizontal (vertical or diagonal, respectively) detail coefficients are reconstructed. `N` must be a strictly positive integer.

Instead of giving the wavelet name, you can give the filters.

For `Y = upcoef2(O,X,Lo_R,Hi_R,N,S)` is the reconstruction low-pass filter and `Hi_R` is the reconstruction high-pass filter.

`Y = upcoef2(O,X,'wname',N)` or `Y = upcoef2(O,X,Lo_R,Hi_R,N)` returns the computed result without any truncation.

`Y = upcoef2(O,X,'wname')` is equivalent to `Y = upcoef2(O,X,'wname',1)`.

`Y = upcoef2(O,X,Lo_R,Hi_R)` is equivalent to
`Y = upcoef2(O,X,Lo_R,Hi_R,1)`.

## Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load original image.
load woman;
% X contains the loaded image.

% Perform decomposition at level 2
% of X using db4.
[c,s] = wavedec2(X,2,'db4');

% Reconstruct approximation and details
% at level 1, from coefficients.
% This can be done using wrcoef2, or
% equivalently using:
%
% Step 1: Extract coefficients from the
% decomposition structure [c,s].
%
% Step 2: Reconstruct using upcoef2.

siz = s(size(s,1),:);

ca1 = appcoef2(c,s,'db4',1);
a1 = upcoef2('a',ca1,'db4',1,siz);

chd1 = detcoef2('h',c,s,1);
hd1 = upcoef2('h',chd1,'db4',1,siz);

cvd1 = detcoef2('v',c,s,1);
vd1 = upcoef2('v',cvd1,'db4',1,siz);

cdd1 = detcoef2('d',c,s,1);
dd1 = upcoef2('d',cdd1,'db4',1,siz);
```

## More About

### Algorithms

See `upcoef`.

## See Also

`idwt2`

# upwlev

Single-level reconstruction of 1-D wavelet decomposition

## Syntax

```
[NC,NL,cA] = upwlev(C,L,'wname')
```

## Description

`upwlev` is a one-dimensional wavelet analysis function.

`[NC,NL,cA] = upwlev(C,L,'wname')` performs the single-level reconstruction of the wavelet decomposition structure `[C,L]` giving the new one `[NC,NL]`, and extracts the last approximation coefficients vector `cA`.

`[C,L]` is a decomposition at level `n = length(L)-2`, so `[NC,NL]` is the same decomposition at level `n`-1 and `cA` is the approximation coefficients vector at level `n`.

`'wname'` is a string containing the wavelet name, `C` is the original wavelet decomposition vector, and `L` the corresponding bookkeeping vector (for detailed storage information, see `wavedec` ).

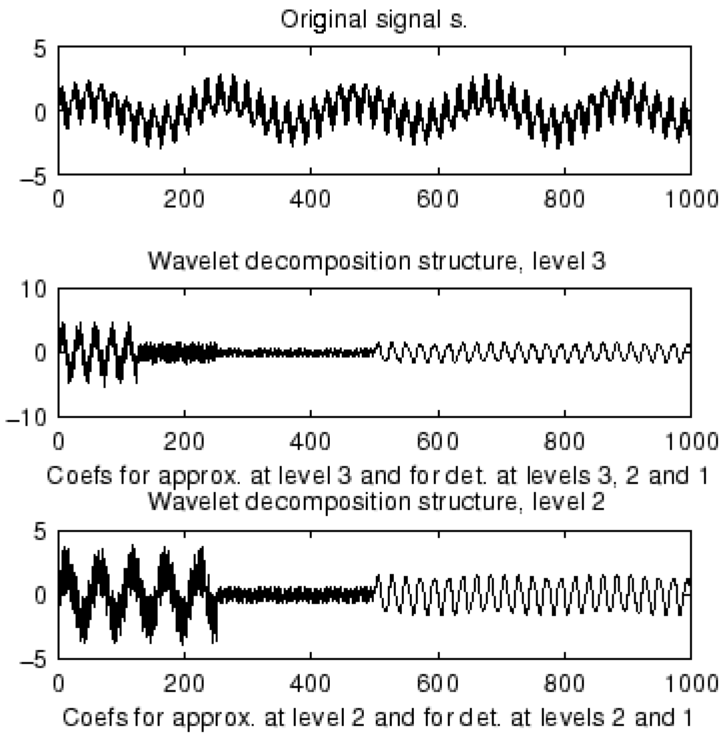
Instead of giving the wavelet name, you can give the filters.

For `[NC,NL,cA] = upwlev(C,L,Lo_R,Hi_R)`, `Lo_R` is the reconstruction low-pass filter and `Hi_R` is the reconstruction high-pass filter.

## Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load original one-dimensional signal.
load sumsin; s = sumsin;

% Perform decomposition at level 3 of s using db1.
[c,l] = wavedec(s,3,'db1');
```

```
subplot(311); plot(s);
title('Original signal s.');
subplot(312); plot(c);
title('Wavelet decomposition structure, level 3')
xlabel(['Coefs for approx. at level 3 ' ...
        'and for det. at levels 3, 2 and 1'])

% One step reconstruction of the wavelet decomposition
% structure at level 3 [c,l], so the new structure [nc,nl]
% is the wavelet decomposition structure at level 2.
[nc,nl] = upwlev(c,l,'db1');
subplot(313); plot(nc);
title('Wavelet decomposition structure, level 2')
xlabel(['Coefs for approx. at level 2 ' ...
        'and for det. at levels 2 and 1'])

% Editing some graphical properties,
% the following figure is generated.
```



## More About

- `upcoef`
- `wavedec`

### See Also

`idwt`

## upwlev2

Single-level reconstruction of 2-D wavelet decomposition

### Syntax

```
[NC,NS,cA] = upwlev2(C,S,'wname')
[NC,NS,cA] = upwlev2(C,S,Lo_R,Hi_R)
```

### Description

`upwlev2` is a two-dimensional wavelet analysis function.

`[NC,NS,cA] = upwlev2(C,S,'wname')` performs the single-level reconstruction of wavelet decomposition structure `[C,S]` giving the new one `[NC,NS]`, and extracts the last approximation coefficients matrix `cA`.

`[C,S]` is a decomposition at level `n = size(S,1)-2`, so `[NC,NS]` is the same decomposition at level `n`-1 and `cA` is the approximation matrix at level `n`.

`'wname'` is a string containing the wavelet name, `C` is the original wavelet decomposition vector, and `S` the corresponding bookkeeping matrix (for detailed storage information, see `wavedec2`).

Instead of giving the wavelet name, you can give the filters.

For `[NC,NS,cA] = upwlev2(C,S,Lo_R,Hi_R)`, `Lo_R` is the reconstruction low-pass filter and `Hi_R` is the reconstruction high-pass filter.

### Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load original image.
load woman;
% X contains the loaded image.
% Perform decomposition at level 2
% of X using db1.
[c,s] = wavedec2(X,2,'db1');
sc = size(c)

sc =
    1   65536

val_s = s

val_s =
    64   64
    64   64
    128  128
    256  256

% One step reconstruction of wavelet
% decomposition structure [c,s].
[nc,ns] = upwlev2(c,s,'db1');
snc = size(nc)

snc =
    1   65536

val_ns = ns

val_ns =
    128   128
    128   128
    256   256
```

## See Also

`idwt2` | `upcoef2` | `wavedec2`

# wave2lp

Laurent polynomials associated with wavelet

## Syntax

```
[Hs,Gs,Ha,Ga] = wave2lp(W)
```

## Description

`[Hs,Gs,Ha,Ga] = wave2lp(W)` returns the four Laurent polynomials associated with the wavelet *W* (see `liftwave`).

The pairs `(Hs,Gs)` and `(Ha,Ga)` are the synthesis and the analysis pair respectively.

The `H`-polynomials (`G`-polynomials) are low-pass (high-pass) polynomials.

For an orthogonal wavelet, `Hs` = `Ha` and `Gs` = `Ga`.

## Examples

```
% Get Laurent polynomials associated to the "lazy" wavelet.
[Hs,Gs,Ha,Ga] = wave2lp('lazy')

Hs(z) = 1

Gs(z) = z^(-1)

Ha(z) = 1

Ga(z) = z^(-1)

% Get Laurent polynomials associated to the db1 wavelet.
[Hs,Gs,Ha,Ga] = wave2lp('db1')

Hs(z) = + 0.7071 + 0.7071*z^(-1)

Gs(z) = - 0.7071 + 0.7071*z^(-1)
```

```
Ha(z) = + 0.7071 + 0.7071*z^(-1)

Ga(z) = - 0.7071 + 0.7071*z^(-1)

% Get Laurent polynomials associated to the bior1.3 wavelet.
[Hs,Gs,Ha,Ga] = wave2lp('bior1.3')

Hs(z) = + 0.7071 + 0.7071*z^(-1)

Gs(z) = ...
    + 0.08839*z^(+2) + 0.08839*z^(+1) - 0.7071 + 0.7071*z^(-1) -
0.08839*z^(-2)  ...
    - 0.08839*z^(-3)

Ha(z) = ...
    - 0.08839*z^(+2) + 0.08839*z^(+1) + 0.7071 + 0.7071*z^(-1) +
0.08839*z^(-2)  ...
    - 0.08839*z^(-3)

Ga(z) = - 0.7071 + 0.7071*z^(-1)
```

## See Also

`laurpoly`

## wavedec

Multilevel 1-D wavelet decomposition

### Syntax

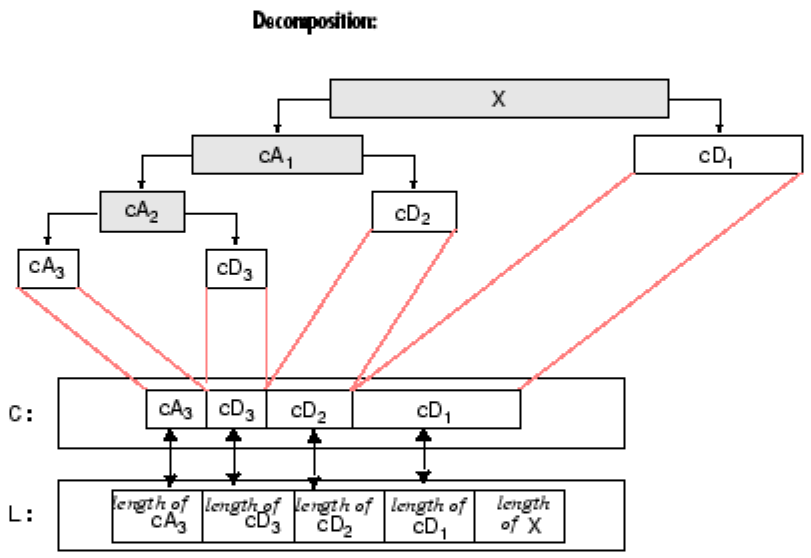
```
[C,L] = wavedec(X,N,'wname')
[C,L] = wavedec(X,N,Lo_D,Hi_D)
```

### Description

`wavedec` performs a multilevel one-dimensional wavelet analysis using either a specific wavelet (`'wname'`) or a specific wavelet decomposition filters (`Lo_D` and `Hi_D`, see `wfilters`).

**Note** `wavedec` supports only Type 1 (orthogonal) or Type 2 (biorthogonal) wavelets.

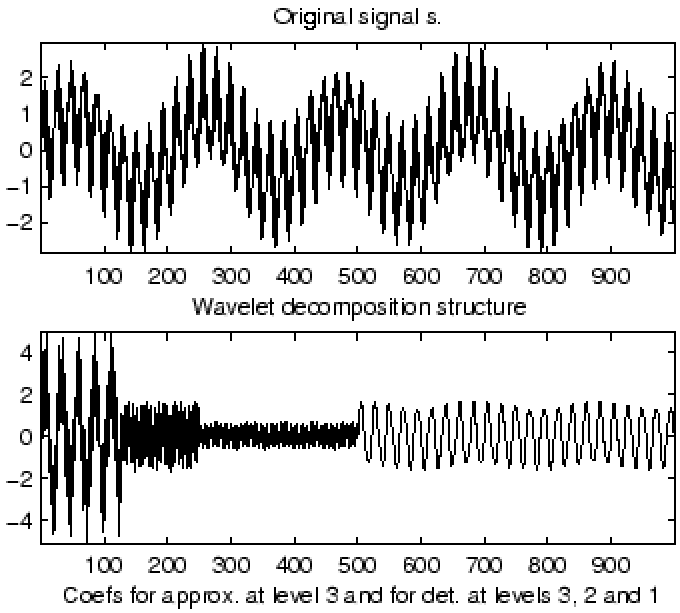
`[C,L] = wavedec(X,N,'wname')` returns the wavelet decomposition of the signal `X` at level `N`, using `'wname'`. `N` must be a strictly positive integer (see `wmaxlev` for more information). The output decomposition structure contains the wavelet decomposition vector `C` and the bookkeeping vector `L`. The structure is organized as in this level-3 decomposition example.



`[C,L] = wavedec(X,N,Lo_D,Hi_D)` returns the decomposition structure as above, given the low- and high-pass decomposition filters you specify.

## Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load original one-dimensional signal.
load sumsin; s = sumsin;
% Perform decomposition at level 3 of s using db1.
[c,l] = wavedec(s,3,'db1');
% Using some plotting commands,
% the following figure is generated.
```

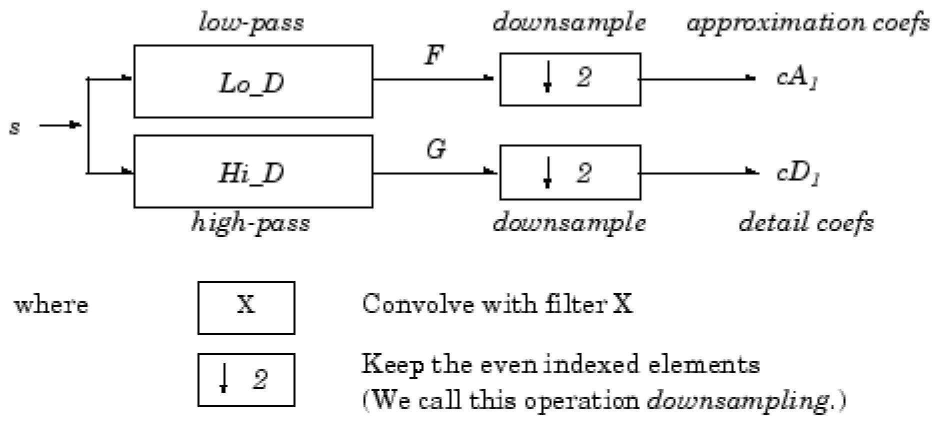


## More About

### Algorithms

Given a signal $s$ of length $N$, the DWT consists of $\log_2 N$ stages at most. The first step produces, starting from $s$, two sets of coefficients: approximation coefficients $CA_1$, and detail coefficients $CD_1$. These vectors are obtained by convolving $s$ with the low-pass filter `Lo_D` for approximation, and with the high-pass filter `Hi_D` for detail, followed by dyadic decimation (downsampling).

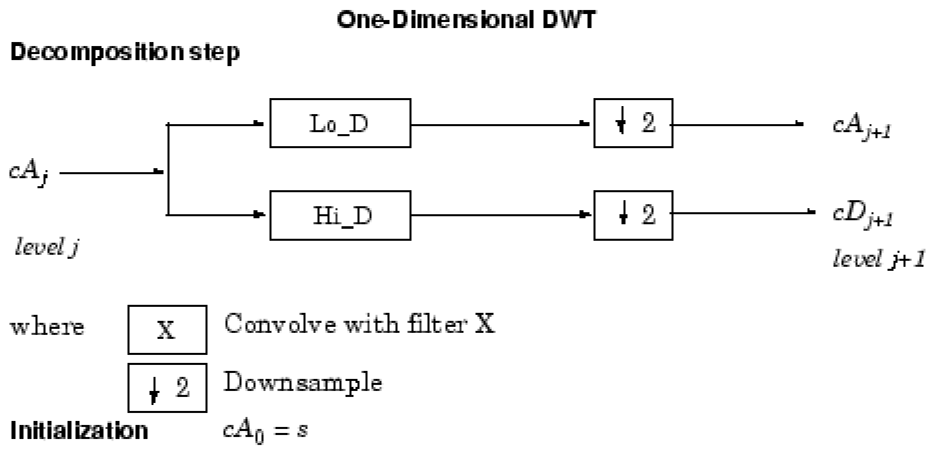
More precisely, the first step is



The length of each filter is equal to $2N$. If $n$ = length($s$), the signals $F$ and $G$ are of length $n + 2N - 1$ and the coefficients $cA_1$ and $cD_1$ are of length

$$\text{floor}\left(\frac{n-1}{2}\right) + N$$

The next step splits the approximation coefficients $cA_1$ in two parts using the same scheme, replacing $s$ by $cA_1$, and producing $cA_2$ and $cD_2$, and so on



The wavelet decomposition of the signal $s$ analyzed at level $j$ has the following structure: [$cA_j$, $cD_j$, ..., $cD_1$].

This structure contains, for $J = 3$, the terminal nodes of the following tree:

$$\begin{tikzcd}
 & & & s \arrow[dl] \arrow[dr] & \\
 & & cA_1 \arrow[dl] \arrow[dr] & & cD_1 \\
 & cA_2 \arrow[dl] \arrow[dr] & & cD_2 & \\
cA_3 & & cD_3 & &
\end{tikzcd}$$

## References

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics. SIAM Ed.

Mallat, S. (1989), “A theory for multiresolution signal decomposition: the wavelet representation,” *IEEE Pattern Anal. and Machine Intell.*, vol. 11, no. 7, pp 674–693.

Meyer, Y. (1990), *Ondelettes et opérateurs*, Tome 1, Hermann Ed. (English translation: *Wavelets and operators*, Cambridge Univ. Press. 1993.)

## See Also

`dwt` | `waveinfo` | `waverec` | `wfilters` | `wmaxlev`

# wavedec2

Multilevel 2-D wavelet decomposition

## Syntax

```
[C,S] = wavedec2(X,N,'wname')
[C,S] = wavedec2(X,N,Lo_D,Hi_D)
```

## Description

`wavedec2` is a two-dimensional wavelet analysis function.

`[C,S] = wavedec2(X,N,'wname')` returns the wavelet decomposition of the matrix `X` at level `N`, using the wavelet named in string `'wname'` (see `wfilters` for more information).

Outputs are the decomposition vector `C` and the corresponding bookkeeping matrix `S`.

`N` must be a strictly positive integer (see `wmaxlev` for more information).

Instead of giving the wavelet name, you can give the filters.

For `[C,S] = wavedec2(X,N,Lo_D,Hi_D)`, `Lo_D` is the decomposition low-pass filter and `Hi_D` is the decomposition high-pass filter.

Vector `C` is organized as

```
C = [ A(N) | H(N) | V(N) | D(N) | ...
H(N-1) | V(N-1) | D(N-1) | ... | H(1) | V(1) | D(1) ].
```

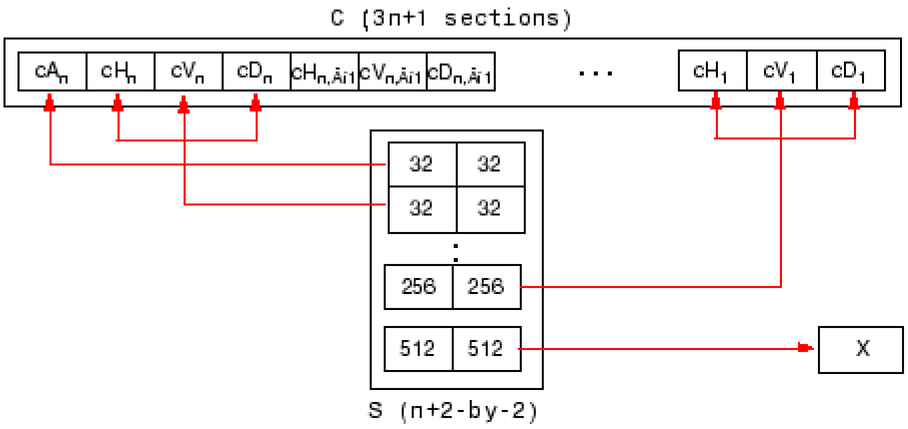
where `A`, `H`, `V`, `D`, are row vectors such that

- `A` = approximation coefficients
- `H` = horizontal detail coefficients
- `V` = vertical detail coefficients
- `D` = diagonal detail coefficients

- Each vector is the vector column-wise storage of a matrix.

Matrix S is such that

- `S(1,:)` = size of approximation coefficients(N).
- `S(i,:)` = size of detail coefficients(`N-i+2`) for `i` = 2, ...`N+1` and `S(N+2,:) = size(X)`.



## Examples

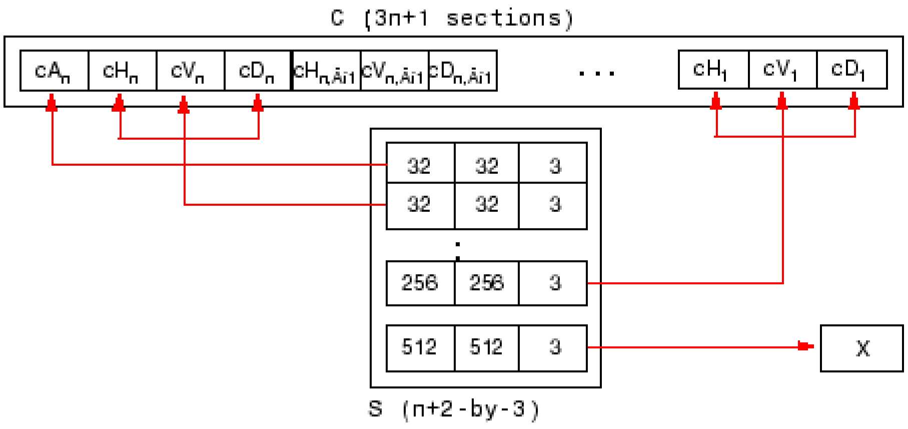
```
% The current extension mode is zero-padding (see dwtmode).

% Load original image.
load woman;
% X contains the loaded image.

% Perform decomposition at level 2
% of X using db1.
[c,s] = wavedec2(X,2,'db1');

% Decomposition structure organization.
sizex = size(X)

sizex =
    256  256
sizec = size(c)
sizec =
    1  65536
    val_s = s

val_s =
    64  64
    64  64
    128  128
    256  256
```

## More About

### Tips

When X represents an indexed image, X, as well as the output arrays cA,cH,cV, and cD are m-by-n matrices. When X represents a truecolor image, it is an m-by-n-by-3 array, where each m-by-n matrix represents a red, green, or blue color plane concatenated along the third dimension. The size of vector C and the size of matrix S depend on the type of analyzed image.

For a truecolor image, the decomposition vector C and the corresponding bookkeeping matrix S can be represented as follows.



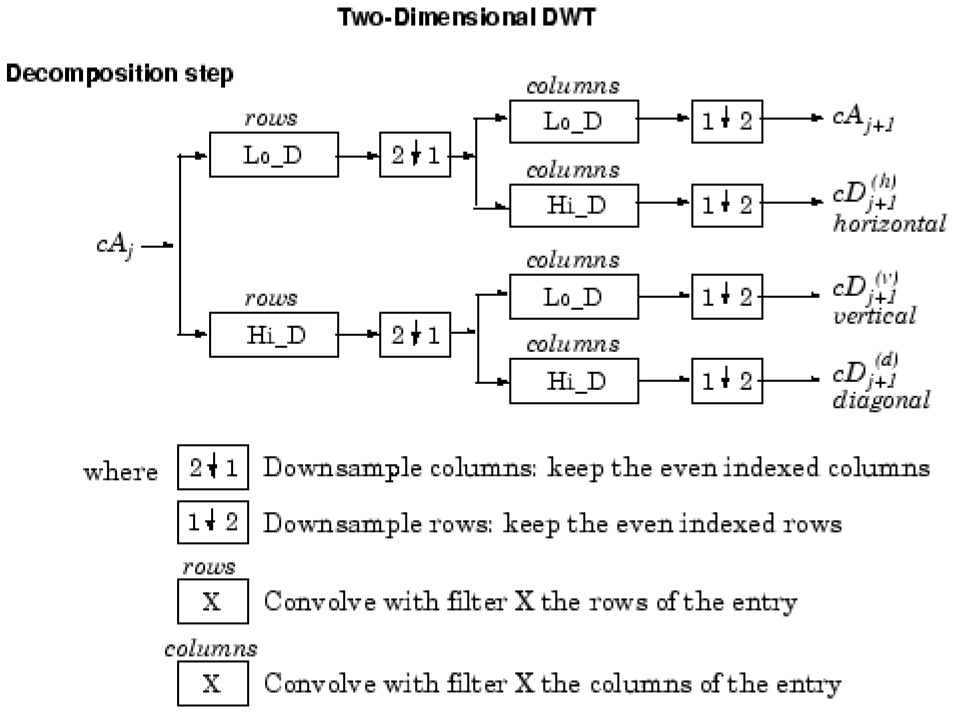
For more information on image formats, see the `image` and `imfinfo` reference pages.

## Algorithms

For images, an algorithm similar to the one-dimensional case is possible for two-dimensional wavelets and scaling functions obtained from one-dimensional ones by tensor product.

This kind of two-dimensional DWT leads to a decomposition of approximation coefficients at level *j* in four components: the approximation at level *j*+1, and the details in three orientations (horizontal, vertical, and diagonal).

The following chart describes the basic decomposition step for images:


Two-Dimensional DWT
Decomposition step
$cA_j$
rows
Lo_D
Hi_D
2↓1
columns
Lo_D
Hi_D
1↓2
$cA_{j+1}$
$cD^{(h)}_{j+1}$ horizontal
$cD^{(v)}_{j+1}$ vertical
$cD^{(d)}_{j+1}$ diagonal
where
2↓1 Downsample columns: keep the even indexed columns
1↓2 Downsample rows: keep the even indexed rows
rows X Convolve with filter X the rows of the entry
columns X Convolve with filter X the columns of the entry

**Initialization** $cA_0 = s$ for the decomposition initialization

So, for *J*=2, the two-dimensional wavelet tree has the form

$$\begin{tikzcd}
 & & & s \arrow[dl] \arrow[d] \arrow[dr] \arrow[drr] & & \\
 & & cA_1 \arrow[dll] \arrow[dl] \arrow[d] \arrow[dr] & cD_1^h & cD_1^d & cD_1^v \\
cA_2 & cD_2^h & cD_2^d & cD_2^v & &
\end{tikzcd}$$

## References

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics. SIAM Ed.

Mallat, S. (1989), "A theory for multiresolution signal decomposition: the wavelet representation," *IEEE Pattern Anal. and Machine Intell.*, vol. 11, no. 7, pp. 674–693.

Meyer, Y. (1990), *Ondelettes et opérateurs*, Tome 1, Hermann Ed. (English translation: *Wavelets and operators*, Cambridge Univ. Press. 1993.)

## See Also

`dwt` | `waveinfo` | `waverec2` | `wfilters` | `wmaxlev`

# wavedec3

Multilevel 3-D wavelet decomposition

## Syntax

```
WDEC = wavedec3(X,N,'wname')
WDEC = wavedec3(X,N,'wname','mode','ExtM')
WDEC = wavedec3(X,N,{LoD,HiD,LoR,HiR})
```

## Description

`wavedec3` is a three-dimensional wavelet analysis function.

`WDEC = wavedec3(X,N,'wname')` returns the wavelet decomposition of the 3-D array `X` at level `N`, using the wavelet named in string `'wname'` or the particular wavelet filters you specify. It uses the default extension mode `'sym'`. See `dwtmode`. `N` must be a positive integer.

`WDEC = wavedec3(X,N,'wname','mode','ExtM')` uses the specified DWT extension mode .

`WDEC = wavedec3(X,N,{LoD,HiD,LoR,HiR})` uses the decomposition and reconstruction filters you specify in a cell array.

`WDEC` is the output decomposition structure, with the following fields:

| | |
|---|---|
| `sizeINI` | Size of the three-dimensional array `X` |
| `level` | Level of the decomposition |
| `mode` | Name of the wavelet transform extension mode |
| `filters` | Structure with 4 fields, `LoD`, `HiD`, `LoR`, `HiR`, which contain the filters used for the DWT. |
| `dec` | `N` x 1 cell array containing the coefficients of the decomposition. `N` is equal to `7*WDEC.level+1`. |

| | |
|---|---|
| | dec{1} contains the lowpass component (approximation) at the level of the decomposition. The approximation is equivalent to the filtering operations 'LLL'.<br><br>dec{k+2},...,dec{k+8} with k = 0,7,14,...,7*(WDEC.level-1) contain the 3-D wavelet coefficients for the multiresolution starting with the coarsest level when k=0.<br><br>For example, if WDEC.level=3, dec{2},...,dec{8} contain the wavelet coefficients for level 3 (k=0), dec{9},...,dec{15} contain the wavelet coefficients for level 2 (k=7), and dec{16},...,dec{22} contain the wavelet coefficients for level 1 (k=7*(WDEC.level-1)).<br><br>At each level, the wavelet coefficients in dec{k +2},...,dec{k+8} are in the following order: 'HLL','LHL','HHL','LLH','HLH','LHH','HHH'.<br><br>The strings give the order in which the separable filtering operations are applied from left to right. For example, 'LHH' means that the lowpass (scaling) filter with downsampling is applied to the rows of X, followed by the highpass (wavelet) filter with downsampling applied to the columns of X. Finally, the highpass filter with downsampling is applied to the 3rd dimension of X. |
| sizes | Successive sizes of the decomposition components |

## Examples

### 3-D Wavelet Transform

Find the 3-D DWT of a volume.

Construct 8-by-8-by-8 matrix. Obtain the 3-D discrete wavelet transform at level 1 using the Haar wavelet and the default whole-point symmetric extension mode.

```
% Matrix of integers 1:64
M = magic(8);
% Make data 3-D
```

```
X = repmat(M,[1 1 8]);
% Decompose X at level 1 using db1.
wd1 = wavedec3(X,1,'db1');
```

### 3-D Wavelet Transform Using Specified Decomposition and Reconstruction Filters

Specify the decomposition and reconstruction filters as a cell array.

Construct 8-by-8-by-8 matrix. Obtain the 3-D discrete wavelet transform down to level 2 using the Daubechies extremal phase wavelet with two vanishing moments. Input the decomposition and reconstruction filters as a cell array. Use the periodic extension mode.

```
% Matrix of integers 1:64
M = magic(8);
% Make data 3-D
X = repmat(M,[1 1 8]);
[LoD,HiD,LoR,HiR] = wfilters('db2');
wd2 = wavedec3(X,2,{LoD,HiD,LoR,HiR},'mode','per');
```

### Coefficient Order in 3-D Wavelet Transform

Compare the output of `wavedec3` and `dwt3` to illustrate the ordering of the 3-D wavelet coefficients described in the `dec` field description.

```
X = reshape(1:512,8,8,8);
dwtOut = dwt3(X,'db1','mode','per');
wdec = wavedec3(X,1,'db1','mode','per');
max(abs((wdec.dec{4}(:)-dwtOut.dec{2,2,1}(:))))
max(abs((wdec.dec{5}(:)-dwtOut.dec{1,1,2}(:))))
```

## See Also

`dwt3` | `dwtmode` | `waveinfo` | `waverec3` | `wfilters` | `wmaxlev`

## wavedemo

Wavelet Toolbox software examples

### Syntax

### Description

`wavedemo` opens a GUI that allows you to choose between several Wavelet Toolbox examples.

# wavefun

Wavelet and scaling functions

## Syntax

```
[PHI,PSI,XVAL] = wavefun('wname',ITER)
[PHI1,PSI1,PHI2,PSI2,XVAL] = wavefun('wname',ITER)
[PHI,PSI,XVAL] = wavefun('wname',ITER)
[PSI,XVAL] = wavefun('wname',ITER)
[...] = wavefun(wname,A,B)
[...] = wavefun('wname',max(A,B))
[...] = wavefun('wname',0)
[...] = wavefun('wname',8,0)
[...] = wavefun('wname')
[...] = wavefun('wname',8)
```

## Description

The function `wavefun` returns approximations of the wavelet function `'wname'` and the associated scaling function, if it exists. The positive integer `ITER` determines the number of iterations computed; thus, the refinement of the approximations.

*For an orthogonal wavelet*:

`[PHI,PSI,XVAL] = wavefun('wname',ITER)` returns the scaling and wavelet functions on the points grid `XVAL`.

*For a biorthogonal wavelet*:

`[PHI1,PSI1,PHI2,PSI2,XVAL] = wavefun('wname',ITER)` returns the scaling and wavelet functions both for decomposition `(PHI1,PSI1)` and for reconstruction `(PHI2,PSI2)`.

*For a Meyer wavelet*:

`[PHI,PSI,XVAL] = wavefun('wname',ITER)`

*For a wavelet without scaling function (e.g., Morlet, Mexican Hat, Gaussian derivatives wavelets or complex wavelets)*:

`[PSI,XVAL] = wavefun('wname',ITER)`

`[...] = wavefun(wname,A,B)`, where `A` and `B` are positive integers, is equivalent to `[...] = wavefun('wname',max(A,B))`, and draws plots.

When `A` is set equal to the special value 0,

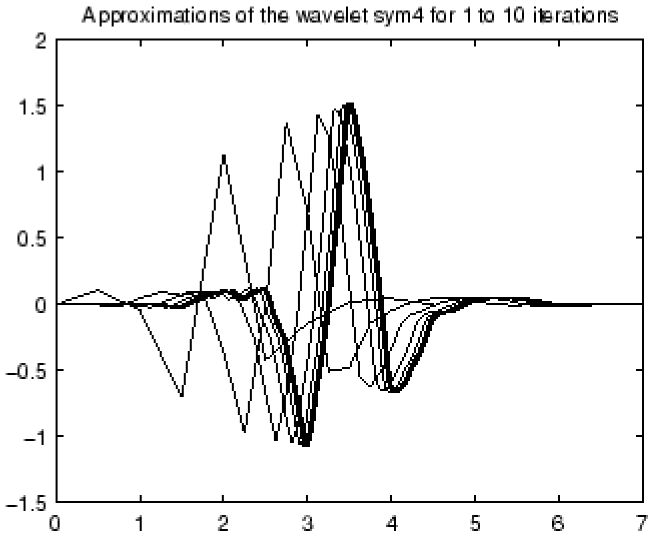
- `[...] = wavefun('wname',0)` is equivalent to
- `[...] = wavefun('wname',8,0)`.
- `[...] = wavefun('wname')` is equivalent to
- `[...] = wavefun('wname',8)`.

The output arguments are optional.

## Examples

On the following graph, 10 piecewise linear approximations of the `sym4` wavelet obtained after each iteration of the cascade algorithm are shown.

```
% Set number of iterations and wavelet name.
iter = 10;
wav = 'sym4';

% Compute approximations of the wavelet function using the
% cascade algorithm.
for i = 1:iter
    [phi,psi,xval] = wavefun(wav,i);
    plot(xval,psi);
    hold on
end
title(['Approximations of the wavelet ',wav, ...
       ' for 1 to ',num2str(iter),' iterations']);
hold off
```



## More About

### Algorithms

For compactly supported wavelets defined by filters, in general no closed form analytic formula exists.

The algorithm used is the cascade algorithm. It uses the single-level inverse wavelet transform repeatedly.

Let us begin with the scaling function $\phi$.

Since $\phi$ is also equal to $\phi_{0,0}$, this function is characterized by the following coefficients in the orthogonal framework:

- $<\phi, \phi_{0,n}> = 1$ only if $n = 0$ and equal to 0 otherwise
- $<\phi, \psi_{-j,k}> = 0$ for positive $j$, and all $k$.

This expansion can be viewed as a wavelet decomposition structure. Detail coefficients are all zeros and approximation coefficients are all zeros except one equal to 1.

Then we use the reconstruction algorithm to approximate the function $\phi$ over a dyadic grid, according to the following result:

For any dyadic rational of the form $x = n2^{-j}$ in which the function is continuous and where $j$ is sufficiently large, we have pointwise convergence and

$$\left|\phi(x) - 2^{\frac{j}{2}}\langle\phi, \phi_{-j, n2^{j-J}}\rangle\right| \le C.2^{-j\alpha}$$

where $C$ is a constant, and α is a positive constant depending on the wavelet regularity.

Then using a good approximation of ϕ on dyadic rationals, we can use piecewise constant or piecewise linear interpolations η on dyadic intervals, for which uniform convergence occurs with similar exponential rate:

$$\|\phi - \eta\|_{\infty} \le C.2^{-j\alpha}$$

So using a $J$-step reconstruction scheme, we obtain an approximation that converges exponentially towards ϕ when $J$ goes to infinity.

Approximations are computed over a grid of dyadic rationals covering the support of the function to be approximated.

Since a scaled version of the wavelet function ψ can also be expanded on the $(\phi_{-1,n)})_n$, the same scheme can be used, after a single-level reconstruction starting with the appropriate wavelet decomposition structure. Approximation coefficients are all zeros and detail coefficients are all zeros except one equal to 1.

For biorthogonal wavelets, the same ideas can be applied on each of the two multiresolution schemes in duality.

**Note** This algorithm may diverge if the function to be approximated is not continuous on dyadic rationals.

## References

Daubechies, I., *Ten lectures on wavelets*, CBMS, SIAM, 1992, pp. 202Äì213.

Strang, G.; T. Nguyen (1996), *Wavelets and Filter Banks*, Wellesley-Cambridge Press.

## See Also

`intwave` | `waveinfo` | `wfilters`

# wavefun2

Wavelet and scaling functions 2-D

## Syntax

```
[PHI,PSI,XVAL] = wavefun('wname',ITER)
[S,W1,W2,W3,XYVAL] = wavefun2('wname',ITER,'plot')
[S,W1,W2,W3,XYVAL] = wavefun2(wname,A,B)
[S,W1,W2,W3,XYVAL] = wavefun2('wname',max(A,B))
[S,W1,W2,W3,XYVAL] = wavefun2('wname',0)
[S,W1,W2,W3,XYVAL] = wavefun2('wname',4,0)
[S,W1,W2,W3,XYVAL] = wavefun2('wname')
[S,W1,W2,W3,XYVAL] = wavefun2('wname',4)
```

## Description

For an orthogonal wavelet `'wname'`, `wavefun2` returns the scaling function and the three wavelet functions resulting from the tensor products of the one-dimensional scaling and wavelet functions.

If `[PHI,PSI,XVAL] = wavefun('wname',ITER)`, the scaling function `S` is the tensor product of `PHI` and `PSI`.

The wavelet functions `W1`, `W2`, and `W3` are the tensor products (`PHI`,`PSI`), (`PSI`,`PHI`), and (`PSI`,`PSI`), respectively.

The two-dimensional variable `XYVAL` is a $2^{\mathrm{ITER}}$ x $2^{\mathrm{ITER}}$ points grid obtained from the tensor product (XVAL,XVAL).

The positive integer `ITER` determines the number of iterations computed and thus, the refinement of the approximations.

`[S,W1,W2,W3,XYVAL] = wavefun2('wname',ITER,'plot')` computes and also plots the functions.

`[S,W1,W2,W3,XYVAL] = wavefun2(wname,A,B)`, where `A` and `B` are positive integers, is equivalent to

[S,W1,W2,W3,XYVAL] = wavefun2('*wname*',max(A,B)). The resulting functions are plotted.

When A is set equal to the special value 0,

- [S,W1,W2,W3,XYVAL] = wavefun2('*wname*',0) is equivalent to [S,W1,W2,W3,XYVAL] = wavefun2('*wname*',4,0).
- [S,W1,W2,W3,XYVAL] = wavefun2('*wname*') is equivalent to [S,W1,W2,W3,XYVAL] = wavefun2('*wname*',4).

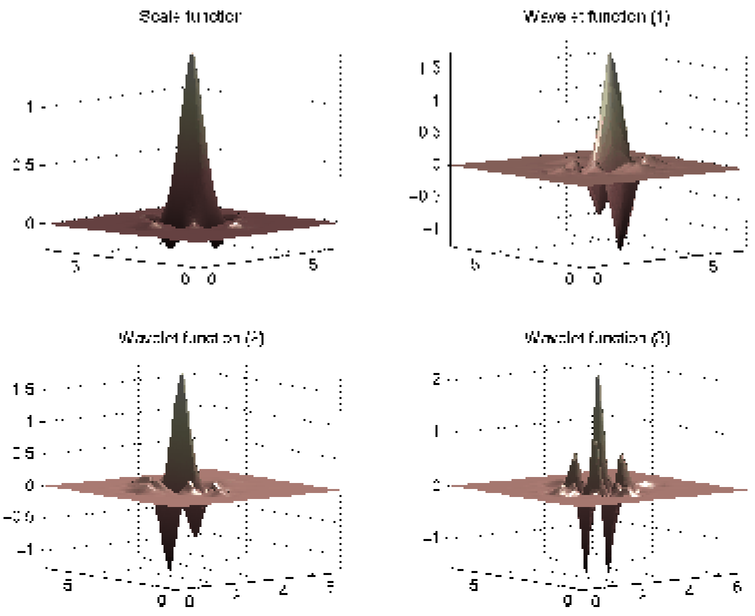
The output arguments are optional.

**Note** The wavefun2 function can only be used with an orthogonal wavelet.

## Examples

On the following graph, a linear approximation of the sym4 wavelet obtained using the cascade algorithm is shown.

```
% Set number of iterations and wavelet name.
iter = 4;
wav = 'sym4';

% Compute approximations of the wavelet and scale functions using
% the cascade algorithm and plot.
[s,w1,w2,w3,xyval] = wavefun2(wav,iter,0);
```



## More About

### Algorithms

See `wavefun` for more information.

## References

Daubechies, I., *Ten lectures on wavelets*, CBMS, SIAM, 1992, pp. 202Äì213.

Strang, G.; T. Nguyen (1996), *Wavelets and Filter Banks*, Wellesley-Cambridge Press.

## See Also

`intwave` | `wavefun` | `waveinfo` | `wfilters`

## waveinfo

Wavelets information

### Syntax

```
waveinfo('wname')
```

### Description

`waveinfo` provides information on all wavelets within the toolbox.

`waveinfo('wname')` provides information on the wavelet family whose short name is specified by the string `'wname'`. Available family short names are listed in the table below.

| Wavelet Family Short Name | Wavelet Family Name |
|---|---|
| `'haar'` | Haar wavelet |
| `'db'` | Daubechies wavelets |
| `'sym'` | Symlets |
| `'coif'` | Coiflets |
| `'bior'` | Biorthogonal wavelets |
| `'rbio'` | Reverse biorthogonal wavelets |
| `'meyr'` | Meyer wavelet |
| `'dmey'` | Discrete approximation of Meyer wavelet |
| `'gaus'` | Gaussian wavelets |
| `'mexh'` | Mexican hat wavelet |
| `'morl'` | Morlet wavelet |
| `'cgau'` | Complex Gaussian wavelets |
| `'shan'` | Shannon wavelets |
| `'fbsp'` | Frequency B-Spline wavelets |
| `'cmor'` | Complex Morlet wavelets |

The family short names can also be user-defined ones (see `wavemngr` for more information).

`waveinfo('wsys')` provides information on wavelet packets.

## Examples

```
waveinfo('db')

DBINFO Information on Daubechies wavelets.
Daubechies Wavelets
General characteristics: Compactly supported
wavelets with extremal phase and highest
number of vanishing moments for a given
support width. Associated scaling filters are
minimum-phase filters.

Family          Daubechies
Short name      db
Order N         N strictly positive integer
Examples        db1 or haar, db4, db15

Orthogonal      yes
Biorthogonal    yes
Compact support yes
DWT             possible
CWT             possible

Support width   2N-1
Filters length  2N
Regularity      about 0.2 N for large N
Symmetry        far from
Number of vanishing moments for psi     N

Reference:   I. Daubechies,
Ten lectures on wavelets CBMS, SIAM, 61, 1994, 194-202.
```

## See Also

`wavemngr`

# waveletfamilies

Wavelet families and family members

## Syntax

```
waveletfamilies('f')
waveletfamilies('n')
waveletfamilies('a')
```

## Description

`waveletfamilies` or `waveletfamilies('f')` displays the names of all available wavelet families.

`waveletfamilies('n')` displays the names of all available wavelets in each family.

`waveletfamilies('a')` displays all available wavelet families with their corresponding properties.

## Examples

```
waveletfamilies

===================================
Haar                    haar
Daubechies              db
Symlets                 sym
Coiflets                coif
BiorSplines             bior
ReverseBior             rbio
Meyer                   meyr
DMeyer                  dmey
Gaussian                gaus
Mexican_hat             mexh
Morlet                  morl
Complex Gaussian        cgau
Shannon                 shan
Frequency B-Spline      fbsp
```

```
Complex Morlet      cmor
===================================

waveletfamilies('n')

===================================
Haar                    haar
===================================
Daubechies              db
------------------------------
db1 db2 db3 db4
db5 db6 db7 db8
db9 db10    db**
===================================
Symlets                 sym
------------------------------
sym2    sym3    sym4    sym5
sym6    sym7    sym8    sym**
===================================
Coiflets                coif
------------------------------
coif1   coif2   coif3   coif4
coif5
===================================
BiorSplines             bior
------------------------------
bior1.1 bior1.3 bior1.5 bior2.2
bior2.4 bior2.6 bior2.8 bior3.1
bior3.3 bior3.5 bior3.7 bior3.9
bior4.4 bior5.5 bior6.8
===================================
ReverseBior             rbio
------------------------------
rbio1.1 rbio1.3 rbio1.5 rbio2.2
rbio2.4 rbio2.6 rbio2.8 rbio3.1
rbio3.3 rbio3.5 rbio3.7 rbio3.9
rbio4.4 rbio5.5 rbio6.8
===================================
Meyer                   meyr
===================================
DMeyer                  dmey
===================================
Gaussian                gaus
------------------------------
```

```
gaus1   gaus2   gaus3   gaus4
gaus5   gaus6   gaus7   gaus8
gaus**
===================================
Mexican_hat             mexh
===================================
Morlet                  morl
===================================
Complex Gaussian        cgau
------------------------------
cgau1   cgau2   cgau3   cgau4
cgau5   cgau**
===================================
Shannon                 shan
------------------------------
shan1-1.5   shan1-1 shan1-0.5   shan1-0.1
shan2-3 shan**
===================================
Frequency B-Spline      fbsp
------------------------------
fbsp1-1-1.5 fbsp1-1-1   fbsp1-1-0.5 fbsp2-1-1
fbsp2-1-0.5 fbsp2-1-0.1 fbsp**
===================================
Complex Morlet          cmor
------------------------------
cmor1-1.5   cmor1-1 cmor1-0.5   cmor1-1
cmor1-0.5   cmor1-0.1   cmor**
===================================

waveletfamilies('a')

Type of Wavelets
----------------
type = 1   - orthogonals wavelets          (F.I.R.)
type = 2   - biorthogonals wavelets        (F.I.R.)
type = 3   - with scale function
type = 4   - without scale function
type = 5   - complex wavelet.
---------------------------------------------------------------

------------------------
Family Name : Haar
haar
1
```

```
no
no
dbwavf

------------------------
Family Name : Daubechies
db
1
1 2 3 4 5 6 7 8 9 10 **
integer
dbwavf

------------------------
Family Name : Symlets
sym
1
2 3 4 5 6 7 8 **
integer
symwavf

------------------------
Family Name : Coiflets
coif
1
1 2 3 4 5
integer
coifwavf

------------------------
Family Name : BiorSplines
bior
2
1.1 1.3 1.5 2.2 2.4 2.6 2.8 3.1 3.3 3.5 3.7 3.9 4.4 5.5 6.8
real
biorwavf

------------------------
Family Name : ReverseBior
rbio
2
1.1 1.3 1.5 2.2 2.4 2.6 2.8 3.1 3.3 3.5 3.7 3.9 4.4 5.5 6.8
real
rbiowavf
```

```
------------------------
Family Name : Meyer
meyr
3
no
no
meyer
-8 8
------------------------
Family Name : DMeyer
dmey
1
no
no
dmey.mat

------------------------
Family Name : Gaussian
gaus
4
1 2 3 4 5 6 7 8 **
integer
gauswavf
-5 5
------------------------
Family Name : Mexican_hat
mexh
4
no
no
mexihat
-8 8
-------------------------
Family Name : Morlet
morl
4
no
no
morlet
-8 8
------------------------
Family Name : Complex Gaussian
cgau
5
```

```
1 2 3 4 5 **
integer
cgauwavf
-5 5
------------------------
Family Name : Shannon
shan
5
1-1.5 1-1 1-0.5 1-0.1 2-3 **
string
shanwavf
-20 20
------------------------
Family Name : Frequency B-Spline
fbsp
5
1-1-1.5 1-1-1 1-1-0.5 2-1-1 2-1-0.5 2-1-0.1 **
string
fbspwavf
-20 20
------------------------
Family Name : Complex Morlet
cmor
5
1-1.5 1-1 1-0.5 1-1 1-0.5 1-0.1 **
string
cmorwavf
-8 8
------------------------
```

## See Also

`wavemngr`

## wavemenu

Wavelet Toolbox GUI tools

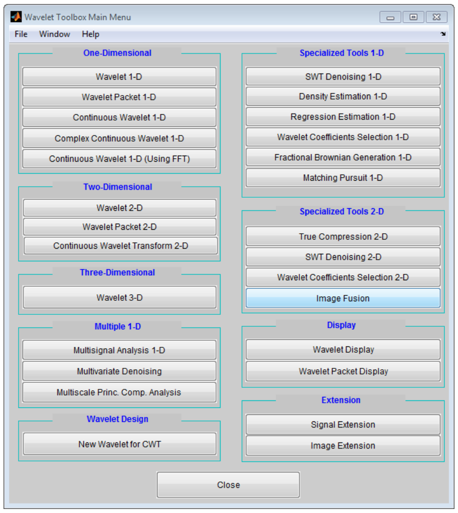
### Syntax

### Description

`wavemenu` opens a menu for accessing the various graphical tools provided in the Wavelet Toolbox software.

### Examples

```
wavemenu
```



# wavemngr

Wavelet manager

## Syntax

```
wavemngr('add',FN,FSN,WT,NUMS,FILE)
wavemngr('add',FN,FSN,WT,NUMS,FILE,B)
wavemngr('add',FN,FSN,WT,{NUMS,TYPNUMS},FILE)
wavemngr('add',FN,FSN,WT,{NUMS,TYPNUMS},FILE,B)
```

## Description

`wavemngr` is a type of wavelets manager. It allows you to add, delete, restore, or read wavelets.

`wavemngr('add',FN,FSN,WT,NUMS,FILE)` or `wavemngr('add',FN,FSN,WT,NUMS,FILE,B)` or `wavemngr('add',FN,FSN,WT,{NUMS,TYPNUMS},FILE)` or `wavemngr('add',FN,FSN,WT,{NUMS,TYPNUMS},FILE,B)`, add a new wavelet family to the toolbox.

`FN` = Family Name (string)

`FSN` = Family Short Name (string of length equal or less than four characters)

`WT` defines the wavelet type:

- `WT` = 1, for orthogonal wavelets
- `WT` = 2, for biorthogonal wavelets
- `WT` = 3, for wavelet with scaling function
- `WT` = 4, for wavelet without scaling function
- `WT` = 5, for complex wavelet without scaling function

If the family contains a single wavelet, `NUMS = ' '`.

Examples:

| | |
|---|---|
| `mexh` | `j` |

| | |
|---|---|
| `morl` | |

If the wavelet is member of a finite family of wavelets, `NUMS` is a string containing a blank separated list of items representing wavelet parameters.

Example:

| | |
|---|---|
| `bior` | `NUMS = '1.1  1.3  ...  4.4  5.5  6.8'` |

If the wavelet is part of an infinite family of wavelets, `NUMS` is a string containing a blank separated list of items representing wavelet parameters, terminated by the special sequence **.

Examples:

| | |
|---|---|
| `db` | `NUMS = '1  2  3  4  5  6  7  8  9  10  **'` |
| `shan` | `NUMS = '1-1.5  1-1  1-0.5  1-0.1  2-3  **'` |

In these last two cases, `TYPNUMS` specifies the wavelet parameter input format: `'integer'` or `'real'` or `'string'`; the default value is `'integer'`.

Examples:

| | |
|---|---|
| `db` | `TYPNUMS = 'integer'` |
| `bior` | `TYPNUMS = 'real'` |
| `shan` | `TYPNUMS = 'string'` |

`FILE` = MAT-file or code file name (string). See usage in the “Examples” section.

`B = [lb ub]` specifies lower and upper bounds of effective support for wavelets of type = 3, 4, or 5.

This option is fully documented in “ Adding Your Own Wavelets” in the User's Guide.

`wavemngr('del',N)`, deletes a wavelet or a wavelet family. `N` is the Family Short Name or the Wavelet Name (in the family). `N` is a string.

`wavemngr('restore')` or `wavemngr('restore',IN2)` restores previous or initial wavelets. If `nargin = 1`, the previous `wavelets.asc` ASCII-file is restored; otherwise the initial `wavelets.asc` ASCII-file is restored. Here `IN2` is a dummy argument.

`OUT1 = wavemngr('read')` returns all wavelet family names.

`OUT1 = wavemngr('read',IN2)` returns all wavelet names, `IN2` is a dummy argument.

`OUT1 = wavemngr('read_asc')` reads `wavelets.asc` ASCII-file and returns all wavelets information.

## Examples

```
% List initial wavelets families.
    wavemngr('read')

ans =
===================================
Haar                haar
Daubechies          db
Symlets             sym
Coiflets            coif
BiorSplines         bior
ReverseBior         rbio
Meyer               meyr
DMeyer              dmey
Gaussian            gaus
Mexican_hat         mexh
Morlet              morl
Complex Gaussian    cgau
Shannon             shan
Frequency B-Spline  fbsp
Complex Morlet      cmor
===================================
% List all wavelets.
wavemngr('read',1)

ans =

===================================
Haar                haar
===================================
Daubechies          db
------------------------------
db1   db2   db3   db4
db5   db6   db7   db8
db9   db10  db**
===================================
Symlets             sym
------------------------------
sym2  sym3  sym4  sym5
sym6  sym7  sym8  sym**
===================================
Coiflets            coif
------------------------------
coif1  coif2  coif3  coif4
coif5
===================================
BiorSplines         bior
------------------------------
bior1.1   bior1.3   bior1.5   bior2.2
bior2.4   bior2.6   bior2.8   bior3.1
bior3.3   bior3.5   bior3.7   bior3.9
bior4.4   bior5.5   bior6.8
===================================
ReverseBior           rbio
------------------------------
rbio1.1   rbio1.3   rbio1.5   rbio2.2
rbio2.4   rbio2.6   rbio2.8   rbio3.1
rbio3.3   rbio3.5   rbio3.7   rbio3.9
rbio4.4   rbio5.5   rbio6.8
===================================
Meyer                 meyr
===================================
DMeyer                dmey
===================================
Gaussian              gaus
------------------------------
gaus1   gaus2   gaus3   gaus4
gaus5   gaus6   gaus7   gaus8
gaus**
===================================
Mexican_hat           mexh
===================================
Morlet                morl
===================================
Complex Gaussian      cgau
------------------------------
cgau1   cgau2   cgau3   cgau4
cgau5   cgau**
===================================
Shannon               shan
```

```
------------------------------
shan1-1.5   shan1-1   shan1-0.5   shan1-0.1
shan2-3   shan**
===================================
Frequency B-Spline    fbsp
------------------------------
fbsp1-1-1.5   fbsp1-1-1   fbsp1-1-0.5   fbsp2-1-1
fbsp2-1-0.5   fbsp2-1-0.1   fbsp**
===================================
Complex Morlet        cmor
------------------------------
cmor1-1.5   cmor1-1   cmor1-0.5   cmor1-1
cmor1-0.5   cmor1-0.1   cmor**
===================================
```

In the following example, new compactly supported orthogonal wavelets are added to the toolbox. These wavelets, which are a slight generalization of the Daubechies wavelets, are based on the use of Bernstein polynomials and are due to Kateb and Lemarié in an unpublished work.

**Note** The files used in this example can be found in the `wavedemo` folder.

```
% Add new family of orthogonal wavelets.
% You must define:
%
%     Family Name:          Lemarie
%     Family Short Name:    lem
%     Type of wavelet:      1 (orth)
%     Wavelets numbers:     1 2 3 4 5
%     File driver:          lemwavf
%
%     The function lemwavf.m must be as follows:
%     function w = lemwavf(wname)
%     where the input argument wname is a string:
%     wname = 'lem1' or 'lem2' ... i.e.,
%     wname = sh.name + number
%     and w the corresponding scaling filter.
%     The addition is obtained using:

wavemngr('add','Lemarie','lem',1,'1 2 3 4 5','lemwavf');

% The ascii file 'wavelets.asc' is saved as
% 'wavelets.prv', then it is modified and
% the MAT file 'wavelets.inf' is generated.

% List wavelets families.
wavemngr('read')

ans =
===================================
Haar                   haar
Daubechies             db
Symlets                sym
Coiflets               coif
BiorSplines            bior
ReverseBior            rbio
Meyer                  meyr
DMeyer                 dmey
Gaussian               gaus
Mexican_hat            mexh
Morlet                 morl
Complex Gaussian       cgau
Shannon                shan
Frequency B-Spline     fbsp
Complex Morlet         cmor
Lemarie                lem
===================================
% Remove the added family.
wavemngr('del','Lemarie');

% List wavelets families.
wavemngr('read')

ans =
===================================
Haar                   haar
Daubechies             db
Symlets                sym
Coiflets               coif
BiorSplines            bior
ReverseBior            rbio
Meyer                  meyr
DMeyer                 dmey
Gaussian               gaus
Mexican_hat            mexh
Morlet                 morl
```

```
Complex Gaussian    cgau
Shannon             shan
Frequency B-Spline  fbsp
Complex Morlet      cmor
===================================
% Restore the previous ascii file
% 'wavelets.prv', then build
% the MAT-file 'wavelets.inf'.
wavemngr('restore');

% List restored wavelets.
wavemngr('read',1)

ans =
===================================
Haar                  haar
===================================
Daubechies            db
------------------------------
db1   db2   db3   db4
db5   db6   db7   db8
db9   db10   db**
===================================
Symlets               sym
------------------------------
sym2   sym3   sym4   sym5
sym6   sym7   sym8   sym**
===================================
Coiflets              coif
------------------------------
coif1   coif2   coif3   coif4
coif5
===================================
BiorSplines           bior
------------------------------
bior1.1   bior1.3   bior1.5   bior2.2
bior2.4   bior2.6   bior2.8   bior3.1
bior3.3   bior3.5   bior3.7   bior3.9
bior4.4   bior5.5   bior6.8
===================================
ReverseBior           rbio
------------------------------
rbio1.1   rbio1.3   rbio1.5   rbio2.2
rbio2.4   rbio2.6   rbio2.8   rbio3.1
```

```
rbio3.3   rbio3.5   rbio3.7   rbio3.9
rbio4.4   rbio5.5   rbio6.8
===================================
Meyer                 meyr
===================================
DMeyer                dmey
===================================
Gaussian              gaus
------------------------------
gaus1   gaus2   gaus3   gaus4
gaus5   gaus6   gaus7   gaus8
gaus**
===================================
Mexican_hat           mexh
===================================
Morlet                morl
===================================
Complex Gaussian      cgau
------------------------------
cgau1   cgau2   cgau3   cgau4
cgau5   cgau**
===================================
Shannon               shan
------------------------------
shan1-1.5   shan1-1   shan1-0.5   shan1-0.1
shan2-3   shan**
===================================
Frequency B-Spline    fbsp
------------------------------
fbsp1-1-1.5   fbsp1-1-1   fbsp1-1-0.5   fbsp2-1-1
fbsp2-1-0.5   fbsp2-1-0.1   fbsp**
===================================
Complex Morlet        cmor
------------------------------
cmor1-1.5   cmor1-1   cmor1-0.5   cmor1-1
cmor1-0.5   cmor1-0.1   cmor**
===================================
Lemarie     lem
------------------------------
lem1    lem2    lem3    lem4    lem5
===================================
% Restore initial wavelets.
%
% Restore the initial ascii file
% 'wavelets.ini' and initial
% MAT-file 'wavelets.bin'.
wavemngr('restore',0);

% List wavelets families.
wavemngr('read')

ans =
===================================
Haar                haar
Daubechies          db
Symlets             sym
Coiflets            coif
BiorSplines         bior
ReverseBior         rbio
Meyer               meyr
DMeyer              dmey
Gaussian            gaus
Mexican_hat         mexh
Morlet              morl
Complex Gaussian    cgau
Shannon             shan
Frequency B-Spline  fbsp
Complex Morlet      cmor
===================================
% Add new family of orthogonal wavelets.
wavemngr('add','Lemarie','lem',1,'1 2 3','lemwavf');

% All command line capabilities are available for
% the new wavelets.
%
% Example 1: compute the four associated filters.
[Lo_D,Hi_D,Lo_R,Hi_R] = wfilters('lem3');

% Example 2: compute scale and wavelet functions.
[phi,psi,xval] = wavefun('lem3');

% Add a new family of orthogonal wavelets: special form
% for the GUI mode.
%
% The file lemwavf allows you to compute the filter for
% any order. If you want to get a popup of the form
% 1 2 3 **, associated with the family, then wavelets are
% appended for GUI mode using:
```

```
wavemngr('restore',0);
wavemngr('add','Lemarie','lem',1,'1 2 3 **','lemwavf');

% After this sequence, all GUI capabilities are available for
% the new wavelets.
% Note that the last command allows a short cut in the
% order definition only if possible orders are integers.
```

**Caution** `wavemngr` works on the current folder. If you add a new wavelet family, it is available in this folder only. Refer to, “ Adding Your Own Wavelets”, in the User's Guide.

## Limitations

`wavemngr` allows you to add a new wavelet. You must verify that it is truly a wavelet. No check is performed either about this point or about the type of the new wavelet.

## wavenames

Wavelet names for LWT

### Syntax

`W = wavenames(T)`

### Description

`W = wavenames(T)` returns a cell array that contains the name of all wavelets of type *T*. The valid values for *T* are

- `'all'` — all wavelets
- `'lazy'` — “lazy” wavelet
- `'orth'` — orthogonal wavelets
- `'bior'` — biorthogonal wavelets

`W = wavenames` is equivalent to `W = wavenames('all')`.

## waverec

Multilevel 1-D wavelet reconstruction

### Syntax

```
X = waverec(C,L,Lo_R,Hi_R)
X = waverec(C,L,'wname')
X = appcoef(C,L,'wname',0)
```

### Description

`waverec` performs a multilevel one-dimensional wavelet reconstruction using either a specific wavelet (`'wname'`, see `wfilters`) or specific reconstruction filters (`Lo_R` and `Hi_R`). .

**Note** `waverec` supports only Type 1 (orthogonal) or Type 2 (biorthogonal) wavelets.

`X = waverec(C,L,'wname')` reconstructs the signal `X` based on the multilevel wavelet decomposition structure `[C,L]` and wavelet `'wname'`. (For information about the decomposition structure, see `wavedec`.)

`X = waverec(C,L,Lo_R,Hi_R)` reconstructs the signal `X` as above, using the reconstruction filters you specify. `Lo_R` is the reconstruction low-pass filter and `Hi_R` is the reconstruction high-pass filter.

Note that `X = waverec(C,L,'wname')` is equivalent to `X = appcoef(C,L,'wname',0)`.

### Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load original one-dimensional signal.
load leleccum; s = leleccum(1:3920); ls = length(s);
```

```
% Perform decomposition of signal at level 3 using db5.
[c,l] = wavedec(s,3,'db5');

% Reconstruct s from the wavelet decomposition structure [c,l].
a0 = waverec(c,l,'db5');

% Check for perfect reconstruction.
err = norm(s-a0)
err =
    3.2079e-09
```

## See Also

appcoef | idwt | wavedec

# waverec2

Multilevel 2-D wavelet reconstruction

## Syntax

```
X = waverec2(C,S,'wname')
X = waverec2(C,S,Lo_R,Hi_R)
waverec2(wavedec2(X,N,'wname'),'wname')
X = waverec2(C,S,'wname')
X = appcoef2(C,S,'wname',0)
```

## Description

`X = waverec2(C,S,'wname')` performs a multilevel wavelet reconstruction of the matrix `X` based on the wavelet decomposition structure `[C,S]`. For detailed storage information, see `wavedec2`. `'wname'` is a string containing the name of the wavelet. See `wfilters` for more information.

Instead of specifying the wavelet name, you can specify the filters.

- `X = waverec2(C,S,Lo_R,Hi_R)`, `Lo_R` is the reconstruction low-pass filter
- `Hi_R` is the reconstruction high-pass filter.

`waverec2` is the inverse function of `wavedec2` in the sense that the abstract statement `waverec2(wavedec2(X,N,'wname'),'wname')` returns `X`.

`X = waverec2(C,S,'wname')` is equivalent to `X = appcoef2(C,S,'wname',0)`.

## Examples

```
% The current extension mode is zero-padding (see dwtmode).
% Load original image.
load woman;
% X contains the loaded image.
% Perform decomposition at level 2
% of X using sym4.
```

```
[c,s] = wavedec2(X,2,'sym4');
% Reconstruct X from the wavelet
% decomposition structure [c,s].
a0 = waverec2(c,s,'sym4');
% Check for perfect reconstruction.
max(max(abs(X-a0)))
ans =
    2.5565e-10
```

## More About

### Tips

If `C` and `S` are obtained from an indexed image analysis or a truecolor image analysis, `X` is an `m`-by-`n` matrix or an `m`-by-`n`-by-3 array, respectively.

For more information on image formats, see the `image` and `imfinfo` reference pages.

## See Also

`appcoef2` | `idwt2` | `wavedec2`

## waverec3

Multilevel 3-D wavelet reconstruction

### Syntax

```
X = waverec3(WDEC)
C = waverec3(WDEC,TYPE,N)
X = waverec3(WDEC,'a',0)
X = waverec3(WDEC,'ca',0)
C = waverec3(WDEC,TYPE)
C = waverec3(WDEC,TYPE,N)
```

### Description

`waverec3` performs a multilevel 3-D wavelet reconstruction starting from a multilevel 3-D wavelet decomposition.

`X = waverec3(WDEC)` reconstructs the 3-D array `X` based on the multilevel wavelet decomposition structure `WDEC`. You can also use `waverec3` to extract coefficients from a 3-D wavelet decomposition.

`WDEC` is a structure with the fields shown in the table.

`C = waverec3(WDEC,TYPE,N)` reconstructs the multilevel components at level `N` of a 3-D wavelet decomposition. `N` must be a positive integer less than or equal to the level of the decomposition.

Valid values for `TYPE` are:

- A group of three characters `'xyz'`, one per direction, with `'x'`,`'y'` and `'z'` selected in the set {`'a'`, `'d'`, `'l'`, `'h'`} or in the corresponding uppercase set {`'A'`,`'D'`, `'L'`, `'H'`}), where `'A'` (or `'L'`) is a low-pass filter and `'D'` (or `'H'`) is a high-pass filter.
- The char `'d'` (or `'h'` or `'D'` or `'H'`) gives the sum of all the components different from the low-pass.
- The char `'a'` (or `'l'` or `'A'` or `'L'`) gives the low-pass component (the approximation at level `N`).

For extraction, the valid values for TYPE are the same but prefixed by `'c'` or `'C'`.

`X = waverec3(WDEC,'a',0)` or `X = waverec3(WDEC,'ca',0)` is equivalent to `X = waverec3(WDEC)`. `X` is a reconstruction of the coefficients in `WDEC` at level 0.

`C = waverec3(WDEC,TYPE)` is equivalent to `C = waverec3(WDEC,TYPE,N)` with `N` equal to the level of the decomposition.

| | |
|---|---|
| `sizeINI` | Size of the three-dimensional array `X` |
| `level` | Level of the decomposition |
| `mode` | Name of the wavelet transform extension mode |
| `filters` | Structure with 4 fields, `LoD`, `HiD`, `LoR`, and `HiR`, which contain the filters used for DWT |
| `dec` | `N` x 1 cell array containing the coefficients of the decomposition. `N` is equal to `7*WDEC.level+1`.<br><br>`dec{1}` contains the lowpass component (approximation) at the level of the decomposition. The approximation is equivalent to the filtering operations `'LLL'`.<br><br>`dec{k+2},...,dec{k+8}` with `k = 0,7,14,...,7*(WDEC.level-1)` contain the 3-D wavelet coefficients for the multiresolution starting with the coarsest level when `k=0`.<br><br>For example, if `WDEC.level=3`, `dec{2},...,dec{8}` contain the wavelet coefficients for level 3 (`k=0`), `dec{9},...,dec{15}` contain the wavelet coefficients for level 2 (`k=7`), and `dec{16},...,dec{22}` contain the wavelet coefficients for level 1 (`k=7*(WDEC.level-1)`).<br><br>At each level, the wavelet coefficients in `dec{k+2},...,dec{k+8}` are in the following order: `'HLL'`,`'LHL'`,`'HHL'`,`'LLH'`,`'HLH'`,`'LHH'`,`'HHH'`.<br><br>The strings give the order in which the separable filtering operations are applied from left to right. For example, `'LHH'` means that the lowpass (scaling) filter with downsampling is applied to the rows of `X`, followed by the highpass (wavelet) |

| | |
|---|---|
| | filter with downsampling applied to the columns of X. Finally, the highpass filter with downsampling is applied to the 3rd dimension of X. |
| `sizes` | Successive sizes of the decomposition components |

## Examples

### Perfect Reconstruction with 3-D Discrete Wavelet Transform

Construct a 3-D matrix, obtain the wavelet transform down to level 2 using the `db2` wavelet, and reconstruct the matrix to verify perfect reconstruction.

Create 3-D matrix.

```
M = magic(8);
X = repmat(M,[1 1 8]);
```

Obtain the 3-D discrete wavelet transform of the matrix and reconstruct the input based on the 3-D approximation and detail coefficients.

```
wd = wavedec3(X,2,'db2');
XR = waverec3(wd);
```

Verify perfect reconstruction using the wavelet decomposition down to level 2.

```
err1 = max(abs(X(:)-XR(:)))
```

Verify that the data matrix is the sum of the approximation and the details from levels 2 and 1.

```
A = waverec3(wd,'LLL');
% Reconstruct the sum of components different from
% the lowpass component.
D = waverec3(wd,'d');
% Check that X = A + D.
err2 = max(abs(X(:)-A(:)-D(:)))
```

### Compare `waverec3` and `idwt3`

Compare level-1 reconstructions based on the filtering operations `'LLH'` using `idwt3` and `waverec3`.

```
dwtOut = dwt3(X,'db2');
Xr = idwt3(dwtOut,'LLH');
Xrec = waverec3(wd,'LLH',1);
norm(Xr(:)-Xrec(:))
```

## See Also

`idwt3` | `waveinfo` | `wavedec3`

## wavsupport

Wavelet support

### Syntax

```
[LB,UB] = wavsupport(wname)
```

### Description

`[LB,UB] = wavsupport(wname)` returns the lower bound, LB, and upper bound, UB, of the support for the wavelet specified by wname. wname is any valid wavelet. For real-valued wavelets with and without scaling functions and complex-valued wavelets without scaling functions (wavelets type 3,4, and 5), the bounds indicate the effective support of the wavelet. For orthogonal and biorthogonal wavelets (type 1 and type 2), the lower and upper bounds are `-0.5*(LF-1)` and `0.5*(LF-1)`, where `LF` is the length of the wavelet filter.

### Examples

Support of Haar wavelet:

```
[LB, UB] = wavsupport('haar');
LowerBound = -0.5*(2-1);
UpperBound = 0.5*(2-1);
% Compare [LB,UB] and [LowerBound, UpperBound]
```

Effective support of complex-valued Gaussian wavelet:

```
[LB,UB] = wavsupport('cgau3');
```

### See Also

`wavemngr`

# wbmpen

Penalized threshold for wavelet 1-D or 2-D de-noising

## Syntax

```
THR = wbmpen(C,L,SIGMA,ALPHA)
wbmpen(C,L,SIGMA,ALPHA,ARG)
```

## Description

`THR = wbmpen(C,L,SIGMA,ALPHA)` returns global threshold `THR` for de-noising. `THR` is obtained by a wavelet coefficients selection rule using a penalization method provided by Birgé-Massart.

`[C,L]` is the wavelet decomposition structure of the signal or image to be de-noised.

`SIGMA` is the standard deviation of the zero mean Gaussian white noise in de-noising model (see `wnoisest` for more information).

`ALPHA` is a tuning parameter for the penalty term. It must be a real number greater than 1. The sparsity of the wavelet representation of the de-noised signal or image grows with `ALPHA`. Typically `ALPHA` = 2.

`THR` minimizes the penalized criterion given by

let $t^*$ be the minimizer of

```
crit(t) = -sum(c(k)^2,k≤t) + 2*SIGMA^2*t*(ALPHA + log(n/t))
```

where `c(k)` are the wavelet coefficients sorted in decreasing order of their absolute value and `n` is the number of coefficients; then THR=|c($t^*$)|.

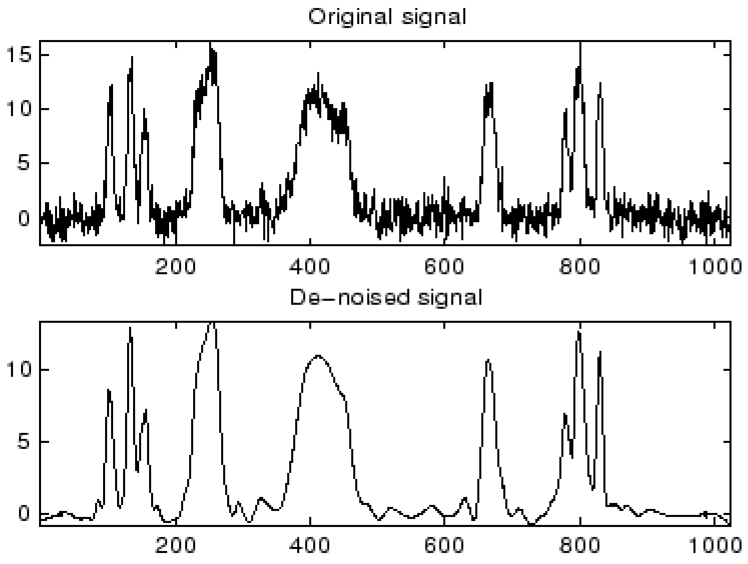
`wbmpen(C,L,SIGMA,ALPHA,ARG)` computes the global threshold and, in addition, plots three curves:

- `2*SIGMA^2*t*(ALPHA + log(n/t))`

- `sum(c(k)^2,k¬≤t)`
- `crit(t)`

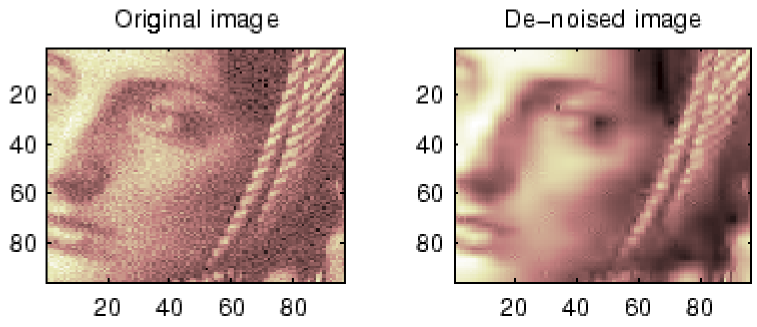
## Examples

```
% Example 1: Signal de-noising.
% Load noisy bumps signal.
load noisbump; x = noisbump;

% Perform a wavelet decomposition of the signal
% at level 5 using sym6.
wname = 'sym6'; lev = 5;
[c,l] = wavedec(x,lev,wname);
% Estimate the noise standard deviation from the
% detail coefficients at level 1, using wnoisest.
sigma = wnoisest(c,l,1);

% Use wbmpen for selecting global threshold
% for signal de-noising, using the tuning parameter.
alpha = 2;
thr = wbmpen(c,l,sigma,alpha)
thr =

    2.7681

% Use wdencmp for de-noising the signal using the above
% threshold with soft thresholding and approximation kept.
keepapp = 1;
xd = wdencmp('gbl',c,l,wname,lev,thr,'s',keepapp);

% Plot original and de-noised signals.
figure(1)
subplot(211), plot(x), title('Original signal')
subplot(212), plot(xd), title('De-noised signal')
```



```
% Example 2: Image de-noising.
% Load original image.
load noiswom;
nbc = size(map,1);

% Perform a wavelet decomposition of the image
% at level 3 using coif2.
wname = 'coif2'; lev = 3;
[c,s] = wavedec2(X,lev,wname);

% Estimate the noise standard deviation from the
% detail coefficients at level 1.
det1 = detcoef2('compact',c,s,1);
sigma = median(abs(det1))/0.6745;

% Use wbmpen for selecting global threshold
% for image de-noising.
alpha = 1.2;
thr = wbmpen(c,l,sigma,alpha)

thr =

   36.0621
```

```
% Use wdencmp for de-noising the image using the above
% thresholds with soft thresholding and approximation kept.
keepapp = 1;
xd = wdencmp('gbl',c,s,wname,lev,thr,'s',keepapp);

% Plot original and de-noised images.
figure(2)
colormap(pink(nbc));
subplot(221), image(wcodemat(X,nbc))
title('Original image')
subplot(222), image(wcodemat(xd,nbc))
title('De-noised image')
```



## See Also

wden | wdencmp | wpbmpen | wpdencmp

# wcodemat

Extended pseudocolor matrix scaling

## Syntax

```
Y = wcodemat(X)
Y = wcodemat(X,NBCODES)
Y = wcodemat(X,NBCODES,OPT)
Y = wcodemat(X,NBCODES,OPT,ABSOL)
```

## Description

`wcodemat` rescales an input matrix to a specified range for display. If the specified range is the full range of the current colormap, `wcodemat` is similar in behavior to `imagesc`.

`Y = wcodemat(X)` rescales the matrix `X` to integers in the range [1,16].

`Y = wcodemat(X,NBCODES)` rescales the input `X` as integers in the range [1,`NBCODES`] . The default value of `NBCODES` is 16.

`Y = wcodemat(X,NBCODES,OPT)` rescales the matrix along the dimension specified by `OPT`. Valid strings for `OPT` are: `'column'` (or `'c'`), `'row'` (or `'r'`), and `'mat'` (or `'m'`). `'rows'` scales `X` row-wise, `'column'` scales `X` column-wise, and `'mat'` scales `X` globally. The default value of `OPT` is `'mat'`.

`Y = wcodemat(X,NBCODES,OPT,ABSOL)` rescales the input matrix `X` based on the absolute values of the entries in `X` if `ABSOL` is nonzero, or based on the signed values of `X` if `ABSOL` is equal to zero. The default value of `ABSOL` is 1.

## Examples

Scale level-one approximation coefficients globally to the full range of the colormap.

```
load woman;
% Get the range of the colormap
NBCOL = size(map,1);
```

```
% Obtain the 2D dwt using the Haar wavelet
[cA1,cH1,cV1,cD1] = dwt2(X,'db1');
% Display without scaling
image(cA1);
colormap(map);
title('Unscaled Image');
figure;
% Display with scaling
image(wcodemat(cA1,NBCOL));
colormap(map);
title('Scaled Image');
```

# wcoher

Wavelet coherence

## Syntax

```
WCOH = wcoher(Sig1,Sig2,Scales,wname)
WCOH = wcoher(...,Name,Value)
[WCOH,WCS] = wcoher(...)
[WCOH,WCS,CWT_S1,CWT_S2] = wcoher(...)
[...] = wcoh(...,'plot')
```

## Description

`WCOH = wcoher(Sig1,Sig2,Scales,wname)` returns the wavelet coherence for the input signals Sig1 and Sig2 using the wavelet specified in wname at the scales in Scales. The input signals must be real-valued and equal in length.

`WCOH = wcoher(...,Name,Value)` returns the wavelet coherence with additional options specified by one or more Name,Value pair arguments.

`[WCOH,WCS] = wcoher(...)` returns the wavelet cross spectrum.

`[WCOH,WCS,CWT_S1,CWT_S2] = wcoher(...)` returns the continuous wavelet transforms of Sig1 and Sig2.

`[...] = wcoh(...,'plot')` displays the modulus and phase of the wavelet cross spectrum.

## Input Arguments

**`Sig1`**

A real-valued one-dimensional input signal. Sig1 is a row or column vector.

**`Sig2`**

A real-valued one-dimensional input signal. Sig2 is a row or column vector.

**Scales**

Scales is a vector of real-valued, positive scales at which to compute the wavelet coherence.

**wname**

Wavelet used in the wavelet coherence. wname is any valid wavelet name.

## Name-Value Pair Arguments

**'asc'**

Scale factor for arrows in quiver plot. `wcoher` represents the phase using `quiver`. asc corresponds to the scale input argument in `quiver`.

**Default:** 1

**'nas'**

Number of arrows in scale. Together with the number of scales, nas determines the spacing between the y coordinates in the input to `quiver`. The y input to `quiver` is `1:length(Scales)/(nas-1):Scales(end)`

**Default:** 20

**'nsw'**

Length of smoothing window in scale. nsw is a positive integer that specifies the length of a moving average filter in scale.

**Default:** 1

**'ntw'**

Length of smoothing window in time. ntw is a positive integer that specifies the length of a moving average filter in time.

**Default:** `min[20,0.05*length(Sig1)]`

**'plot'**

Type of plot. plot is one of the following strings:

- `'cwt'`

  Displays the continuous wavelet transforms of signals 1 and 2.

- `'wcs'`

  Displays the wavelet cross spectrum.

- `'wcoh'`

  Displays the phase of the wavelet cross spectrum.

- `'all'`

  Displays all plots in separate figures.

## Output Arguments

**`WCOH`**

Wavelet coherence.

**`WCS`**

Wavelet cross spectrum.

**`CWT_S1`**

Continuous wavelet transform of signal 1.

**`CWT_S2`**

Continuous wavelet transform of signal 2.

## Examples

Wavelet coherence of sine waves in noise with delay:

```
t  = linspace(0,1,2048);
x = sin(16*pi*t)+0.5*randn(1,2048);
y = sin(16*pi*t+pi/4)+0.5*randn(1,2048);
```

```
wname  = 'cgau3';
scales = 1:512;
ntw = 21; % smoothing parameter
% Display the modulus and phased of the wavelet cross spectrum.
wcoher(x,y,scales,wname,'ntw',ntw,'plot');
```

Sine wave and Doppler signal:

```
t  = linspace(0,1,1024);
x = -sin(8*pi*t) + 0.4*randn(1,1024);
x = x/max(abs(x));
y = wnoise('doppler',10);
wname  = 'cgau3';
scales = 1:512;
ntw = 21; % smoothing parameter
% Display of the CWT of the two signals.
wcoher(x,y,scales,wname,'ntw',ntw,'plot','cwt');
% Display of the wavelet cross spectrum.
wcoher(x,y,scales,wname,'ntw',ntw,'nsw',1,'plot','wcs');
% Display of the modulus and phased of the wavelet cross spectrum.
wcoher(x,y,scales,wname,'ntw',ntw,'plot');
```

## More About

### Wavelet Cross Spectrum

The wavelet cross spectrum of two time series, $x$ and $y$ is:

$$C_{xy}(a,b) = S(C_x^*(a,b)C_y(a,b))$$

where $C_x(a,b)$ and $C_y(a,b)$ denote the continuous wavelet transforms of $x$ and $y$ at scales $a$ and positions $b$. The superscript * is the complex conjugate and $S$ is a smoothing operator in time and scale.

For real-valued time series, the wavelet cross spectrum is real-valued if you use a real-valued analyzing wavelet, and complex-valued if you use a complex-valued analyzing wavelet.

### Wavelet Coherence

The wavelet coherence of two time series $x$ and $y$ is:

$$\frac{S(C_x^*(a,b)C_y(a,b))}{\sqrt{S(|C_x(a,b)|^2)}\sqrt{S(|C_y(a,b)|^2)}}$$

where $C_x(a,b)$ and $C_y(a,b)$ denote the continuous wavelet transforms of $x$ and $y$ at scales $a$ and positions $b$. The superscript * is the complex conjugate and $S$ is a smoothing operator in time and scale.

For real-valued time series, the wavelet coherence is real-valued if you use a real-valued analyzing wavelet, and complex-valued if you use a complex-valued analyzing wavelet.

- Wavelet Coherence

## References

Grinsted, A, J.C. Moore, and S. Jevrejeva. “Application of the cross wavelet transform and wavelet coherence to geophysical time series. *Nonlinear Processes in Geophysics*. 11, 2004, pp. 561-566.

Torrence. C., and G. Compo. “A Practical Guide to Wavelet Analysis”. *Bulletin of the American Meteorological Society*, 79, pp. 61-78.

## See Also

`cwt`

## wcompress

True compression of images using wavelets

## Syntax

```
wcompress('c',X,SAV_FILENAME,COMP_METHOD)
wcompress(...,'ParName1',ParVal1,'ParName2',ParVal2,...)
[COMPRAT,BPP] = wcompress('c',...)
XC = wcompress('u',SAV_FILENAME)
XC = wcompress('u',SAV_FILENAME,'plot')
XC = wcompress('u',SAV_FILENAME,'step')
```

## Description

The `wcompress` command performs either compression or uncompression of grayscale or truecolor images.

More theoretical information on true compression is in “True Compression for Images” of the Wavelet Toolbox User's Guide.

### Compression

`wcompress('c',X,SAV_FILENAME,COMP_METHOD)` compresses the image `X` using the compression method `COMP_METHOD`.

The compressed image is saved in the file `SAV_FILENAME`. `X` can be either a 2-D array containing an indexed image or a 3-D array of `uint8` containing a truecolor image.

wcompress`('c',FILENAME,...)` loads the image `X` from the file `FILENAME` which is a MATLAB Supported Format (MSF) file: MAT-file or other image files (see `imread`).

wcompress`('c',I,...)` converts the indexed image `X = I{1}` to a truecolor image `Y` using the colormap `map = I{2}` and then compresses `Y`.

The valid compression methods are divided in three categories.

1 Progressive Coefficients Significance Methods (**PCSM**):

| MATLAB Name | Compression Method Name |
|---|---|
| `'ezw'` | Embedded Zerotree Wavelet |
| `'spiht'` | Set Partitioning In Hierarchical Trees |
| `'stw'` | Spatial-orientation Tree Wavelet |
| `'wdr'` | Wavelet Difference Reduction |
| `'aswdr'` | Adaptively Scanned Wavelet Difference Reduction |
| `'spiht_3d'` | Set Partitioning In Hierarchical Trees 3D for truecolor images |

For more details on these methods, see the references and especially Walker and also Said and Pearlman.

1 Coefficients Thresholding Methods (**CTM-1**):

| MATLAB Name | Compression Method Name |
|---|---|
| `'lvl_mmc'` | Subband thresholding of coefficients and Huffman encoding |

For more details on this method, see the Strang and Nguyen reference.

1 Coefficients Thresholding Methods (**CTM-2**):

| MATLAB Name | Compression Method Name |
|---|---|
| `'gbl_mmc_f'` | Global thresholding of coefficients and fixed encoding |
| `'gbl_mmc_h'` | Global thresholding of coefficients and Huffman encoding |

**Note** The Discrete Wavelet Transform uses the periodized extension mode. Each of the two dimensions of the image must be a power of 2.

All the compression methods use parameters which have default values. You can change these values using the following syntax:

```
wcompress(...,'ParName1',ParVal1,'ParName2',ParVal2,...)
```

Some of the parameters are related to display or to data transform functionalities. The others are linked to the compression process itself.

## Data transform parameters

- `'ParName'` = `'wname'` or `'WNAME'` sets the wavelet name.

  `ParVal` is a string (see `waveletfamilies`). The default for is *bior4.4*
- `'ParName'` = `'level'` or `'LEVEL'` sets the level of decomposition.

  `ParVal` is an integer such that: `1` ≤ `level` ≤ `levmax` which is the maximum possible level (see `wmaxlev`).

  The default level depends on the method:

  - for **PCSM** methods `level` is equal to `levmax`.

  - for **CTM** methods level is equal to `fix(levmax/2)`
- `ParName'` = `'it'` or `'IT'` sets Image type Transform.

  `ParVal` must be one of the following strings:

  `'n'` : no transformation (default), image type (truecolor or grayscale) is automatically detected.

  `'g'` : grayscale transformation type.

  `'c'` : color transformation type (RGB `uint8`).
- `'ParName'` = `'cc'` or `'CC'` sets Color Conversion parameter if `X` is a truecolor image.

  `ParVal` must be one of the following strings:

  `'rgb'` or `'none'` : No conversion (default).

  `'yuv'` : YUV color space transform.

  `'klt'` : Karhunen-Loeve transform.

  `'yiq'` : YIQ color space transform.

  `'xyz'` : CIEXYZ color space transform.

## Parameter for Progressive Coefficients Significance Methods (PCSM)

- `'ParName'` = `'maxloop'` or `'MAXLOOP'` sets the maximum number of steps for the compression algorithm.

  `ParVal` must be a positive integer or `Inf` (default is 10).

## Parameters for Coefficients Thresholding Methods (CTM-1)

Either of the following parameters may be used:

- `'ParName'` = `'bpp'` or `'BPP'` sets the bit-per-pixel ratio.

  `ParVal` must be such that $0 \leq$ `ParVal` $\leq 8$ (grayscale) or 24 (truecolor).
- `'ParName'` = `'comprat'` or `'COMPRAT'` sets the compression ratio.

  `ParVal` must be such that $0 \leq$ `ParVal` $\leq 100$.

## Parameters for Coefficients Thresholding Methods (CTM-2)

Two parameters may be used. The first is related to the threshold and the second is the number of classes for quantization.

The first one may be chosen among the five following parameters:

- `'ParName'` = `'threshold'` or `'THRESHOLD'` sets the threshold value for compression.

  `ParVal` must be a positive (or zero) real number.
- `'ParName'` = `'nbcfs'` or `'NBCFS'` sets the number of preserved coefficients in the wavelet decomposition.

  `ParVal` must be an integer such that: $0 \leq$ `ParVal` $\leq$ total number of coefficients of wavelet decomposition.
- `'ParName'` = `'percfs'` or `'PERCFS'` sets the percentage of preserved coefficients in the wavelet decomposition.

  `ParVal` must be a real number such that: $0 \leq$ `ParVal` $\leq 100$.
- `'ParName'` = `'bpp'` or `'BPP'` sets the bit-per-pixel ratio.

`ParVal` must be such that: `0` ≤ `ParVal` ≤ `8` (grayscale) or `24` (truecolor)

- `'ParName'` = `'comprat'` or `'COMPRAT'` sets the compression ratio.

  `ParVal` must be such that: `0` ≤ `ParVal` ≤ `100`.

The second parameter sets the number of classes for quantization:

- `'ParName'` = `'nbclas'` or `'NBCLAS'` sets the number of classes.

  `ParVal` must be a real number such that: `2` ≤ `ParVal` ≤ `200`.

### Display parameter

- `'ParName'` = `'plotpar'` or `'PLOTPAR'` sets the plot parameter.

  `ParVal` must be one of the following strings or numbers:

  `'plot'` or `0`: plots only the compressed image.

  `'step'` or `1`: displays each step of the encoding process (only for **PCSM** methods).

`[COMPRAT,BPP] = wcompress('c',...)` returns the compression ratio `COMPRAT` and the bit_per_pixel ratio `BPP`.

### Uncompression

`XC = wcompress('u',SAV_FILENAME)` uncompresses the file `SAV_FILENAME` and returns the image `XC`. Depending on the initial compressed image, `XC` can be a 2-D array containing either an indexed image or a 3-D array of `uint8` containing a truecolor image.

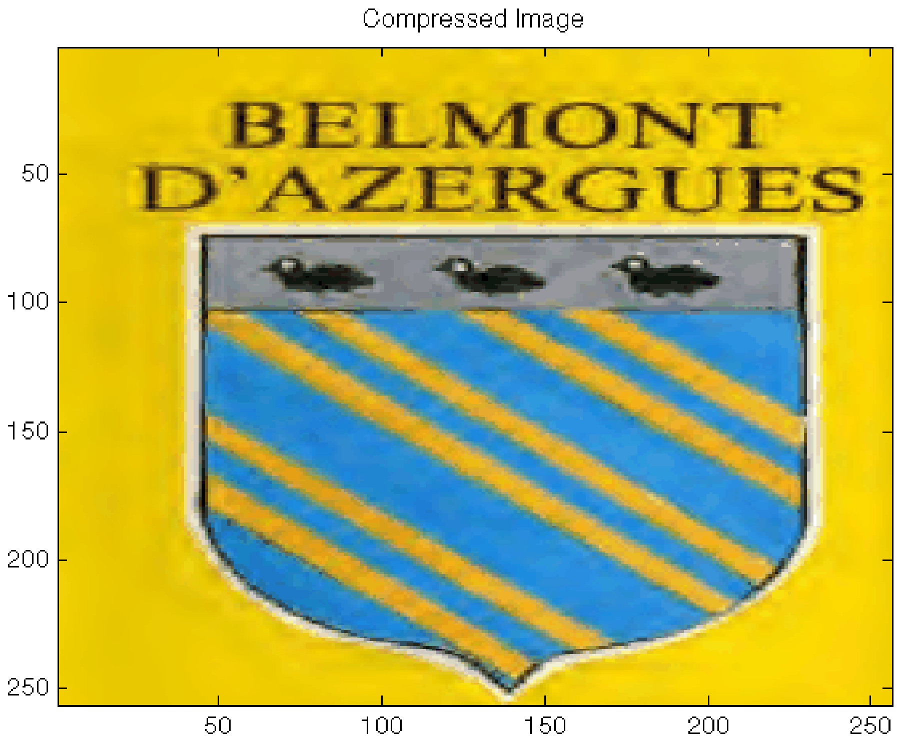
`XC = wcompress('u',SAV_FILENAME,'plot')` plots the uncompressed image.

`XC = wcompress('u',SAV_FILENAME,'step')` shows the step-by-step uncompression, only for **PCSM** methods.

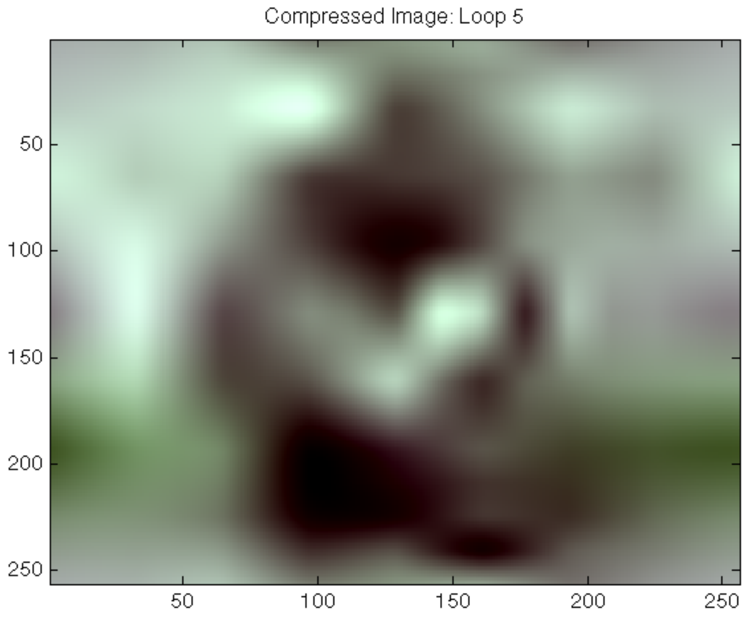
## Examples

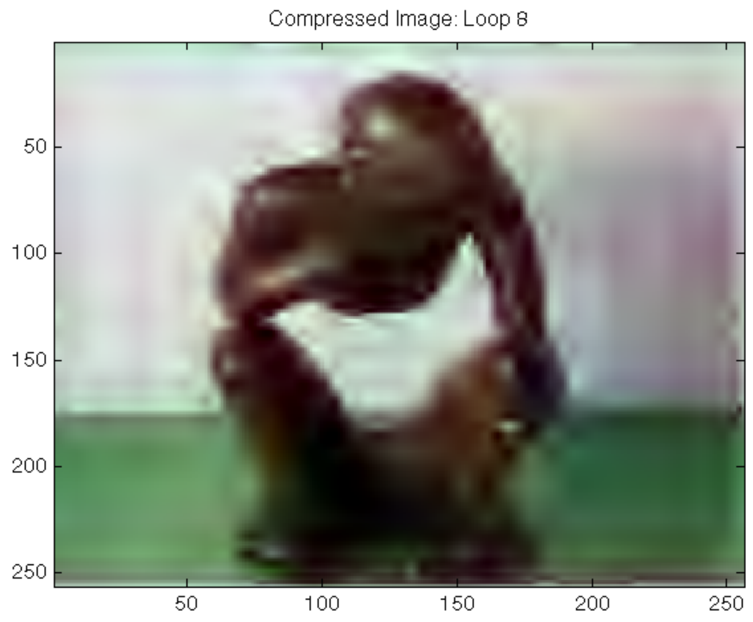
```
% Example 1: Compression and uncompression using
% basic parameters.
```

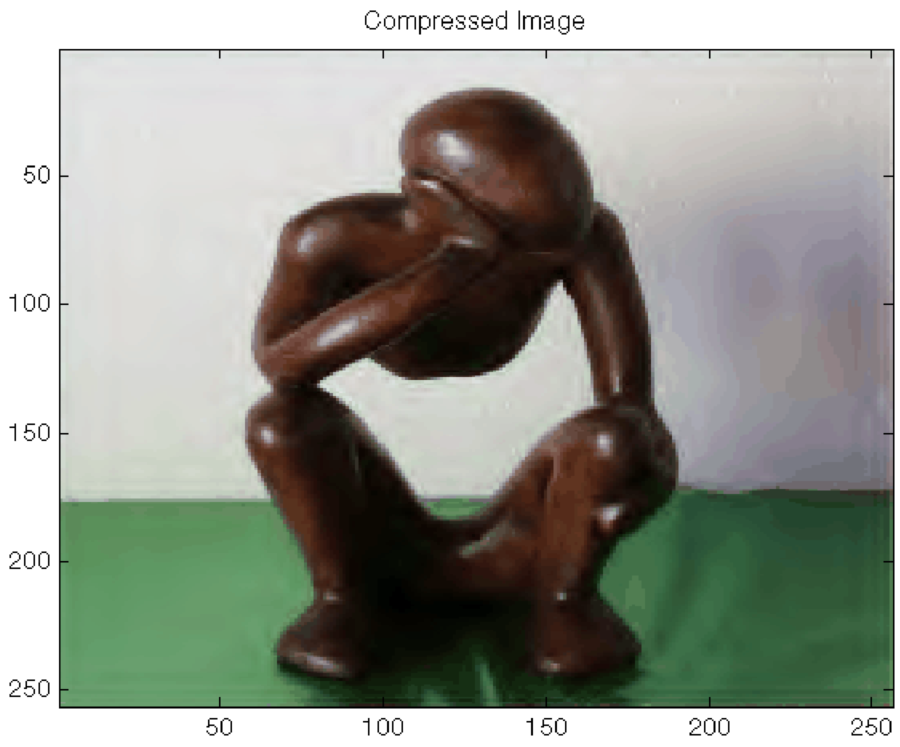
```
%
% This example demonstrates first how to compress the jpeg
% image arms.jpg using the 'stw' compression method and
% save it to the file: 'comp_arms.wtc'.

wcompress('c','arms.jpg','comp_arms.wtc','stw');

% Then, it shows how to load the stored image from
% the file 'comp_arms.wtc' and to display the step by
% step uncompression leading to the final image below.

wcompress('u','comp_arms.wtc','step');
```



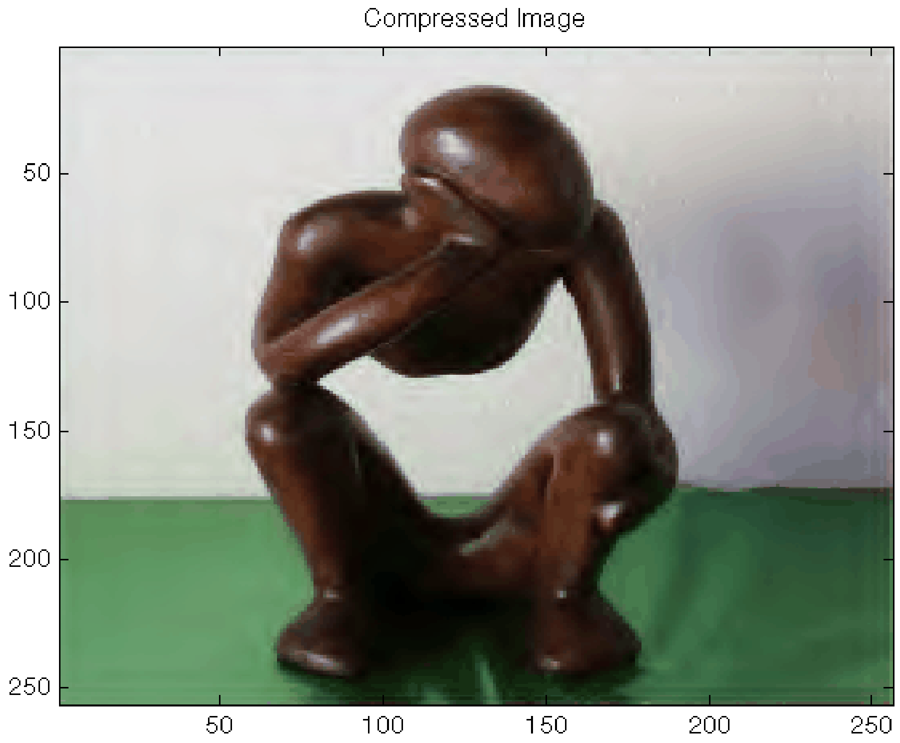
```
% Example 2: Compression and uncompression using
% advanced parameters.
%
% This example demonstrates how to compress a jpeg
% image using the 'aswdr' compression method and
% save it to the file: 'woodstatue.wtc'.
% During the compression process 3 parameters are used:
% - Conversion color (cc) set to Karhunen-Loeve transform 'klt'
% - Maximum number of loops (maxloop) set to 11
% - Plot type (plotpar) set to step by step display
% By the way two performance indicators are displayed:
% the compression ratio (cr) and the bit-per-pixel ratio (bpp).

[cr,bpp] = wcompress('c','woodstatue.jpg','woodstatue.wtc', ...
             'aswdr','cc','klt','maxloop',11,'plotpar','step')

cr =

    3.0701

bpp =

    0.7368
```




Compressed Image: Loop 8
50
100
150
200
250
50
100
150
200
250

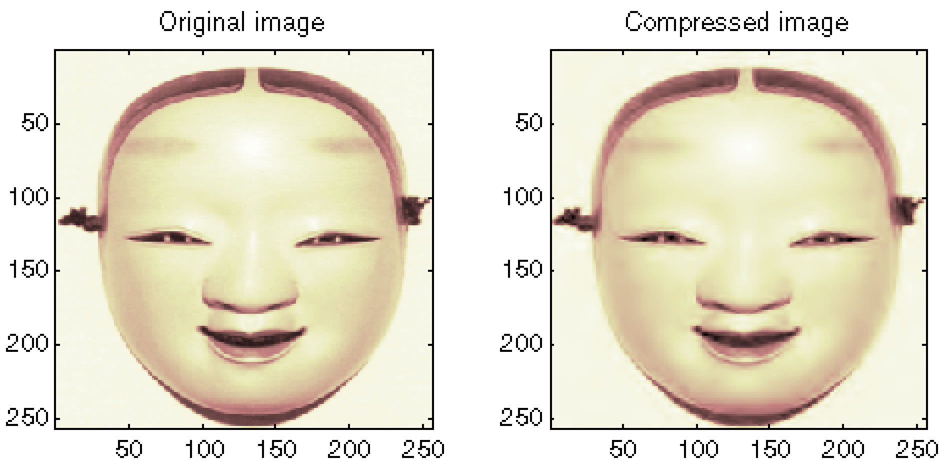


```
% Then, it shows how to load the stored image from the
% file 'woodstatue.wtc' and to display the step by step
% uncompression process.

wcompress('u','woodstatue.wtc','step');
```

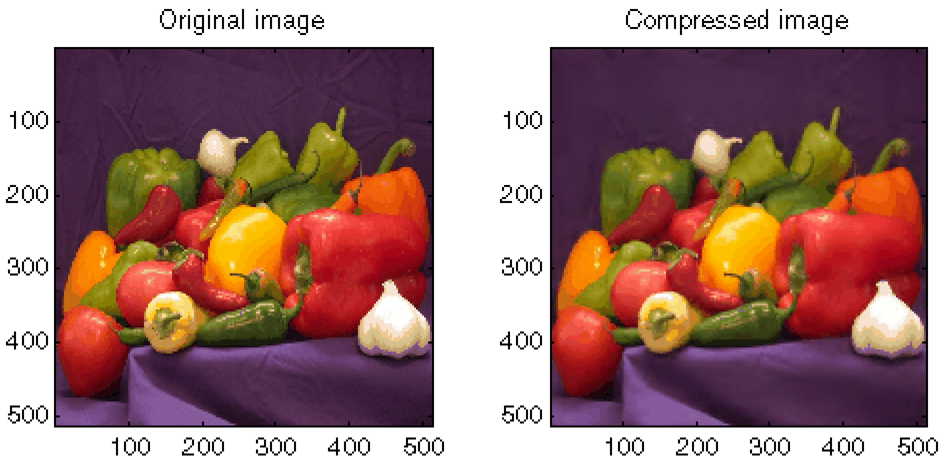


```
delete('woodstatue.wtc')

% Example 3: Compression and uncompression of a grayscale image
% and computed MSE and PSNR error values.
%
% Two measures are commonly used to quantify the error between
% two images: the Mean Square Error(MSE) and the Peak Signal
% to Noise Ratio (PSNR) which is expressed in decibels.
%
% This example demonstrates how to compress the mask image using
% the 'spiht' compression method and save it to the 'mask.wtc'
% file.

load mask;
[cr,bpp] = wcompress('c',X,'mask.wtc','spiht','maxloop',12)

cr =
    2.8336
```

```
bpp =

    0.2267

% Then, it shows how to load the stored image from the file
% 'mask.wtc', uncompress it and delete the file 'mask.wtc'.

Xc = wcompress('u','mask.wtc');
delete('mask.wtc')

% The orginal and compressed images are displayed.
colormap(pink(255))
subplot(1,2,1); image(X);  title('Original image')
axis square
subplot(1,2,2); image(Xc); title('Compressed image')
axis square
```



```
% Finally the MSE and the PSNR are computed.

D = abs(X-Xc).^2;
mse  = sum(D(:))/numel(X)

mse =
   33.6564

psnr = 10*log10(255*255/mse)
psnr =

   32.8601
% Example 4: Compression and uncompression of a truecolor image
% and computed MSE and PSNR error values.
% Compression parameters are the same as those used for example 3,
% but using the 'spiht_3d' method give better performance yet.

X = imread('wpeppers.jpg');
[cr,bpp] = wcompress('c',X,'wpeppers.wtc','spiht','maxloop',12)

cr =
    1.6527

bpp =
    0.3966

Xc = wcompress('u','wpeppers.wtc');
delete('wpeppers.wtc')
subplot(1,2,1); image(X);  title('Original image'), axis square
subplot(1,2,2); image(Xc); title('Compressed image'), axis square
```



```
D = abs(double(X)-double(Xc)).^2;
mse  = sum(D(:))/numel(X)

mse =

    26.7808
```

```
psnr = 10*log10(255*255/mse)

psnr =

   33.8526
```

## References

Christophe, E., C. Mailhes, P. Duhamel (2006), “Adaptation of zerotrees using signed binary digit representations for 3 dimensional image coding,” EURASIP *Journal on Image and Video Processing*, 2007, to appear in the special issue on Wavelets in Source Coding, Communications, and Networks, Paper ID 54679.

Misiti, M., Y. Misiti, G. Oppenheim, J.-M. Poggi (2007), *Wavelets and their applications*, ISTE DSP Series.

Said A., W.A. Pearlman (1996), “A new, fast, and efficient image codec based on set partitioning in hierarchical trees,” *IEEE Trans. on Circuits and Systems for Video Technology*, Vol. 6, No. 3, pp. 243–250.

Shapiro J.M. (1993), “Embedded image coding using zerotrees of wavelet coefficients”,P *IEEE Trans. Signal Proc.*, Vol. 41, No. 12, pp. 3445–3462.

Strang, G.; T. Nguyen (1996), *Wavelets and Filter Banks*, Wellesley-Cambridge Press.

Walker J.S. (1999), “Wavelet-Based Image Compression,” University of Wisconsin, Eau Claire, Wisconsin, USA, , Sub-chapter of CRC Press book: *Transform and Data Compression. A Primer on Wavelets and Their Scientific Applications*.

## See Also

`imread` | `imwrite` | `wmaxlev`

# wdcbm

Thresholds for wavelet 1-D using Birgé-Massart strategy

## Syntax

```
[THR,NKEEP] = wdcbm(C,L,ALPHA,M)
wdcbm(C,L,ALPHA)
wdcbm(C,L,ALPHA,L(1))
```

## Description

`[THR,NKEEP] = wdcbm(C,L,ALPHA,M)` returns level-dependent thresholds `THR` and numbers of coefficients to be kept `NKEEP`, for de-noising or compression. `THR` is obtained using a wavelet coefficients selection rule based on the Birgé-Massart strategy.

`[C,L]` is the wavelet decomposition structure of the signal to be de-noised or compressed, at level `j = length(L)-2`. `ALPHA` and `M` must be real numbers greater than 1.

`THR` is a vector of length `j`; `THR(i)` contains the threshold for level i.

`NKEEP` is a vector of length `j`; `NKEEP(i)` contains the number of coefficients to be kept at level i.

j, `M` and `ALPHA` define the strategy:

- At level j+1 (and coarser levels), everything is kept.
- For level i from 1 to j, the $n_i$ largest coefficients are kept with $n_i$ = `M` / $(j+2-i)^{ALPHA}$.

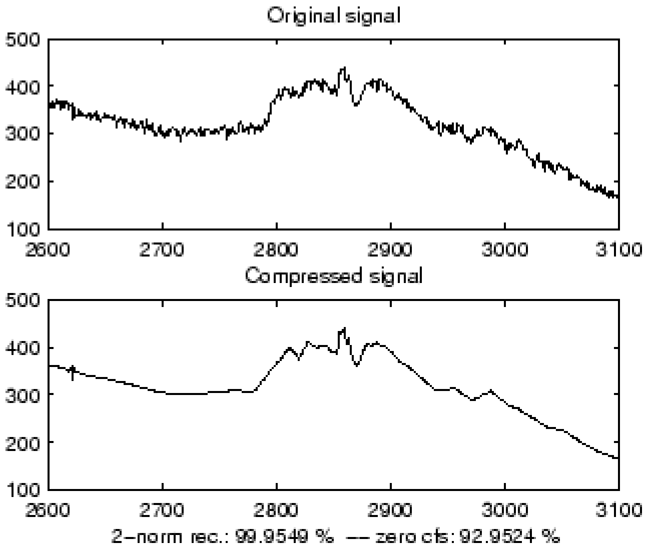
Typically `ALPHA` = 1.5 for compression and `ALPHA` = 3 for de-noising.

A default value for `M` is `M` = `L`(1), the number of the coarsest approximation coefficients, since the previous formula leads for i = j+1, to $n_{j+1}$ = `M` = `L`(1). Recommended values for `M` are from `L`(1) to 2*`L`(1).

`wdcbm(C,L,ALPHA)` is equivalent to `wdcbm(C,L,ALPHA,L(1))`.

## Examples

```
% Load electrical signal and select a part of it.
load leleccum; indx = 2600:3100;
x = leleccum(indx);

% Perform a wavelet decomposition of the signal
% at level 5 using db3.
wname = 'db3'; lev = 5;
[c,l] = wavedec(x,lev,wname);

% Use wdcbm for selecting level dependent thresholds
% for signal compression using the adviced parameters.
alpha = 1.5; m = l(1);
[thr,nkeep] = wdcbm(c,l,alpha,m)

thr =
   19.5569   17.1415   20.2599   42.8959   15.0049

nkeep =
     1     2     3     4     7

% Use wdencmp for compressing the signal using the above
% thresholds with hard thresholding.
[xd,cxd,lxd,perf0,perfl2] = ...
                  wdencmp('lvd',c,l,wname,lev,thr,'h');

% Plot original and compressed signals.
subplot(211), plot(indx,x), title('Original signal');
subplot(212), plot(indx,xd), title('Compressed signal');
xlab1 = ['2-norm rec.: ',num2str(perfl2)];
xlab2 = [' %  -- zero cfs: ',num2str(perf0), ' %'];
xlabel([xlab1 xlab2]);
```



## References

Birgé, L.; P. Massart (1997), "From model selection to adaptive estimation," in D. Pollard (ed), *Festchrift for L. Le Cam*, Springer, pp. 55–88.

## See Also

`wden` | `wdencmp` | `wpdencmp`

# wdcbm2

Thresholds for wavelet 2-D using Birgé-Massart strategy

## Syntax

```
[THR,NKEEP] = wdcbm2(C,S,ALPHA,M)
wdcbm2(C,S,ALPHA)
wdcbm2(C,S,ALPHA,prod(S(1,:)))
```

## Description

`[THR,NKEEP] = wdcbm2(C,S,ALPHA,M)` returns level-dependent thresholds `THR` and numbers of coefficients to be kept `NKEEP`, for de-noising or compression. `THR` is obtained using a wavelet coefficients selection rule based on the Birgé-Massart strategy.

`[C,S]` is the wavelet decomposition structure of the image to be de-noised or compressed, at level `j = size(S,1)-2`.

`ALPHA` and `M` must be real numbers greater than 1.

`THR` is a matrix 3 by `j`; `THR(:,i)` contains the level dependent thresholds in the three orientations: horizontal, diagonal, and vertical, for level i.

`NKEEP` is a vector of length `j`; `NKEEP(i)` contains the number of coefficients to be kept at level i.

j, `M` and `ALPHA` define the strategy:

- At level `j`+1 (and coarser levels), everything is kept.
- For level i from 1 to `j`, the $n_i$ largest coefficients are kept with $n_i$ = `M` $(j+2-i)^{ALPHA}$.

Typically `ALPHA` = 1.5 for compression and `ALPHA` = 3 for de-noising.

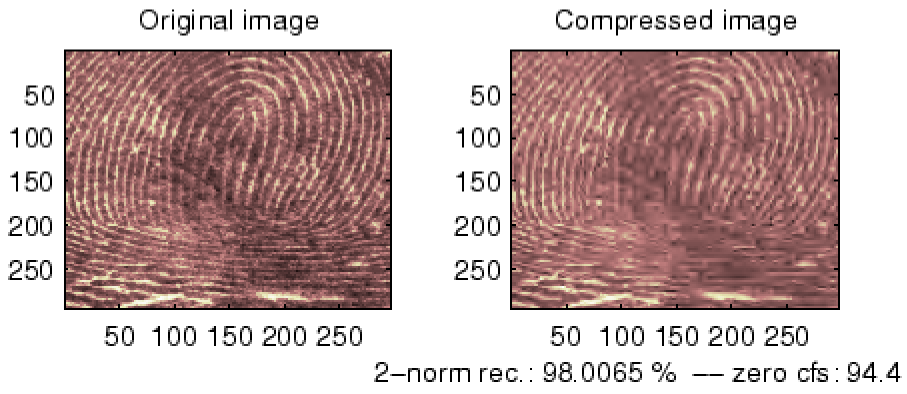
A default value for `M` is `M` = `prod(S(1,:))`, the length of the coarsest approximation coefficients, since the previous formula leads for i = j+1, to $n_{j+1}$ = `M` = `prod(S(1,:))`.

Recommended values for `M` are from `prod(S(1,:))` to `6*prod(S(1,:))`.

`wdcbm2(C,S,ALPHA)` is equivalent to `wdcbm2(C,S,ALPHA,prod(S(1,:)))`.

## Examples

```
% Load original image.
load detfingr;
nbc = size(map,1);

% Perform a wavelet decomposition of the image
% at level 3 using sym4.
wname = 'sym4'; lev = 3;
[c,s] = wavedec2(X,lev,wname);

% Use wdcbm2 for selecting level dependent thresholds
% for image compression using the adviced parameters.
alpha = 1.5; m = 2.7*prod(s(1,:));
[thr,nkeep] = wdcbm2(c,s,alpha,m)

thr =
   21.4814   46.8354   40.7907
   21.4814   46.8354   40.7907
   21.4814   46.8354   40.7907

nkeep =
         624         961        1765

% Use wdencmp for compressing the image using the above
% thresholds with hard thresholding.
[xd,cxd,sxd,perf0,perfl2] = ...
                  wdencmp('lvd',c,s,wname,lev,thr,'h');

% Plot original and compressed images.
colormap(pink(nbc));
subplot(221), image(wcodemat(X,nbc)),
title('Original image')
subplot(222), image(wcodemat(xd,nbc)),
title('Compressed image')
xlab1 = ['2-norm rec.: ',num2str(perfl2)];
xlab2 = [' %  -- zero cfs: ',num2str(perf0), ' %'];
xlabel([xlab1 xlab2]);
```



## References

Birgé, L.; P. Massart (1997). "From model selection to adaptive estimation," in D. Pollard (ed), *Festchrift for L. Le Cam*, Springer, pp. 55–88.

### See Also

`wdencmp` | `wpdencmp`

# wdecenergy

Multisignal 1-D decomposition energy distribution

## Syntax

```
[E,PEC,PECFS] = wdecenergy(DEC)
[E,PEC,PECFS,IDXSORT,LONGS] = wdecenergy(DEC,'sort')
[E,PEC,PECFS] = wdecenergy(DEC,OPTSORT,IDXSIG)
[E,PEC,PECFS,IDXSORT,LONGS] = wdecenergy(DEC,OPTSORT,IDXSIG)
```

## Description

`[E,PEC,PECFS] = wdecenergy(DEC)` computes the vector E that contains the energy (L2-Norm) of each decomposed signal, the matrix PEC that contains the percentage of energy for each wavelet component (approximation and details) of each signal, and the matrix PECFS that contains the percentage of energy for each coefficient.

- E(i) is the energy (L2-norm) of the ith signal.
- PEC(i,1) is the percentage of energy for the approximation of level MAXLEV = DEC.level of the ith signal.
- PEC(i,j), j=2,...,MAXLEV+1 is the percentage of energy for the detail of level (MAXLEV+1-j) of the ith signal.
- PECFS(i,j), is the percentage of energy for jth coefficients of the ith signal.

`[E,PEC,PECFS,IDXSORT,LONGS] = wdecenergy(DEC,'sort')` returns PECFS sorted (by row) in ascending order and an index vector IDXSORT.

- Replacing 'sort' by 'ascend' returns the same result.
- Replacing 'sort' by 'descend' returns PECFS sorted in descending order.

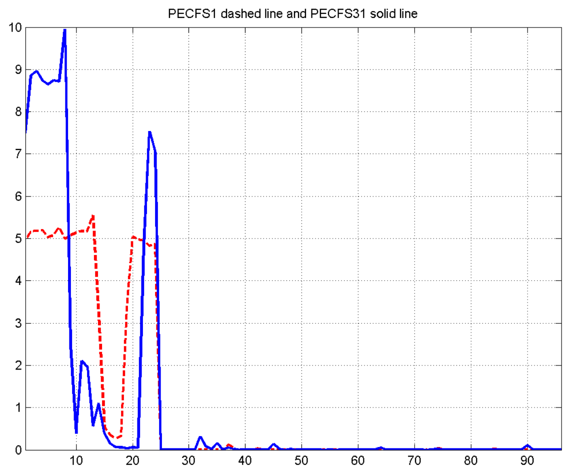
LONGS is a vector containing the lengths of each family of coefficients.

`[E,PEC,PECFS] = wdecenergy(DEC,OPTSORT,IDXSIG)` returns the values for the signals whose indices are given by the `IDXSIG` vector.

`[E,PEC,PECFS,IDXSORT,LONGS] = wdecenergy(DEC,OPTSORT,IDXSIG)` returns the values for the signals whose indices are given by the IDXSIG vector, the index vector IDXSORT, and LONGS, which is a vector containing the lengths of each family of coefficients. Valid values for OPTSORT are 'none', 'sort', 'ascend', 'descend'.

## Examples

```
% Load original 1D-multisignal.
load thinker
% Perform a decomposition at level 2 using wavelet db2.
dec = mdwtdec('r',X,2,'db2');
% Compute the energy distribution.
[E,PEC,PECFS] = wdecenergy(dec);
% Display the total energy and the distribution of energy
% for each wavelet component (A2, D2, D1).
E31 = E(31)
perA2D2D1 = PEC(31,:)
% Compare the coefficient energy distribution
% for signal 1 and signal 31.
PECFS_1  = PECFS(1,:);
PECFS_31 = PECFS(31,:);
figure;
plot(PECFS_1,'--r','linewidth',2); hold on
plot(PECFS_31,'b','linewidth',2);
grid; set(gca,'Xlim',[1,size(PECFS,2)])
title('PECFS1 dashed line and PECFS31 solid line')
```



## See Also

`mdwtdec` | `mdwtrec`

## wden

Automatic 1-D de-noising

### Syntax

```
[XD,CXD,LXD] = wden(X,TPTR,SORH,SCAL,N,'wname')
[XD,CXD,LXD] = wden(C,L,TPTR,SORH,SCAL,N,'wname')
```

### Description

`wden` is a one-dimensional de-noising function.

`wden` performs an automatic de-noising process of a one-dimensional signal using wavelets.

`[XD,CXD,LXD] = wden(X,TPTR,SORH,SCAL,N,'wname')` returns a de-noised version `XD` of input signal `X` obtained by thresholding the wavelet coefficients.

Additional output arguments `[CXD,LXD]` are the wavelet decomposition structure (see `wavedec` for more information) of the de-noised signal `XD`.

`TPTR` string contains the threshold selection rule:

- `'rigrsure'` uses the principle of Stein's Unbiased Risk.
- `'heursure'` is an heuristic variant of the first option.
- `'sqtwolog'` for the universal threshold $\sqrt{2\ln(\bullet)}$
- `'minimaxi'` for minimax thresholding (see `thselect` for more information)

`SORH` (`'s'` or `'h'`) is for soft or hard thresholding (see `wthresh` for more information).

`SCAL` defines multiplicative threshold rescaling:

`'one'` for no rescaling

`'sln'` for rescaling using a single estimation of level noise based on first-level coefficients

`'mln'` for rescaling done using level-dependent estimation of level noise

Wavelet decomposition is performed at level `N` and `'wname'` is a string containing the name of the desired orthogonal wavelet (see `wmaxlev` and `wfilters` for more information).

`[XD,CXD,LXD] = wden(C,L,TPTR,SORH,SCAL,N,'wname')` returns the same output arguments, using the same options as above, but obtained directly from the input wavelet decomposition structure `[C,L]` of the signal to be de-noised, at level `N` and using `'wname'` orthogonal wavelet.

The underlying model for the noisy signal is basically of the following form:

$$s(n) = f(n) + \sigma e(n)$$

where time $n$ is equally spaced.

In the simplest model, suppose that $e(n)$ is a Gaussian white noise $N(0,1)$ and the noise level σ a is supposed to be equal to 1.

The de-noising objective is to suppress the noise part of the signal $s$ and to recover $f$.

The de-noising procedure proceeds in three steps:

1. Decomposition. Choose a wavelet, and choose a level `N`. Compute the wavelet decomposition of the signal s at level `N`.
2. Detail coefficients thresholding. For each level from 1 to `N`, select a threshold and apply soft thresholding to the detail coefficients.
3. Reconstruction. Compute wavelet reconstruction based on the original approximation coefficients of level `N` and the modified detail coefficients of levels from 1 to `N`.

More details about threshold selection rules are in “Denoising and Nonparametric Function Estimation”, in the User's Guide, and in the help of the `thselect` function. Let us point out that

- The detail coefficients vector is the superposition of the coefficients of $f$ and the coefficients of $e$, and that the decomposition of $e$ leads to detail coefficients that are standard Gaussian white noises.
- Minimax and SURE threshold selection rules are more conservative and are more convenient when small details of function $f$ lie in the noise range. The two other rules remove the noise more efficiently. The option `'heursure'` is a compromise.

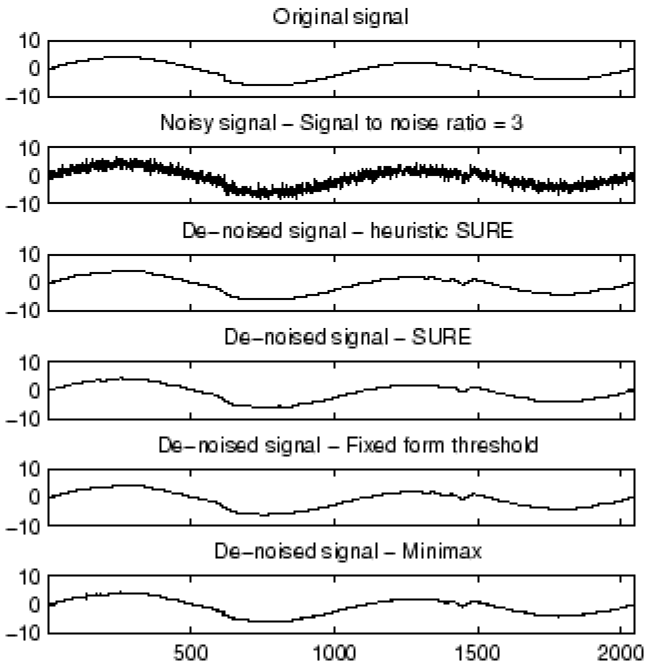
In practice, the basic model cannot be used directly. This section examines the options available, to deal with model deviations. The remaining parameter `scal` has to be specified. It corresponds to threshold rescaling methods.

- Option `scal` = `'one'` corresponds to the basic model.
- In general, you can ignore the noise level that must be estimated. The detail coefficients $CD_1$ (the finest scale) are essentially noise coefficients with standard deviation equal to σ. The median absolute deviation of the coefficients is a robust estimate of σ. The use of a robust estimate is crucial because if level 1 coefficients contain *f* details, these details are concentrated in few coefficients to avoid signal end effects, which are pure artifacts due to computations on the edges.
- The option `scal = 'sln'` handles threshold rescaling using a single estimation of level noise based on the first-level coefficients.
- When you suspect a nonwhite noise *e*, thresholds must be rescaled by a level-dependent estimation of the level noise. The same kind of strategy is used by estimating $\sigma_{lev}$ level by level. This estimation is implemented in the file `wnoisest`, which handles the wavelet decomposition structure of the original signal *s* directly.
- The option `scal` = `'mln'` handles threshold rescaling using a level-dependent estimation of the level noise.

## Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Set signal to noise ratio and set rand seed.
snr = 3; init = 2055615866;

% Generate original signal and a noisy version adding
% a standard Gaussian white noise.
[xref,x] = wnoise(3,11,snr,init);

% De-noise noisy signal using soft heuristic SURE thresholding
% and scaled noise option, on detail coefficients obtained
% from the decomposition of x, at level 5 by sym8 wavelet.
lev = 5;
xd = wden(x,'heursure','s','one',lev,'sym8');

% Plot signals.
subplot(611), plot(xref), axis([1 2048 -10 10]);
title('Original signal');
```

```
subplot(612), plot(x), axis([1 2048 -10 10]);
title(['Noisy signal - Signal to noise ratio = ',...
num2str(fix(snr))]);
subplot(613), plot(xd), axis([1 2048 -10 10]);
title('De-noised signal - heuristic SURE');

% De-noise noisy signal using soft SURE thresholding
xd = wden(x,'heursure','s','one',lev,'sym8');

% Plot signal.
subplot(614), plot(xd), axis([1 2048 -10 10]);
title('De-noised signal - SURE');

% De-noise noisy signal using fixed form threshold with
% a single level estimation of noise standard deviation.
xd = wden(x,'sqtwolog','s','sln',lev,'sym8');

% Plot signal.
subplot(615), plot(xd), axis([1 2048 -10 10]);
title('De-noised signal - Fixed form threshold');

% De-noise noisy signal using minimax threshold with
% a multiple level estimation of noise standard deviation.
xd = wden(x,'minimaxi','s','sln',lev,'sym8');

% Plot signal.
subplot(616), plot(xd), axis([1 2048 -10 10]);
title('De-noised signal - Minimax');

% If many trials are necessary, it is better to perform
% decomposition once and threshold it many times:

% decomposition.
[c,l] = wavedec(x,lev,'sym8');

% threshold the decomposition structure [c,l].
xd = wden(c,l,'minimaxi','s','sln',lev,'sym8');

% Editing some graphical properties,
% the following figure is generated.
```



## References

Antoniadis, A.; G. Oppenheim, Eds. (1995), *Wavelets and statistics*, 103, Lecture Notes in Statistics, Springer Verlag.

Donoho, D.L. (1993), “Progress in wavelet analysis and WVD: a ten minute tour,” in *Progress in wavelet analysis and applications*, Y. Meyer, S. Roques, pp. 109–128. Frontières Ed.

Donoho, D.L.; I.M. Johnstone (1994), “Ideal spatial adaptation by wavelet shrinkage,” *Biometrika*, Vol. 81, pp. 425–455.

Donoho, D.L. (1995), “De-noising by soft-thresholding,” *IEEE Trans. on Inf. Theory*, 42 3, pp. 613– 627.

Donoho, D.L.; I.M. Johnstone, G. Kerkyacharian, D. Picard (1995), “Wavelet shrinkage: asymptotia,” *Jour. Roy. Stat. Soc.*, *series B*, Vol. 57, No. 2, pp. 301–369.

## See Also

`thselect` | `wavedec` | `wdencmp` | `wfilters` | `wthresh`

## wdencmp

De-noising or compression

### Syntax

```
[XC,CXC,LXC,PERF0,PERFL2] =
wdencmp('gbl',X,'wname',N,THR,SORH,KEEPAPP)
wdencmp('gbl',C,L,'wname',N,THR,SORH,KEEPAPP)
[XC,CXC,LXC,PERF0,PERFL2] = wdencmp('lvd',X,'wname',N,THR,SORH)
[XC,CXC,LXC,PERF0,PERFL2] = wdencmp('lvd',C,L,'wname',N,THR,SORH)
[XC,CXC,LXC,PERF0,PERFL2] = wdencmp('lvd',X,'wname',N,THR,SORH)
[XC,CXC,LXC,PERF0,PERFL2] = wdencmp('lvd',C,L,'wname',N,THR,SORH)
```

### Description

`wdencmp` is a one- or two-dimensional de-noising and compression-oriented function.

`wdencmp` performs a de-noising or compression process of a signal or an image, using wavelets.

`[XC,CXC,LXC,PERF0,PERFL2] = wdencmp('gbl',X,'wname',N,THR,SORH,KEEPAPP)` returns a de-noised or compressed version `XC` of input signal `X` (one- or two-dimensional) obtained by wavelet coefficients thresholding using global positive threshold `THR`.

Additional output arguments `[CXC,LXC]` are the wavelet decomposition structure of `XC` (see `wavedec` or `wavedec2` for more information). `PERF0` and `PERFL2` are $L^2$ -norm recovery and compression score in percentage.

`PERFL2` = 100 * (vector-norm of `CXC` / vector-norm of `C`)$^2$ if `[C,L]` denotes the wavelet decomposition structure of `X`.

If `X` is a one-dimensional signal and `'wname'` an orthogonal wavelet, `PERFL2` is reduced to

$$\frac{100\|XC\|^2}{\|X\|^2}$$

Wavelet decomposition is performed at level `N` and `'wname'` is a string containing wavelet name (see `wmaxlev` and `wfilters` for more information). `SORH` (`'s'` or `'h'`) is for soft or hard thresholding (see `wthresh` for more information). If `KEEPAPP` = 1, approximation coefficients cannot be thresholded, otherwise it is possible.

`wdencmp('gbl',C,L,'wname',N,THR,SORH,KEEPAPP)` has the same output arguments, using the same options as above, but obtained directly from the input wavelet decomposition structure `[C,L]` of the signal to be de-noised or compressed, at level `N` and using `'wname'` wavelet.

For the one-dimensional case and `'lvd'` option, `[XC,CXC,LXC,PERF0,PERFL2] = wdencmp('lvd',X,'wname',N,THR,SORH)` or `[XC,CXC,LXC,PERF0,PERFL2] = wdencmp('lvd',C,L,'wname',N,THR,SORH)` have the same output arguments, using the same options as above, but allowing level-dependent thresholds contained in vector `THR` (`THR` must be of length `N`). In addition, the approximation is kept. Note that, with respect to `wden` (automatic de-noising), `wdencmp` allows more flexibility and you can implement your own de-noising strategy.

For the two-dimensional case and `'lvd'` option, `[XC,CXC,LXC,PERF0,PERFL2] = wdencmp('lvd',X,'wname',N,THR,SORH)` or `[XC,CXC,LXC,PERF0,PERFL2] = wdencmp('lvd',C,L,'wname',N,THR,SORH)`.

`THR` must be a matrix 3 by `N` containing the level-dependent thresholds in the three orientations, horizontal, diagonal, and vertical.

Like denoising, the compression procedure contains three steps:

1. Decomposition.
2. Detail coefficient thresholding. For each level from 1 to `N`, a threshold is selected and hard thresholding is applied to the detail coefficients.
3. Reconstruction.

The difference with the denoising procedure is found in step 2.

## Examples

### Denoise Image Using Default Global Threshold

Denoise an image in additive white Gaussian noise using the Donoho-Johnstone universal threshold.

Load the image and add white Gaussian noise.

```
load sinsin;
Y = X + 18*randn(size(X));
```

Use `ddencmp` to obtain the threshold and denoise the image. Plot the original image, noisy image, and denoised result.

```
[thr,sorh,keepapp] = ddencmp('den','wv',Y);
xd = wdencmp('gbl',Y,'sym4',2,thr,sorh,keepapp);
subplot(221)
imagesc(X); title('Original Image');
subplot(222);
imagesc(Y); title('Noisy Image');
subplot(223)
imagesc(xd); title('Denoised Image');
```

### Denoise 1-D Signal Using Default Global Threshold

Denoise 1-D electricity consumption data using the Donoho-Johnstone global threshold.

Load the signal and select a segment for denoising.

```
load leleccum; indx = 2600:3100;
x = leleccum(indx);
```

Use `ddencmp` to determine the default global threshold and denoise the signal. Plot the original and denoised signals.

```
[thr,sorh,keepapp] = ddencmp('den','wv',x);
xd = wdencmp('gbl',x,'db3',2,thr,sorh,keepapp);
subplot(211)
plot(x); title('Original Signal');
subplot(212)
plot(xd); title('Denoised Signal');
```

## References

DeVore, R.A.; B. Jawerth, B.J. Lucier (1992), “Image compression through wavelet transform coding,” *IEEE Trans. on Inf. Theory*, vol. 38, No 2, pp. 719–746.

Donoho, D.L. (1993), “Progress in wavelet analysis and WVD: a ten minute tour,” in Progress in wavelet analysis and applications, Y. Meyer, S. Roques, pp. 109–128. Frontières Ed.

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Donoho, D.L.; I.M. Johnstone, “Ideal de-noising in an orthonormal basis chosen from a library of bases,” C.R.A.S. Paris, t. 319, Ser. I, pp. 1317–1322.

Donoho, D.L. (1995), “De-noising by soft-thresholding,” *IEEE Trans. on Inf. Theory*, 41, 3, pp. 613–627.

## See Also

`ddencmp` | `wavedec` | `wavedec2` | `wbmpen` | `wcompress` | `wdcbm2` | `wden` | `wpdencmp` | `wthresh`

## wenergy

Energy for 1-D wavelet or wavelet packet decomposition

### Syntax

```
[Ea,Ed] = wenergy(C,L)
E = wenergy(T)
```

### Description

For a one-dimensional wavelet decomposition `[C,L]` (see `wavedec` for details), `[Ea,Ed] = wenergy(C,L)` returns `Ea`, which is the percentage of energy corresponding to the approximation and `Ed`, which is the vector containing the percentages of energy corresponding to the details.

For a wavelet packet tree *T* (see `wptree`, `wpdec`, `wpdec2`), `E = wenergy(T)` returns a vector `E`, which contains the percentages of energy corresponding to the terminal nodes of the tree *T*. In this case, `wenergy` is a method of the `wptree` object *T*, which overloads the previous `wenergy` function.

### Examples

```
% Example 1: 1-D wavelet decomposition
%-------------------------------------
load noisbump
[C,L] = wavedec(noisbump,4,'sym4');
[Ea,Ed] = wenergy(C,L)

Ea =

   88.2860

Ed =

    2.1560    1.2286    1.4664    6.8630
```

```
% Example 2: 1-D wavelet packet decomposition
%-------------------------------------------
load noisbump
T = wpdec(noisbump,3,'sym4');
E = wenergy(T)

E =

95.0329    1.4664    0.6100    0.6408    0.5935    0.5445    0.5154
0.5965
```

# wenergy2

Energy for 2-D wavelet decomposition

## Syntax

```
[Ea,Eh,Ev,Ed] = wenergy2(C,S)
[Ea,EDetail] = wenergy2(C,S)
```

## Description

For a two-dimensional wavelet decomposition `[C,S]` (see `wavedec2` for details), `[Ea,Eh,Ev,Ed] = wenergy2(C,S)` returns `Ea`, which is the percentage of energy corresponding to the approximation, and vectors `Eh`, `Ev`, `Ed`, which contain the percentages of energy corresponding to the horizontal, vertical, and diagonal details, respectively.

`[Ea,EDetail] = wenergy2(C,S)` returns `Ea`, and `EDetail`, which is the sum of vectors `Eh`, `Ev`, and `Ed`.

## Examples

```
load detail
[C,S] = wavedec2(X,2,'sym4');
[Ea,Eh,Ev,Ed] = wenergy2(C,S)

Ea =
   89.3520

Eh =
    1.8748    2.7360

Ev =
    1.5860    2.6042

Ed =
    0.7539    1.0932
```

```
[Ea,EDetails] = wenergy2(C,S)

Ea =
   89.3520

EDetails =
    4.2147    6.4334
```

## wentropy

Entropy (wavelet packet)

### Syntax

```
E = wentropy(X,T,P)
E = wentropy(X,T)
E = wentropy(X,T,0)
```

### Description

`E = wentropy(X,T,P)` returns the entropy `E` of the vector or matrix input `X`. In both cases, output `E` is a real number.

`E = wentropy(X,T)` is equivalent to `E = wentropy(X,T,0)`.

*T* is a string containing the type of entropy and `P` is an optional parameter depending on the value of *T*.

| Entropy Type Name (*T*) | Parameter (P) | Comments |
|---|---|---|
| `'shannon'` | | `P` is not used. |
| `'log energy'` | | `P` is not used. |
| `'threshold'` | `0 ≤ P` | `P` is the threshold. |
| `'sure'` | `0 ≤ P` | `P` is the threshold. |
| `'norm'` | `1 ≤ P` | `P` is the power. |
| `'user'` | `string` | `P` is a string containing the file name of your own entropy function, with a single input `X`. |
| `FunName` | `No constraints on P` | `FunName` is any other string except those used for the previous Entropy Type Names listed above.<br><br>`FunName` contains the file name of your own entropy function, with `X` as input |

| Entropy Type Name (*T*) | Parameter (P) | Comments |
|---|---|---|
| | | and P as additional parameter to your entropy function. |

**Note** The `'user'` option is historical and still kept for compatibility, but it is obsoleted by the last option described in the table above. The `FunName` option do the same as the `'user'` option and in addition gives the possibility to pass a parameter to your own entropy function.

Functionals verifying an additive-type property are well suited for efficient searching of binary-tree structures and the fundamental splitting property of the wavelet packets decomposition. Classical entropy-based criteria match these conditions and describe information-related properties for an accurate representation of a given signal. Entropy is a common concept in many fields, mainly in signal processing. The following example lists different entropy criteria. Many others are available and can be easily integrated. In the following expressions, *s* is the signal and $(s_i)_i$ the coefficients of *s* in an orthonormal basis.

The entropy `E` must be an additive cost function such that $E(0) = 0$ and

$$E(s) = \sum_i E(s_i)$$

- The (nonnormalized) Shannon entropy.

  $E1(s_i) = s_i^2 \log(s_i^2)$ so $E1(s) = -\sum_i s_i^2 \log(s_i^2)$

  with the convention $0log(0) = 0$.

- The concentration in $l^p$ norm entropy with $1 \le p$.

  $E2(s_i) = |s_i|^p$ so $E2(s) = \sum_i |s_i|^p = \|s\|_p^p$

- The “log energy” entropy.

  $E3(s_i) = \log(s_i^2)$ so $E3(s) = \sum_i \log(s_i^2)$

with the convention $log(0) = 0$.

- The threshold entropy.

  $E4(s_i) = 1$ if $|s_i| > p$ and 0 elsewhere so $E4(s) = \#\{i$ such that $|s_i| > p\}$ is the number of time instants when the signal is greater than a threshold $p$.

- The "SURE" entropy.

  E5(s) = n - #{i such that $|s_i| \le p\} + \sum_i min(s_i^2, p^2)$

  For more information, see the section "Wavelet Packets for Compression and De-Noising" of the User's Guide.

## Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Generate initial signal.
x = randn(1,200);

% Compute Shannon entropy of x.
e = wentropy(x,'shannon')

e =
    -142.7607

% Compute log energy entropy of x.
e = wentropy(x,'log energy')
e =
    -281.8975

% Compute threshold entropy of x
% with threshold equal to 0.2.
e = wentropy(x,'threshold',0.2)
e =
    162

% Compute Sure entropy of x
% with threshold equal to 3.
e = wentropy(x,'sure',3)
```

```
e =
 -0.6575

% Compute norm entropy of x with power equal to 1.1.
e = wentropy(x,'norm',1.1)
e =
 160.1583

% Compute user entropy of x with a user defined
% function: userent for example.
% This function must be a code file, with first line
% of the following form:
%
%     function e = userent(x)
%
% where x is a vector and e is a real number.
% Then a new entropy is defined and can be used typing:
%
% e = wentropy(x,'user','userent')
%
% or more directly
%
% e = wentropy(x,'userent')
```

## References

Coifman, R.R.; M.V. Wickerhauser (1992), "Entropy-based Algorithms for best basis selection," *IEEE Trans. on Inf. Theory*, vol. 38, 2, pp. 713–718.

Donoho, D.L.; I.M. Johnstone, "Ideal de-noising in an orthonormal basis chosen from a library of bases," *C.R.A.S. Paris*, *Ser. I*, t. 319, pp. 1317–1322.

# wextend

Extend vector or matrix

## Syntax

## Description

The valid extension types (TYPE) are listed in the table below.

| TYPE | Description |
|---|---|
| 1, '1', '1d' or '1D' | 1-D extension |
| 2, '2', '2d' or '2D' | 2-D extension |
| 'ar' or 'addrow' | Add rows |
| 'ac' or 'addcol' | Add columns |

The valid extension modes (MODE) are listed in the table below.

| MODE | Description |
|---|---|
| 'zpd' | Zero extension |
| 'sp0' | Smooth extension of order 0 |
| 'spd' (or 'sp1') | Smooth extension of order 1 |
| 'sym' or 'symh' | Symmetric-padding (half-point): boundary value symmetric replication |
| 'symw' | Symmetric-padding (whole-point): boundary value symmetric replication |
| 'asym' or 'asymh' | Antisymmetric-padding (half-point): boundary value antisymmetric replication |
| 'asymw' | Antisymmetric-padding (whole-point): boundary value antisymmetric replication |
| 'ppd' | Periodized extension (1) |

| MODE | Description |
|---|---|
| `'per'` | Periodized extension (2):<br><br>If the signal length is odd, `wextend` adds an extra sample, equal to the last value, on the right and performs extension using the `'ppd'` mode. Otherwise, `'per'` reduces to `'ppd'`. The same kind of rule stands for images. |

With `TYPE = {1, '1', '1d'` or `'1D'}`:

- `LOC = 'l'` (or `'u'`) for left (or up) extension.
- `LOC = 'r'` (or `'d'`) for right (or down) extension.
- `LOC = 'b'` for extension on both sides.
- `LOC = 'n'` null extension.
- The default is `LOC = 'b'`.
- `L` is the length of the extension.

With `TYPE = {'ar', 'addrow'}`:

- `LOC` is a `1D` extension location.
- The default is `LOC = 'b'`.
- `L` is the number of rows to add.

With `TYPE` = {`'ac', 'addcol'`}:

- `LOC` is a `1D` extension location.
- The default is `LOC = 'b'`.
- `L` is the number of columns to add.

With `TYPE` = {`2, '2', '2d'` or `'2D'`}:

- `LOC` = `[LOCROW,LOCCOL]` where `LOCROW` and `LOCCOL` are `1D` extension locations or `'n'` (none).
- The default is `LOC = 'bb'`.
- `L = [LROW,LCOL]` where `LROW` is the number of rows to add and `LCOL` is the number of columns to add.

For more information on symmetric extension modes see “References”.

## Examples

```
% Original signal.
x = [1 2 3]

x =

     1     2     3

% 1-D extension length.
l = 2;

% Zero-padding extensions 1-D.
xextzpd1 = wextend('1','zpd',x,l)
xextzpd1 =

     0     0     1     2     3     0     0

xextzpd2 = wextend('1D','zpd',x,l,'b')

xextzpd2 =

     0     0     1     2     3     0     0

% Symmetric extension 1-D.
xextsym = wextend('1D','sym',x,l)

xextsym =

     2     1     1     2     3     3     2

% Periodic extension 1-D.
xextper = wextend('1D','per',x,l)

xextper =

     3     3     1     2     3     3     1     2

% Original image.
X = [1 2 3;4 5 6]
```

```
X =
     1     2     3
     4     5     6

% 2-D extension length.
l = 2;

% Zero-padding extension 2-D.
Xextzpd = wextend(2,'zpd',X,l)

Xextzpd =
     0     0     0     0     0     0     0
     0     0     0     0     0     0     0
     0     0     1     2     3     0     0
     0     0     4     5     6     0     0
     0     0     0     0     0     0     0
     0     0     0     0     0     0     0

% Symmetric extension 2-D.
Xextsym = wextend('2D','sym',X,l)

Xextsym =
     5     4     4     5     6     6     5
     2     1     1     2     3     3     2
     2     1     1     2     3     3     2
     5     4     4     5     6     6     5
     5     4     4     5     6     6     5
     2     1     1     2     3     3     2
```

## References

Strang, G.; T. Nguyen (1996), *Wavelets and filter banks*, Wellesley- Cambridge Press.

## wfbm

Fractional Brownian motion synthesis

### Syntax

```
FBM = wfbm(H,L)
FBM = wfbm(H,L,'plot')
FBM = wfbm(H,L,NS,W)
FBM = wfbm(H,L,W,NS)
wfbm(H,L,'plot',NS)
wfbm(H,L,'plot',W)
wfbm(H,L,'plot',NS,W)
wfbm(H,L,'plot',W,NS)
```

### Description

`FBM = wfbm(H,L)` returns a fractional Brownian motion signal `FBM` of the Hurst parameter `H` (`0 < H < 1`) and length `L`, following the algorithm proposed by Abry and Sellan.

`FBM = wfbm(H,L,'plot')` generates and plots the `FBM` signal.

`FBM = wfbm(H,L,NS,W)` or `FBM = wfbm(H,L,W,NS)` returns the `FBM` using `NS` reconstruction steps and the sufficiently regular orthogonal wavelet `W`.

`wfbm(H,L,'plot',NS)` or `wfbm(H,L,'plot',W)` or `wfbm(H,L,'plot',NS,W)` or `wfbm(H,L,'plot',W,NS)` generates and plots the `FBM` signal.

`wfbm(H,L)` is equivalent to `WFBM(H,L,6,'db10')`.

`wfbm(H,L,NS)` is equivalent to `WFBM(H,L,NS,'db10')`.

`wfbm(H,L,W)` is equivalent to `WFBM(H,L,W,6)`.

A fractional Brownian motion (`fBm`) is a continuous-time Gaussian process depending on the Hurst parameter `0 < H < 1`. It generalizes the ordinary Brownian motion corresponding to `H = 0.5` and whose derivative is the white noise. The `fBm` is self-similar in distribution and the variance of the increments is given by

```
Var(fBm(t)-fBm(s)) = v |t-s|^(2H)
```

where `v` is a positive constant.

## Examples

According to the value of `H`, the `fBm` exhibits for `H > 0.5`, long-range dependence and for `H < 0.5`, short or intermediate dependence. This example shows each situation using the `wfbm` file, which generates a sample path of this process.

```
% Generate fBm for H = 0.3 and H = 0.7

% Set the parameter H and the sample length
H = 0.3; lg = 1000;
% Generate and plot wavelet-based fBm for H = 0.3
fBm03 = wfbm(H,lg,'plot');

H = 0.7;
% Generate and plot wavelet-based fBm for H = 0.7
fBm07 = wfbm(H,lg,'plot');

% The last step is equivalent to
% Define wavelet and level of decomposition
% w = ' db10'; ns = 6;
% Generate
% fBm07 = wfbm(H,lg,'plot',w,ns);
```

`fBm07` clearly exhibits a stronger low-frequency component and has, locally, less irregular behavior.

## More About

### Algorithms

Starting from the expression of the `fBm` process as a fractional integral of the white noise process, the idea of the algorithm is to build a biorthogonal wavelet depending on a given orthogonal one and adapted to the parameter `H`.

Then the generated sample path is obtained by the reconstruction using the new wavelet starting from a wavelet decomposition at a given level designed as follows: details coefficients are independent random Gaussian realizations and approximation coefficients come from a fractional ARIMA process.

This method was first proposed by Meyer and Sellan and implementation issues were examined by Abry and Sellan.

Nevertheless, the samples generated following this original scheme exhibit too many high-frequency components. To circumvent this undesirable behavior Bardet et al. propose downsampling the obtained sample by a factor 10.

Two internal parameters `delta = 10` (the downsampling factor) and a threshold `prec = 1E-4`, to evaluate series by truncated sums, can be modified by the user for extreme values of `H`.

A complete overview of long-range dependence process generators is available in Bardet et al.

## References

Abry, P.; F. Sellan (1996), “The wavelet-based synthesis for the fractional Brownian motion proposed by F. Sellan and Y. Meyer: Remarks and fast implementation,” *Appl. and Comp. Harmonic Anal.*, 3(4), pp. 377–383.

Bardet, J.-M.; G. Lang, G. Oppenheim, A. Philippe, S. Stoev, M.S. Taqqu (2003), “Generators of long-range dependence processes: a survey,” Theory and applications of long-range dependence, Birkhäuser, pp. 579–623.

### See Also

`wfbmesti`

# wfbmesti

Parameter estimation of fractional Brownian motion

## Syntax

```
HEST = wfbmesti(X)
```

## Description

`HEST = wfbmesti(X)` returns a one-by-three vector `HEST` which contains three estimates of the fractal index `H` of the input signal `X`. The signal `X` is assumed to be a realization of fractional Brownian motion with Hurst index `H`.

The first two elements of the vector are estimates based on the second derivative with the second computed in the wavelet domain.

The third estimate is based on the linear regression in loglog plot, of the variance of detail versus level.

A fractional Brownian motion (`fBm`) is a continuous-time Gaussian process depending on the so-called Hurst parameter `0 < H < 1`. It generalizes the ordinary Brownian motion corresponding to `H = 0.5` and whose derivative is the white noise. The `fBm` is self-similar in distribution and the variance of the increments is

```
Var(fBm(t)-fBm(s)) = v |t-s|^(2H)
```

where `v` is a positive constant.

This special form of the variance of the increments suggests various ways to estimate the parameter `H`. One can find in Bardet et al. a survey of such methods. The `wfbmesti` file provides three different estimates. The first one, due to Istas and Lang, is based on the discrete second-order derivative. The second one is a wavelet-based adaptation and has similar properties. The third one, proposed by Flandrin, estimates `H` using the slope of the loglog plot of the detail variance versus the level. A more recent extension can be found in Abry et al.

## Examples

### Hurst Parameter Estimation

This example shows how to estimate the Hurst index of a fractional Brownian motion. The example simulates 1,000 realizations of fractional Brownian motion with H=0.6. Each realization consists of 10,000 samples. At the end of the simulation, the three estimates of the Hurst index are compared.

Initialize the random number generator for repeatable results. Set the Hurst index equal to 0.6 and the length of the realizations to be 10,000.

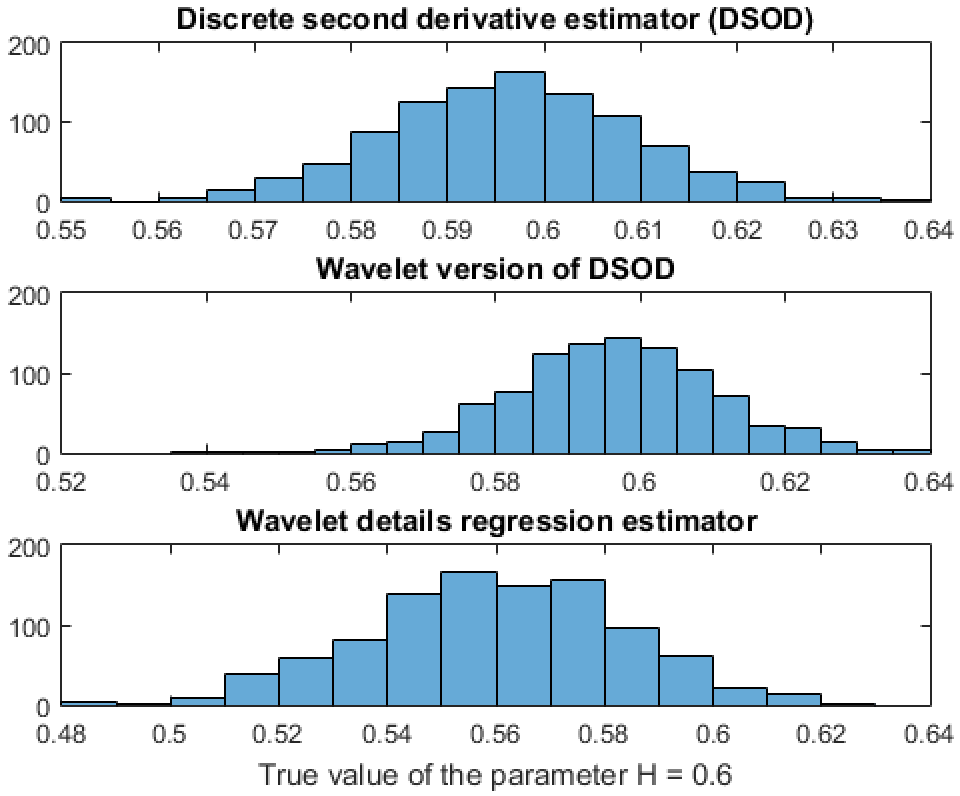
```
rng default;
H = 0.6;
len = 10000;
```

Generate 1,000 realizations of fractional Brownian motion and compute the estimates of the Hurst parameter.

```
n = 1000;
Hest = zeros(n,3);
for ii = 1:n
 fBm06 = wfbm(H,len);
 Hest(ii,:) = wfbmesti(fBm06);
end
```

Compare the estimates.

```
subplot(311), histogram(Hest(:,1));
title('Discrete second derivative estimator (DSOD)')
subplot(312), histogram(Hest(:,2));
title('Wavelet version of DSOD')
subplot(313), histogram(Hest(:,3));
title('Wavelet details regression estimator')
xlabel('True value of the parameter H = 0.6')
```



## References

Abry, P.; P. Flandrin, M.S. Taqqu, D. Veitch (2003), “Self-similarity and long-range dependence through the wavelet lens,” Theory and applications of long-range dependence, Birkhäuser, pp. 527–556.

Bardet, J.-M.; G. Lang, G. Oppenheim, A. Philippe, S. Stoev, M.S. Taqqu (2003), “Semi-parametric estimation of the long-range dependence parameter: a survey,” Theory and applications of long-range dependence, Birkhäuser, pp. 557–577.

Flandrin, P. (1992), “Wavelet analysis and synthesis of fractional Brownian motion,” *IEEE Trans. on Inf. Th.*, 38, pp. 910–917.

Istas, J.; G. Lang (1994), “Quadratic variations and estimation of the local Hölder index of a Gaussian process,” *Ann. Inst. Poincaré*, 33, pp. 407–436.

## See Also

`wfbm`

# wfilters

Wavelet filters

## Syntax

```
[Lo_D,Hi_D,Lo_R,Hi_R] = wfilters('wname')
[F1,F2] = wfilters('wname','type')
```

## Description

`[Lo_D,Hi_D,Lo_R,Hi_R] = wfilters('wname')` computes four filters associated with the orthogonal or biorthogonal wavelet named in the string `'wname'`.

The four output filters are

- `Lo_D`, the decomposition low-pass filter
- `Hi_D`, the decomposition high-pass filter
- `Lo_R`, the reconstruction low-pass filter
- `Hi_R`, the reconstruction high-pass filter

Available orthogonal or biorthogonal wavelet names `'wname'` are listed in the table below.

| **Wavelet Families** | **Wavelets** |
|---|---|
| Daubechies | `'db1'` or `'haar'`, `'db2'`, ... ,`'db10'`, ... , `'db45'` |
| Coiflets | `'coif1'`, ... , `'coif5'` |
| Symlets | `'sym2'`, ... , `'sym8'`, ... ,`'sym45'` |
| Discrete Meyer | `'dmey'` |
| Biorthogonal | `'bior1.1'`, `'bior1.3'`, `'bior1.5'` `'bior2.2'`, `'bior2.4'`, `'bior2.6'`, `'bior2.8'` `'bior3.1'`, `'bior3.3'`, `'bior3.5'`, `'bior3.7'` |

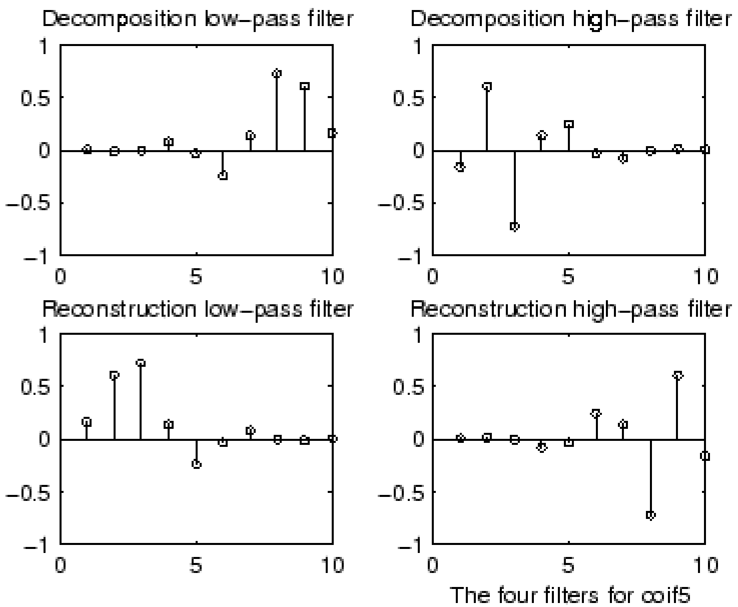
| Wavelet Families | Wavelets |
|---|---|
| | 'bior3.9', 'bior4.4', 'bior5.5', 'bior6.8' |
| Reverse Biorthogonal | 'rbio1.1', 'rbio1.3', 'rbio1.5'<br>'rbio2.2', 'rbio2.4', 'rbio2.6', 'rbio2.8'<br>'rbio3.1', 'rbio3.3', 'rbio3.5', 'rbio3.7'<br>'rbio3.9', 'rbio4.4', 'rbio5.5', 'rbio6.8' |

`[F1,F2] = wfilters('wname','type')` returns the following filters:

| | | |
|---|---|---|
| `Lo_D` and `Hi_D` | (Decomposition filters) | If `'type' = 'd'` |
| `Lo_R` and `Hi_R` | (Reconstruction filters) | If `'type' = 'r'` |
| `Lo_D` and `Lo_R` | (Low-pass filters) | If `'type' = 'l'` |
| `Hi_D` and `Hi_R` | (High-pass filters) | If `'type' = 'h'` |

## Examples

```
% Set wavelet name.
wname = 'db5';

% Compute the four filters associated with wavelet name given
% by the input string wname.
[Lo_D,Hi_D,Lo_R,Hi_R] = wfilters(wname);
subplot(221); stem(Lo_D);
title('Decomposition low-pass filter');
subplot(222); stem(Hi_D);
title('Decomposition high-pass filter');
subplot(223); stem(Lo_R);
title('Reconstruction low-pass filter');
subplot(224); stem(Hi_R);
title('Reconstruction high-pass filter');
xlabel('The four filters for db5')

% Editing some graphical properties,
% the following figure is generated.
```



The four filters for coif5

## References

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics. SIAM Ed.

Mallat, S. (1989), "A theory for multiresolution signal decomposition: the wavelet representation," *IEEE Pattern Anal. and Machine Intell.*, vol. 11, no. 7, pp. 674–693.

## See Also

`biorfilt` | `orthfilt` | `waveinfo`

## wfusimg

Fusion of two images

### Syntax

```
XFUS = wfusimg(X1,X2,WNAME,LEVEL,AFUSMETH,DFUSMETH)
[XFUS,TXFUS,TX1,TX2] = wfusimg(X1,X2,WNAME,LEVEL,AFUSMETH,DFUSMETH)
wfusimg(X1,X2,WNAME,LEVEL,AFUSMETH,DFUSMETH,FLAGPLOT)
```

### Description

The principle of image fusion using wavelets is to merge the wavelet decompositions of the two original images using fusion methods applied to approximations coefficients and details coefficients (see Zeeuw and Misiti et al.).

`XFUS = wfusimg(X1,X2,WNAME,LEVEL,AFUSMETH,DFUSMETH)` returns the fused image `XFUS` obtained by fusion of the two original images `X1` and `X2`. Each fusion method, defined by `AFUSMETH` and `DFUSMETH`, merges in a specific way detailed below, the decompositions of `X1` and `X2`, at level `LEVEL` and using wavelet `WNAME`.

`AFUSMETH` and `DFUSMETH` define the fusion method for approximations and details, respectively.

`[XFUS,TXFUS,TX1,TX2] = wfusimg(X1,X2,WNAME,LEVEL,AFUSMETH,DFUSMETH)` returns, in addition to matrix `XFUS`, three objects of the class WDECTREE associated with `XFUS`, `X1`, and `X2` respectively (see @WDECTREE). `wfusimg(X1,X2,WNAME,LEVEL,AFUSMETH,DFUSMETH,FLAGPLOT)` also plots the objects `TXFUS`, `TX1`, and `TX2`.

`Fusmeth` denotes `AFUSMETH` or `DFUSMETH`. Available fusion methods are

- Simple — `Fusmeth` can be `'max'`, `'min'`, `'mean'`, `'img1'`, `'img2'` or `'rand'`, which merges the two approximations or details structures obtained from `X1` and `X2` elementwise by taking the maximum, the minimum, the mean, the first element, the second element, or a randomly chosen element
- Parameter-dependent — `Fusmeth` is of the following form

  ```
  Fusmeth = struct('name',nameMETH,'param',paramMETH)
  ```

where `nameMETH` can be

| | |
|---|---|
| `'linear'` | |
| `'UD_fusion'` | Up-down fusion |
| `'DU_fusion'` | Down-up fusion |
| `'RL_fusion'` | Right-left fusion |
| `'UserDEF'` | User-defined fusion |

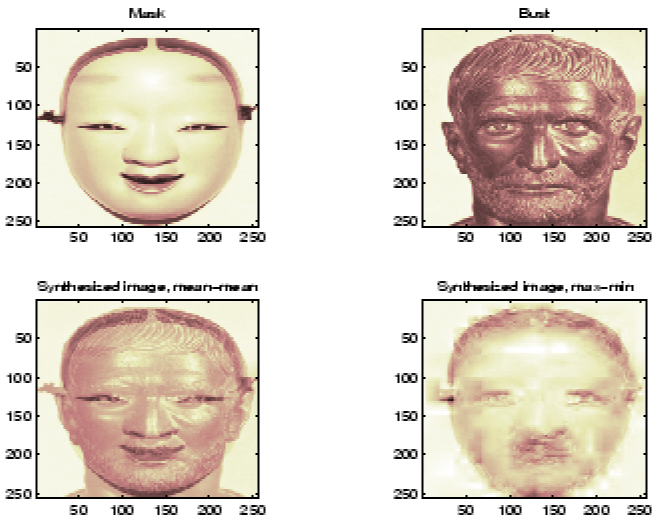
For the description of these options and the `paramMETH` parameter, see `wfusmat`.

## Examples

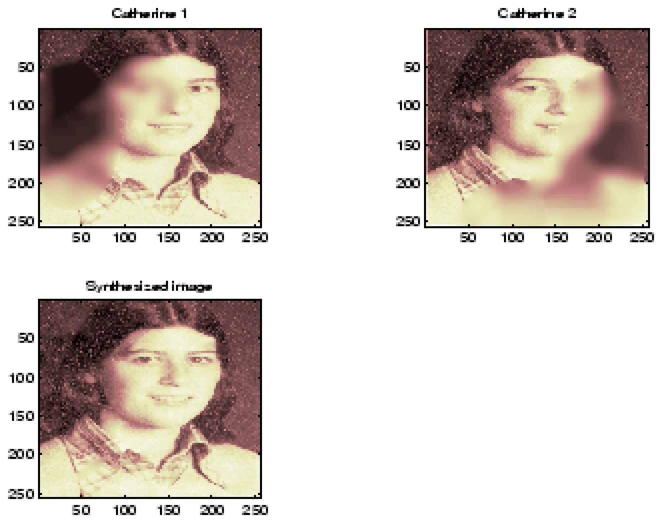
The following three examples examine the process of image fusion

- The first example merges two different images leading to a new image
- The second example restores an image from two fuzzy versions of an original image.
- The third example shows how to make an image fusion using a user defined fusion method.

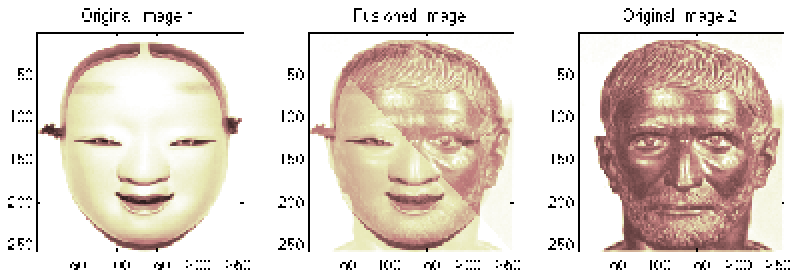
```
% Example 1: Fusion of two different images

% Load two original images: a mask and a bust
load mask; X1 = X;
load bust; X2 = X;

% Merge the two images from wavelet decompositions at level 5
% using db2 by taking two different fusion methods

% fusion by taking the mean for both approximations and details
XFUSmean = wfusimg(X1,X2,'db2',5,'mean','mean');

% fusion by taking the maximum for approximations and the
% minimum for the details
XFUSmaxmin = wfusimg(X1,X2,'db2',5,'max','min');

% Plot original and synthesized images
colormap(map);
subplot(221), image(X1), axis square, title('Mask')
subplot(222), image(X2), axis square, title('Bust')
subplot(223), image(XFUSmean), axis square,
title('Synthesized image, mean-mean')
subplot(224), image(XFUSmaxmin), axis square,
title('Synthesized image, max-min')
```



```
% Example 2: Restoration by fusion of fuzzy images

% Load two fuzzy versions of an original image
load cathe_1; X1 = X;
load cathe_2; X2 = X;

% Merge the two images from wavelet decompositions at level 5
% using sym4 by taking the maximum of absolute value of the
% coefficients for both approximations and details
XFUS = wfusimg(X1,X2,'sym4',5,'max','max');

% Plot original and synthesized images
colormap(map);
subplot(221), image(X1), axis square,
title('Catherine 1')
subplot(222), image(X2), axis square,
title('Catherine 2')
subplot(223), image(XFUS), axis square,
title('Synthesized image')
```



```
% The synthesized image is a restored version of good
% quality of the common underlying original image.

% Example 3: Fusion using a user defined fusion method.
% This example calls a user fusion method defined by the
% file myfus_FUN.m which is listed below at the end of
% the example.

% load two images of the same size.
load mask; A = X;
load bust; B = X;

% Define the fusion method and call the fusion function
Fus_Method = struct('name','userDEF','param','myfus_FUN');
C = wfusmat(A,B,Fus_Method);

figure;
colormap(pink(220))
subplot(1,3,1), image(A), title('Original Image 1'), axis square
subplot(1,3,2), image(C), title('Fusioned Image'), axis square
subplot(1,3,3), image(B), title('Original Image 2'), axis square
```



```
%*****************************
% User defined fusion method.  *
%*****************************
 function C = myfus_FUN(A,B)

D = logical(triu(ones(size(A))));  t = 0.3;
C = A;
C(D)  = t*A(D)+(1-t)*B(D);
C(~D) = t*B(~D)+(1-t)*A(~D);
```

## More About

### Tips

X1 and X2 must be of same size (see `wextend` to resize images) and represent indexed images or truecolor images, which are `m`-by-`n` matrices or `m`-by-`n`-by-3 arrays, respectively.

For more information on image formats, see the `image` and `imfinfo` reference pages.

## References

Zeeuw, P.M. (1998), “Wavelet and image fusion,” CWI, Amsterdam, March 1998, `http:/www.cwi.nl/~pauldz/`

Misiti, M.; Y. Misiti, G. Oppenheim, J.-M. Poggi (2003), “Les ondelettes et leurs applications,” Hermes.

## See Also

`wfusmat` | `wextend`

## wfusmat

Fusion of two matrices or arrayz

### Syntax

```
C = wfusmat(A,B,METHOD)
```

### Description

`C = wfusmat(A,B,METHOD)` returns the fused matrix `C` obtained from the matrices `A` and `B` using the fusion method defined by `METHOD`.

The matrices `A` and `B` must be of the same size. The output matrix `C` is of the same size as `A` and `B`.

Available fusion methods are

- Simple, where `METHOD` is
  - `'max'` : `D = abs(A) ≥ abs(B) ; C = A(D) + B(~D)`
  - `'min'` : `D = abs(A) ≤ abs(B) ; C = A(D) + B(~D)`
  - `'mean'` : `C = (A+B) / 2 ; D = ones(size(A))`
  - `'rand'` : C = A(D) + B(~D); D is a Boolean random matrix
  - `'img1'` : C = A
  - `'img2'` : C = B
- Parameter-dependent, where METHOD is of the following form:

  ```
  METHOD = struct('name',nameMETH,'param',paramMETH)
  ```

  where `nameMETH` can be

  - `'linear'` : `C = A*paramMETH + B*(1-paramMETH),`

    where `0 £ paramMETH ≤ 1`
  - `'UD_fusion'`: Up-down fusion, with paramMETH ≥ 0

```
   x = linspace(0,1,size(A,1));
   P = x.^paramMETH;
```

Then each row of C is computed with

```
C(i,:) = A(i,:)*(1-P(i)) + B(i,:)*P(i);
So C(1,:) = A(1,:) and C(end,:) = A(end,:)
```

- `'DU_fusion'`: Down-up fusion
- `'LR_fusion'`: Left-right fusion (columnwise fusion)
- `'RL_fusion'`: Right-left fusion (columnwise fusion)
- `'UserDEF'`: User-defined fusion, `paramMETH` is a string `'userFUNCTION'` containing a function name such that `C = userFUNCTION(A,B)`.

In addition, `[C,D] = wfusmat(A,B,METHOD)` returns the Boolean matrix `D` when defined, or an empty matrix otherwise.

## wkeep

Keep part of vector or matrix

### Syntax

```
Y = wkeep(X,L,OPT)
Y = wkeep(X,L,FIRST)
Y = wkeep(X,L)
Y = wkeep(X,L,'c')
Y = wkeep(X,S,[FIRSTR FIRSTC])
```

### Description

`wkeep` is a general utility.

For a vector, `Y = wkeep(X,L,OPT)` extracts the vector `Y` from the vector `X`. The length of `Y` is `L`.

If `OPT` is equal to `'c'`, `'l'`, or `'r'`, `Y` is the central, left, or right part of `X`.

`Y = wkeep(X,L,FIRST)` returns the vector `X(FIRST:FIRST+L-1)`.

`Y = wkeep(X,L)` is equivalent to `Y = wkeep(X,L,'c')`.

For a matrix, `Y = wkeep(X,S)` extracts the central part of the matrix `X`. The size of `Y` is `S`.

`Y = wkeep(X,S,[FIRSTR FIRSTC])` extracts the submatrix of matrix `X`, of size `S` and starting from `X(FIRSTR,FIRSTC)`.

### Examples

```
% For a vector.
x = 1:10;
y = wkeep(x,6,'c')
y =
    3     4     5     6     7     8

y = wkeep(x,6)
y =
    3     4     5     6     7     8

y = wkeep(x,7,'c')
y =
    2     3     4     5     6     7     8
y = wkeep(x,6,'l')
y =
    1     2     3     4     5     6

y = wkeep(x,6,'r')
y =
    5     6     7     8     9    10

% For a matrix.
x = magic(5)
x =
    17    24     1     8    15
    23     5     7    14    16
     4     6    13    20    22
    10    12    19    21     3
    11    18    25     2     9

y = wkeep(x,[3 2])
y =
    5     7
    6    13
   12    19
```

# wmaxlev

Maximum wavelet decomposition level

## Syntax

```
L = wmaxlev(S,'wname')
```

## Description

`wmaxlev` is a one- or two-dimensional wavelet or wavelet packets oriented function.

`wmaxlev` can help you avoid unreasonable maximum level values. `L = wmaxlev(S,'wname')` returns the maximum level decomposition of signal or image of size `S` using the wavelet named in the string `'wname'` (see `wfilters` for more information).

`wmaxlev` gives the maximum allowed level decomposition, but in general, a smaller value is taken.

Usual values are 5 for the one-dimensional case, and 3 for the two-dimensional case.

## Examples

```
% For a 1-D signal.
s = 2^10;
w = 'db1';

% Compute maximum level decomposition.
% The rule is the last level for which at least
% one coefficient is correct.
l = wmaxlev(s,w)

l =
    10

% Change wavelet.
w = 'db7';
```

```
% Compute maximum level decomposition.
l = wmaxlev(s,w)

l =
    6

% For a 2-D signal.
s = [2^9 2^7];
w = 'db1';

% Compute maximum level decomposition.
l = wmaxlev(s,w)

l =
    7

% which is the same as:
l = wmaxlev(min(s),w)

l =
    7

% Change wavelet.
w = 'db7';

% Compute maximum level decomposition.
l = wmaxlev(s,w)

l =
    3
```

## See Also

wavedec | wpdec | wavedec2 | wpdec2

# wmpalg

Matching pursuit

## Syntax

```
YFIT = wmpalg(MPALG,Y,MPDICT)
[YFIT,R] = wmpalg(...)
[YFIT,R,COEFF] = wmpalg(...)
[YFIT,R,COEFF,IOPT] = wmpalg(...)
[YFIT,R,COEFF,IOPT,QUAL] = wmpalg(...)
[YFIT,R,COEFF,IOPT,QUAL,X] = wmpalg(...)
[YFIT,R,COEFF,IOPT,QUAL,X] = wmpalg(...,Name,Value)
```

## Description

`YFIT = wmpalg(MPALG,Y,MPDICT)` returns an adaptive greedy approximation, YFIT, of the input signal, Y, in the dictionary, MPDICT. The adaptive greedy approximation uses the matching pursuit algorithm, MPALG. The dictionary, MPDICT, is typically an overcomplete set of vectors constructed using `wmpdictionary`.

`[YFIT,R] = wmpalg(...)` returns the residual, R, which is the difference vector between Y and YFIT at the termination of the matching pursuit.

`[YFIT,R,COEFF] = wmpalg(...)` returns the expansion coefficients, COEFF. The number of expansion coefficients depends on the number of iterations in the matching pursuit.

`[YFIT,R,COEFF,IOPT] = wmpalg(...)` returns the column indices of the retained atoms, IOPT. The length of IOPT equals the length of COEFF and is determined by the number of iterations in the matching pursuit.

`[YFIT,R,COEFF,IOPT,QUAL] = wmpalg(...)` returns the proportion of retained signal energy, QUAL, for each iteration of the matching pursuit. QUAL is the ratio of the $\ell^2$ squared norm of the expansion coefficient vector, COEFF, to the $\ell^2$ squared norm of the input signal, Y.

`[YFIT,R,COEFF,IOPT,QUAL,X] = wmpalg(...)` returns the normalized dictionary, X. X contains the unit vectors in the $\ell^2$ norm corresponding to the columns of MPDICT.

`[YFIT,R,COEFF,IOPT,QUAL,X] = wmpalg(...,Name,Value)` returns an adaptive greedy approximation with additional options specified by one or more Name,Value pair arguments.

## Input Arguments

### `MPALG`

Matching pursuit algorithm as a string. Valid entries are:

- `'BMP'` — Basic matching pursuit
- `'OMP'` — Orthogonal matching pursuit
- `'WMP'` — Weak orthogonal matching pursuit

See “Matching Pursuit Algorithms”.

**Default:** `'BMP'`

### `MPDICT`

Matching pursuit dictionary. MPDICT is a N-by-P matrix where N is equal to the length of the input signal, Y. You can construct MPDICT using `wmpdictionary`. In matching pursuit, MPDICT is commonly a frame, or overcomplete set of vectors. You may use the Name-Value pair `'lstcpt'` to specify a dictionary instead of using MPDICT. If you specify a value for `'lstcpt'`, `wmpalg` calls `wmpdictionary`.

### `Y`

Signal for matching pursuit. Y is 1-D, real-valued row or column vector. The row dimension of MPDICT must match the length of Y.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name,Value arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

**'itermax'**

Positive integer fixing the maximum number of iterations of the matching pursuit algorithm. If you do not specify a `'maxerr'` value, the number of expansion coefficients, COEFF, the number of dictionary vector indices, IOPT, and the length of the QUAL vector equal the value of `'itermax'`.

**Default:** 25

**'lstcpt'**

A cell array of cell arrays with valid subdictionaries. This name-value pair is only valid if you do not input a dictionary in MPDICT. Each cell array describes one subdictionary. Valid subdictionaries are:

- A valid Wavelet Toolbox orthogonal or biorthogonal wavelet family short name with the number of vanishing moments and an optional decomposition level and extension mode. For example, `{'sym4',5}` denotes the Daubechies least-asymmetric wavelet with 4 vanishing moments at level 5 and the default extension mode `'per'`. If you do not specify the optional number level and extension mode, the decomposition level defaults to 5 and the extension mode to `'per'`.
- A valid Wavelet Toolbox orthogonal or biorthogonal wavelet family short name preceded by `wp` with the number of vanishing moments and an optional decomposition level and extension mode. For example, `{'wpsym4',5}` denotes the Daubechies least-asymmetric wavelet packet with 4 vanishing moments at level 5. If you do not specify the optional number level and extension mode, the decomposition level defaults to 5 and the extension mode to `'per'`.
- `'dct'` Discrete cosine transform-II basis. The DCT-II orthonormal basis is:

$$\phi_k(n) = \begin{cases} \dfrac{1}{\sqrt{N}} & k = 0 \\ \sqrt{\dfrac{2}{N}}\cos\left(\frac{\pi}{N}(n+\frac{1}{2})k\right) & k = 1,2,\ldots,N-1 \end{cases}$$

- `'sin'` Sine subdictionary. The sine subdictionary is:

$$\phi_k(t) = \sin(2\pi kt) \quad k = 1,2,\ldots\left\lceil \frac{N}{2} \right\rceil \quad 0 \le t \le 1$$

- `'cos'` Cosine subdictionary. The cosine subdictionary is

$$\phi_k(t) = \cos(2\pi kt) \quad k = 1,2,\ldots\left\lceil \frac{N}{2} \right\rceil \quad 0 \le t \le 1$$

- `'poly'` Polynomial subdictionary. The polynomial subdictionary is:

$$p_n(t) = t^{n-1} \quad n = 1,2,\ldots 20 \qquad 0 \le t \le 1$$

- `'RnIdent'` The shifted Kronecker delta subdictionary. The shifted Kronecker delta subdictionary is:

$$\phi_k(n) = \delta(n-k) \quad k = 0,1,\ldots N$$

If you use the `'lstcpt'` name-value pair to generate your dictionary, you can use the additional `'addbeg'` and `'addend'` name-value pairs to append and addend dictionary atoms. See `wmpdictionary` for details.

**`'maxerr'`**

Cell array containing the name of the norm and the maximum relative error in the norm expressed as a percentage. Valid norms are `'L1'`, `'L2'`, and `'Linf'`. The relative error expressed as a percentage is

$$100\frac{\|R\|}{\|Y\|}$$

where $R$ is the residual at each iteration and $Y$ is the input signal. For example, `{'L1',10}` sets maximum acceptable ratio of the L1 norms of the residual to the input signal to 0.10.

If you specify `'maxerr'`, the matching pursuit terminates when the first of the following conditions is satisfied:

- The number of iterations reaches the minimum of the length of the input signal, Y, or 500:
  `min(length(Y),500)`
- The relative error falls below the percentage you specify with the `'maxerr'` name-value pair.

**'stepplot'**

Number of iterations between successive plots. `'stepplot'` requires a positive integer. This name-value pair is only valid when `'typeplot'` is 2 or 3 (`'movie'` or `'stepwise'`).

**'typeplot'**

Type of plot to produce during the progression of matching pursuit. Valid entries for `'typeplot'` are: `0` or `'none'`, `1` or `'one'`, `2` or `'movie'`, `3` or `'stepwise'`. When `'typeplot'` is `'movie'` or `'stepwise'`, the plot updates based on the value of `'stepplot'`.

**Default:** `0` or `'none'`

**'wmpcfs'**

Optimality factor for weak orthogonal matching pursuit. The optimality factor is a real number in the interval (0,1]. This name-value pair is only valid when MPALG is `'WMP'`.

**Default:** `0.6`

## Output Arguments

**YFIT**

Adaptive greedy approximation of the input signal, Y, in the dictionary

**R**

Residual after matching pursuit terminates

**COEFF**

Expansion coefficients in the dictionary. The selected dictionary atoms weighted by the expansion coefficients yield the approximated signal, YFIT.

**IOPT**

Column indices of the selected dictionary atoms. Using the column indices in IOPT with the expansion coefficients in COEFF, you can form the approximated signal, YFIT.

### QUAL

Proportion of retained signal energy for each iteration in the matching pursuit. QUAL is a vector with each element equal to

$$\frac{||\alpha_k||_2^2}{||Y||_2^2}$$

where $\alpha_k$ is the vector of expansion coefficients after the $k$-th iteration.

### X

The normalized matching pursuit dictionary. X is an N-by-P matrix where N is the length of the input signal, Y. The columns of X have unit norm.

## Examples

### Adaptive Approximation using Orthogonal Matching Pursuit

Approximate the `cuspamax` signal with the dictionary using orthogonal matching pursuit.

Use a dictionary consisting of `sym4` wavelet packets and the DCT-II basis.

```
load cuspamax;
mpdict = wmpdictionary(length(cuspamax),'LstCpt',{{'wpsym4',2},'dct'});
yfit = wmpalg('OMP',cuspamax,mpdict);
plot(cuspamax,'k'); hold on;
plot(yfit,'linewidth',2); legend('Original Signal','Matching Pursuit');
```

### Return Residual, Expansion Coefficients, Selected Atoms, and Approximation Quality

Obtain the expansion coefficients in the dictionary, the column indices of the selected dictionary atoms, and the proportion of retained signal energy.

Create a dictionary consisting of `sym4` wavelet packets and the DCT-II basis. Approximate the `cuspamax` signal with the dictionary using orthogonal matching pursuit.

```
load cuspamax;
mpdict = wmpdictionary(length(cuspamax),'LstCpt',{{'wpsym4',2},'dct'});
```

```
[yfit,r,coeff,iopt,qual] = wmpalg('OMP',cuspamax,mpdict);
```

### Specify the Maximum Number of Iterations

This example shows how to set the maximum number of iterations of the orthogonal matching pursuit to 50.

```
load cuspamax;
lstcpt = {{'wpsym4',1},{'wpsym4',2},'dct'};
mpdict = wmpdictionary(length(cuspamax),'LstCpt',lstcpt);
[yfit,r,coeff,iopt,qual] = wmpalg('OMP',cuspamax,mpdict,'itermax',50);
```

### Stepwise Plot of Weak Orthogonal Matching Pursuit

This example shows how to allow for a suboptimal choice in the update of the orthogonal matching pursuit.

Relax the requirement to be 0.8 times the optimal assignment. Plot the results stepwise and update the plot every 5 iterations.

```
load cuspamax;
lstcpt = {{'wpsym4',1},{'wpsym4',2},'dct'};
mpdict = wmpdictionary(length(cuspamax),'LstCpt',lstcpt);
[yfit,r,coeff,iopt,qual] = wmpalg('WMP',cuspamax,mpdict,'wmpcfs',0.8,...
    'typeplot','stepwise','stepplot',5);
```

### Matching Pursuit of Electricity Consumption Data

Obtain a matching pursuit of electricity consumption measured every minute over a 24-hour period.

Load and plot data. The data shows electricity consumption sampled every minute over a 24-hour period. Because the data is centered, the actual usage values are not interpretable.

```
load elec35_nor;
y = signals(32,:);
plot(y); xlabel('Minutes'); ylabel('Usage');
set(gca,'xlim',[1 1440]);
```

Construct a dictionary for matching pursuit consisting of the Daubechies' extremal–phase wavelet with 2 vanishing moments at level 2, the Daubechies' least-asymmetric wavelet with 4 vanishing moments at levels 1 and 4, the discrete cosine transform-II basis, and the sine basis.

```
dictionary = {{'db4',2},'dct','sin',{'sym4',1},{'sym4',4}};
[mpdict,nbvect] = wmpdictionary(length(y),'lstcpt',dictionary);
```

Implement orthogonal matching pursuit to obtain a signal approximation in the dictionary. Use 35 iterations. Plot the result.

```
[yfit,r,coef,iopt,qual] = wmpalg('OMP',y,mpdict,'itermax',35);
plot(y); hold on;
plot(yfit,'r'); xlabel('Minutes'); ylabel('Usage');
legend('Original Signal','OMP','Location','NorthEast');
set(gca,'xlim',[1 1440]);
```

Using the expansion coefficients in `coef` and the atom indices in `iopt`, construct the signal approximation, `yhat`, directly from the dictionary. Compare `yhat` with `yfit` returned by `wmpalg`.

```
[~,I] = sort(iopt);
X = mpdict(:,iopt(I));
yhat = X*coef(I);
max(abs(yfit-yhat))
```

- "Matching Pursuit — Command Line"
- "Matching Pursuit — Interactive Analysis"

## More About

- "Sparse Representation in Redundant Dictionaries"
- "Matching Pursuit Algorithms"

## References

[1] Cai, T.T. and Wang,L. "Orthogonal Matching Pursuit for Sparse Signal Recovery with Noise". *IEEE® Transactions on Information Theory*, vol. 57, 7, 4680–4688, 2011.

[2] Donoho, D., Elad, M., and Temlyakov, V. "Stable Recovery of Sparse Overcomplete Representations in the Presence of Noise". *IEEE Transactions on Information Theory*. Vol. 52, 1, 6–18, 2004.

[3] Mallat, S. and Zhang, Z. "Matching Pursuits with Time-Frequency Dictionaries". *IEEE Transactions on Signal Processing*, vol. 41, 12, 3397–3415, 1993

[4] bTropp, J.A. “Greed is good: Algorithmic results for sparse approximation”. *IEEE Transactions on Information Theory*, 50, pp. 2231–2242, 2004.

## See Also

`wavemenu` | `wmpdictionary`

# wmpdictionary

Dictionary for matching pursuit

## Syntax

```
MPDICT = wmpdictionary(N)
[MPDICT,NBVECT] = wmpdictionary(N)
[MPDICT,NBVECT]= wmpdictionary(N,Name,Value)
[MPDICT,NBVECT,LST] = wmpdictionary(N,Name,Value)
[MPDICT,NBVECT,LST,LONGS] = wmpdictionary(N,Name,Value)
```

## Description

`MPDICT = wmpdictionary(N)` returns the N-by-P dictionary, MPDICT, for the default subdictionaries `{{'sym4',5},{'wpsym4',5},'dct','sin'}`. The column dimension of MPDICT depends on N.

`[MPDICT,NBVECT] = wmpdictionary(N)` returns the row vector, NBVECT, which contains the number of vectors in each subdictionary. The order of the elements in NBVECT corresponds to the order of the subdictionaries and any prepended or appended subdictionaries. The sum of the elements in NBVECT is the column dimension of MPDICT.

`[MPDICT,NBVECT]= wmpdictionary(N,Name,Value)` returns the dictionary, MPDICT, using additional options specified by one or more Name,Value pair arguments.

`[MPDICT,NBVECT,LST] = wmpdictionary(N,Name,Value)` returns the cell array, LST, with descriptions of the subdictionaries.

`[MPDICT,NBVECT,LST,LONGS] = wmpdictionary(N,Name,Value)` returns the cell array, LONGS, containing the number of vectors in each subdictionary. LONGS is only useful for wavelet subdictionaries. In wavelet subdictionaries, the corresponding element in LONGS gives the number of scaling functions at the coarsest level and wavelet functions by level. See “Visualize Haar Wavelet Dictionary” on page 1-557 for an example using LONGS.

## Input Arguments

**`N`**

A positive integer equal to the length of your input signal. The dictionary atoms are constructed to have N elements. N equals the row dimension of the dictionary, MPDICT.

### Name-Value Pair Arguments

Specify optional comma-separated pairs of Name,Value arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

**`'addbeg'`**

Prepended subdictionary. The prepended subdictionary is an N-by-M matrix where N is the length of the input signal. `wmpdictionary` does not check that the M column vectors of the prepended dictionary form a basis. If you do not specify a value for lstcpt, the subdictionary is prepended to the default dictionary. The column vectors in the prepended subdictionary do not have to be unit-norm.

**`'addend'`**

Appended subdictionary. The appended subdictionary is a N-by-M matrix where N is the length of the input signal. `wmpdictionary` does not check that the M column vectors of the prepended dictionary form a basis. If you do not specify a value for lstcpt, the subdictionary is appended to the default dictionary. The column vectors in the appended subdictionary do not have to be unit-norm.

**`'lstcpt'`**

A cell array of cell arrays with valid subdictionaries. Each cell array describes one subdictionary. Valid subdictionaries are:

- A valid Wavelet Toolbox orthogonal or biorthogonal wavelet family short name with the number of vanishing moments and an optional decomposition level and extension mode. For example, `{'sym4',5}` denotes the Daubechies least-asymmetric wavelet with 4 vanishing moments at level 5 and the default extension mode `'per'`. If you do not specify the optional level and extension mode, the decomposition level defaults to 5 and the extension mode to `'per'`.

- A valid Wavelet Toolbox orthogonal or biorthogonal wavelet family short name preceded by `wp` with the number of vanishing moments and an optional decomposition level and extension mode. For example, `{'wpsym4',5}` denotes the Daubechies least-asymmetric wavelet packet with 4 vanishing moments at level 5. If you do not specify the optional level and extension mode, the decomposition level defaults to 5 and the extension mode to `'per'`.
- `'dct'` Discrete cosine transform-II basis. The DCT-II orthonormal basis is:

$$\phi_k(n) = \begin{cases} \dfrac{1}{\sqrt{N}} & k = 0 \\ \sqrt{\dfrac{2}{N}}\cos\left(\frac{\pi}{N}\left(n+\frac{1}{2}\right)k\right) & k = 1,2,\ldots,N-1 \end{cases}$$

- `'sin'` Sine subdictionary. The sine subdictionary is

$$\phi_k(t) = \sin(2\pi kt) \quad k = 1,2,\ldots\left\lceil \frac{N}{2} \right\rceil \quad 0 \le t \le 1$$

  where $t$ is a linearly-spaced $N$-point vector.
- `'cos'` Cosine subdictionary. The cosine subdictionary is

$$\phi_k(t) = \cos(2\pi kt) \quad k = 1,2,\ldots\left\lceil \frac{N}{2} \right\rceil \quad 0 \le t \le 1$$

  where $t$ is a linearly-spaced $N$-point vector.
- `'poly'` Polynomial subdictionary. The polynomial subdictionary is:

$$p_n(t) = t^{n-1} \quad n = 1,2,\ldots 20 \qquad 0 \le t \le 1$$

  where $t$ is a linearly-spaced $N$-point vector.
- `'RnIdent'` The shifted Kronecker delta subdictionary. The shifted Kronecker delta subdictionary is:

$$\phi_k(n) = \delta(n-k) \quad k = 0,1,\ldots N$$

**Default:** `{{'sym4',5},{'wpsym4',5},'dct','sin'}`

## Output Arguments

### `MPDICT`

Matching pursuit dictionary. MPDICT is an N-by-P matrix with the row dimension, N, equal to the length of the input signal. The column dimension of the matrix depends on the size of the concatenated subdictionaries.

### `NBVECT`

Number of vectors in subdictionaries. NBVECT is a row vector containing the number of elements in each subdictionary. The order of the elements in NBVECT corresponds to the order of the subdictionaries and any prepended or appended subdictionaries.

### `LST`

Cell array describing the dictionary. LST is a 1-by-N cell array where N is the number of subdictionaries. Each element of the cell array contains a description of a subdictionary. If you specify a prepended or appended subdictionary, the first element of LST is `'AddBeg'` or `'AddEnd'`. If you specify a level for the wavelet or wavelet packet, the corresponding element of LST is a 1-by-2 cell array containing the wavelet or wavelet packet name in the first element and the level in the second element.

### `LONGS`

Cell array containing the number of elements for each subdictionary. LONGS is useful only for wavelet subdictionaries. If you specify a wavelet subdictionary, the corresponding element of LONGS provides the number of scaling functions at the coarsest level and the number of wavelets at each level. See “Visualize Haar Wavelet Dictionary” on page 1-557 for an example using LONGS.

## Examples

### Default Dictionary

Create the default dictionary to represent a signal of length 100.

```
mpdict = wmpdictionary(100);
```

### Discrete Cosine Transform and Kronecker Delta Dictionary

Create a DCT and shifted Kronecker delta dictionary to represent a signal of length 100.

```
mpdict = wmpdictionary(100,'lstcpt',{'dct','RnIdent'});
```

### Haar Wavelet Packets and Discrete Cosine Transform Dictionary

Create a Haar wavelet packet (level 2) and DCT dictionary. Return the number of atoms in each subdictionary.

```
[mpdict,nbvect] = wmpdictionary(100,'lstcpt',{{'wphaar',2},'dct'});
```

### Visualize Haar Wavelet Dictionary

Use the output argument, LONGS, to visualize a dictionary.

Create a Haar wavelet dictionary consisting of level-2 scaling functions and level-1 and level-2 wavelet functions.

```
[mpdict,~,~,longs] = wmpdictionary(100,'lstcpt',{{'haar',2}});
for nn = 1:size(mpdict,2)

    if (nn <= longs{1}(1))
        plot(mpdict(:,nn),'k','linewidth',2); grid on;
    xlabel('Translation');
        title('Haar Scaling Function - Level 2');
    elseif (nn>longs{1}(1) & nn<= longs{1}(1)+longs{1}(2))
         plot(mpdict(:,nn),'r','linewidth',2); grid on;
         xlabel('Translation');
        title('Haar Wavelet - Level 2');
    else
        title('Haar Wavelet - Level 1');
        plot(mpdict(:,nn),'b','linewidth',2); grid on;
        title('Haar Wavelet - Level 1');
        xlabel('Translation');
    end
        pause(0.2);
end
```

- “Matching Pursuit — Command Line”
- “Matching Pursuit — Interactive Analysis”

## More About

### Matching Pursuit

Matching pursuit refers to a number of greedy or weak-greedy algorithms for computing an adaptive nonlinear expansion of a signal in a *dictionary*. In the majority of matching pursuit applications, a dictionary is an overcomplete set of vectors. The elements of the dictionary are referred to as *atoms* and are typically constructed to have certain time/frequency or time/scale properties. Matching pursuit takes the NP-hard problem of finding the best nonlinear expansion in a dictionary and implements it in an energy-perserving formulation that guarantees convergence. See “Matching Pursuit Algorithms” for more details.

- “Sparse Representation in Redundant Dictionaries”
- “Matching Pursuit Algorithms”

## References

[1] Cai, T.T. and L. Wang “Orthogonal Matching Pursuit for Sparse Signal Recovery with Noise”. *IEEE Transactions on Information Theory*, vol. 57, 7, 4680–4688, 2011.

[2] Donoho, D., M. Elad, and V. Temlyakov “Stable Recovery of Sparse Overcomplete Representations in the Presence of Noise”. *IEEE Transactions on Information Theory*, 52,1, 6–18, 2004.

[3] Mallat, S. and Z. Zhang “Matching Pursuits with Time-Frequency Dictionaries”. *IEEE Transactions on Signal Processing*, vol. 41, 12, 3397–3415, 1993

[4] Tropp, J.A. “Greed is good: Algorithmic results for sparse approximation”. *IEEE Transactions on Information Theory*, 50, pp. 2231–2242, 2004.

## See Also

`wavemenu` | `wmpalg`

# wmspca

Multiscale Principal Component Analysis

## Syntax

```
[X_SIM,QUAL,NPC,DEC_SIM,PCA_Params] = wmspca(X,LEVEL,WNAME,NPC)
[...] = wmspca(X,LEVEL,WNAME,'mode',EXTMODE,NPC)
[...] = wmspca(DEC,NPC)
[...] = wmspca(X,LEVEL,WNAME,'mode',EXTMODE,NPC)
```

## Description

`[X_SIM,QUAL,NPC,DEC_SIM,PCA_Params] = wmspca(X,LEVEL,WNAME,NPC)` or `[...] = wmspca(X,LEVEL,WNAME,'mode',EXTMODE,NPC)` returns a simplified version `X_SIM` of the input matrix `X` obtained from the wavelet-based multiscale principal component analysis (PCA).

The input matrix `X` contains `P` signals of length `N` stored columnwise (`N` > `P`).

### Wavelet Decomposition Parameters

The wavelet decomposition is performed using the decomposition level `LEVEL` and the wavelet `WNAME`.

`EXTMODE` is the extended mode for the DWT (See `dwtmode`).

If a decomposition `DEC` obtained using `mdwtdec` is available, you can use

`[...] = wmspca(DEC,NPC)` instead of

`[...] = wmspca(X,LEVEL,WNAME,'mode',EXTMODE,NPC)`.

### Principal Components Parameter: NPC

If `NPC` is a vector, then it must be of length `LEVEL+2`. It contains the number of retained principal components for each PCA performed:

- `NPC(d)` is the number of retained noncentered principal components for details at level `d`, for 1 <= `d` <= `LEVEL`.
- `NPC(LEVEL+1)` is the number of retained non-centered principal components for approximations at level LEVEL.
- `NPC(LEVEL+2)` is the number of retained principal components for final PCA after wavelet reconstruction.

`NPC` must be such that 0 <= `NPC(d)` <= `P` for 1 <= `d` <= `LEVEL`+2.

If `NPC = 'kais'` (respectively, `'heur'`), then the number of retained principal components is selected automatically using Kaiser's rule (or the heuristic rule).

- Kaiser's rule keeps the components associated with eigenvalues greater the mean of all eigenvalues.
- The heuristic rule keeps the components associated with eigenvalues greater than 0.05 times the sum of all eigenvalues.

If `NPC = 'nodet'`, then the details are “killed” and all the approximations are retained.

## Output Parameters

`X_SIM` is a simplified version of the matrix `X`.

`QUAL` is a vector of length `P` containing the quality of column reconstructions given by the relative mean square errors in percent.

`NPC` is the vector of selected numbers of retained principal components.

`DEC_SIM` is the wavelet decomposition of `X_SIM`

`PCA_Params` is a structure array of length `LEVEL+2` such that:

- `PCA_Params(d).pc` is a `P`-by-`P` matrix of principal components.

  The columns are stored in descending order of the variances.
- `PCA_Params(d).variances` is the principal component variances vector.
- `PCA_Params(d).npc = NPC`

## Examples

### Wavelet Principal Component Analysis of Noisy Multivariate Signal

Use wavelet multiscale principal component analysis to denoise a multivariate signal.

Load the dataset consisting of 4 signals of length 1024. Plot the original signals and the signals with additive noise.

```
load ex4mwden;
kp = 0;
for i = 1:4
    subplot(4,2,kp+1), plot(x_orig(:,i)); axis tight;
    title(['Original signal ',num2str(i)])
    subplot(4,2,kp+2), plot(x(:,i)); axis tight;
    title(['Noisy signal ',num2str(i)])
    kp = kp + 2;
end
```

Perform the first multiscale wavelet PCA using the Daubechies' least-asymmetric wavelet with 4 vanishing moments, `sym4`. Obtain the multiresolution decomposition down to level 5. Use the heuristic rule to decide how many principal components to retain.

```
level = 5;
wname = 'sym4';
npc = 'heur';
[x_sim, qual, npc] = wmspca(x,level,wname,npc);
```

Plot the result and examine the quality of the approximation.

```
qual
kp = 0;
for i = 1:4
    subplot(4,2,kp+1), plot(x(:,i)); axis tight;
    title(['Noisy signal ',num2str(i)])
    subplot(4,2,kp+2), plot(x_sim(:,i)); axis tight;
    title(['First PCA ',num2str(i)])
    kp = kp + 2;
end
```

The quality results are all close to 100%. The `npc` vector gives the number of principal components retained at each level.

Suppress the noise by removing the principal components at levels 1–3. Perform the multiscale PCA again.

```
npc(1:3) = zeros(1,3);
[x_sim, qual, npc] = wmspca(x,level,wname,npc);
```

Plot the result.

```
kp = 0;
for i = 1:4
    subplot(4,2,kp+1), plot(x(:,i)); axis tight;
    title(['Noisy signal ',num2str(i)])
    subplot(4,2,kp+2), plot(x_sim(:,i)); axis tight;
    title(['Second PCA ',num2str(i)])
    kp = kp + 2;
end
```

## More About

### Algorithms

The multiscale principal components generalizes the usual PCA of a multivariate signal seen as a matrix by performing simultaneously a PCA on the matrices of details of different levels. In addition, a PCA is performed also on the coarser approximation coefficients matrix in the wavelet domain as well as on the final reconstructed matrix. By selecting conveniently the numbers of retained principal components, interesting simplified signals can be reconstructed.

## References

Aminghafari, M.; Cheze, N.; Poggi, J-M. (2006), “Multivariate de-noising using wavelets and principal component analysis,” *Computational Statistics & Data Analysis*, 50, pp. 2381–2398.

Bakshi, B. (1998), “Multiscale PCA with application to MSPC monitoring,” *AIChE J.*, 44, pp. 1596–1610.

## See Also

`wmulden`

# wmulden

Wavelet multivariate de-noising

## Syntax

```
[X_DEN,NPC,NESTCOV,DEC_DEN,PCA_Params,DEN_Params] = ...
wmulden(X,LEVEL,WNAME,NPC_APP,NPC_FIN,TPTR,SORH)
[...] = wmulden(X,LEVEL,WNAME,'mode',EXTMODE,NPC_APP,...)
[...] = wmulden(DEC,NPC_APP)
[...] = wmulden(X,LEVEL,WNAME,'mode',EXTMODE,NPC_APP)
[DEC,PCA_Params] = wmulden('estimate',DEC,NPC_APP,NPC_FIN)
[X_DEN,NPC,DEC_DEN,PCA_Params] = wmulden('execute',DEC,PC_Params)
```

## Description

`[X_DEN,NPC,NESTCOV,DEC_DEN,PCA_Params,DEN_Params] = ...`
`wmulden(X,LEVEL,WNAME,NPC_APP,NPC_FIN,TPTR,SORH)` or
`[...] = wmulden(X,LEVEL,WNAME,'mode',EXTMODE,NPC_APP,...)` returns a de-noised version `X_DEN` of the input matrix `X`. The strategy combines univariate wavelet de-noising in the basis where the estimated noise covariance matrix is diagonal with noncentered Principal Component Analysis (PCA) on approximations in the wavelet domain or with final PCA.

The input matrix `X` contains `P` signals of length N stored columnwise where `N` > `P`.

### Wavelet Decomposition Parameters

The wavelet decomposition is performed using the decomposition level `LEVEL` and the wavelet `WNAME`.

`EXTMODE` is the extended mode for the DWT (See `dwtmode`).

If a decomposition `DEC` obtained using `mdwtdec` is available, you can use

`[...] = wmulden(DEC,NPC_APP)` instead of

`[...] = wmulden(X,LEVEL,WNAME,'mode',EXTMODE,NPC_APP).`

## Principal Components Parameters: NPC_APP and NPC_FIN

The input selection methods `NPC_APP` and `NPC_FIN` define the way to select principal components for approximations at level `LEVEL` in the wavelet domain and for final PCA after wavelet reconstruction, respectively.

If `NPC_APP` (or `NPC_FIN`) is an integer, it contains the number of retained principal components for approximations at level `LEVEL` (or for final PCA after wavelet reconstruction).

`NPC_XXX` must be such that 0 <= `NPC_XXX` <= `P`

`NPC_APP` or `NPC_FIN = 'kais'` or `'heur'` selects the number of retained principal components using Kaiser's rule or the heuristic rule automatically.

- Kaiser's rule keeps the components associated with eigenvalues greater than the mean of all eigenvalues.
- The heuristic rule keeps the components associated with eigenvalues greater than 0.05 times the sum of all eigenvalues.

`NPC_APP` or `NPC_FIN` = `'none'` is equivalent to `NPC_APP` or `NPC_FIN` = `P`.

## De-noising Parameters: TPTR and SORH

The default values for the de-noising parameters TPTR and SORH are:

`TPTR = 'sqtwolog'` and `SORH = 's'`

- Valid values for `TPTR` are

  ```
  'rigsure', 'heursure', 'sqtwolog', 'minimaxi',
  'penalhi', 'penalme', 'penallo'
  ```

- Valid values for SORH are:

  ```
  's' (soft) or 'h' (hard)
  ```

For additional information, see `wden` and `wbmpen`.

## Output Parameters

X_DEN is a de-noised version of the input matrix X.

NPC is the vector of selected numbers of retained principal components.

NESTCOV is the estimated noise covariance matrix obtained using the minimum covariance determinant (MCD) estimator.

DEC_DEN is the wavelet decomposition of X_DEN.

PCA_Params is a structure such that:

```
PCA_Params.NEST = {pc_NEST,var_NEST,NESTCOV}
PCA_Params.APP  = {pc_APP,var_APP,npc_APP}
PCA_Params.FIN  = {pc_FIN,var_FIN,npc_FIN}
```

where:

- pc_XXX is a P-by-P matrix of principal components.

  The columns are stored in descending order of the variances.
- var_XXX is the principal component variances vector.
- NESTCOV is the covariance matrix estimate for detail at level 1.

DEN_Params is a structure such that:

- DEN_Params.thrVAL is a vector of length LEVEL which contains the threshold values for each level.
- DEN_Params.thrMETH is a string containing the name of the de-noising method (TPTR).
- DEN_Params.thrTYPE is a character variable containing the type of the thresholding (SORH).

## Special Cases

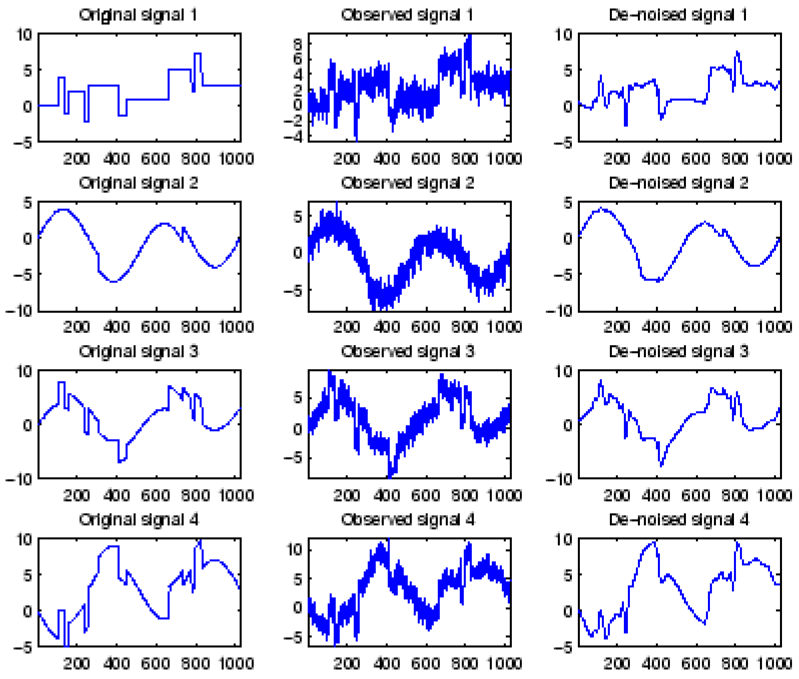
[DEC,PCA_Params] = wmulden('estimate',DEC,NPC_APP,NPC_FIN) returns the wavelet decomposition DEC and the Principal Components Estimates PCA_Params.

[X_DEN,NPC,DEC_DEN,PCA_Params] = wmulden('execute',DEC,PC_Params) uses the principal components estimates PCA_Params previously computed.

The input value DEC can be replaced by X, LEVEL, and WNAME.

## Examples

```
%  Load a multivariate signal x together with
%  the original signals (x_orig) and true noise
%  covariance matrix (covar).

load ex4mwden

%  Set the de-noising method parameters.
level = 5;
wname = 'sym4';
tptr  = 'sqtwolog';
sorh  = 's';

% Set the PCA parameters to select the number of
% retained principal components automatically by
% Kaiser's rule.

npc_app = 'kais';
npc_fin = 'kais';

% Perform multivariate de-noising.
[x_den, npc, nestco] = wmulden(x, level, wname, npc_app, ...
                                   npc_fin, tptr, sorh);

% Display the original and de-noised signals.
kp = 0;
for i = 1:4
    subplot(4,3,kp+1), plot(x_orig(:,i));
    title(['Original signal ',num2str(i)])
    subplot(4,3,kp+2), plot(x(:,i));
    title(['Observed signal ',num2str(i)])
    subplot(4,3,kp+3), plot(x_den(:,i));
    title(['De-noised signal ',num2str(i)])
    kp = kp + 3;
end
```



```
% The results are good: the first function, which is
% irregular, is correctly recovered while the second
% function, more regular, is well de-noised.

% The second output argument gives the numbers
% of retained principal components for PCA for
% approximations and for final PCA.

npc

npc =

     2     2

% The third output argument contains the estimated
% noise covariance matrix using the MCD based
% on the matrix of finest details.

nestco
```

```
nestco =

    1.0784    0.8333    0.6878    0.8141
    0.8333    1.0025    0.5275    0.6814
    0.6878    0.5275    1.0501    0.7734
    0.8141    0.6814    0.7734    1.0967

% The estimation is satisfactory since the values are close
% to the true values given by covar.

covar

covar =

    1.0000    0.8000    0.6000    0.7000
    0.8000    1.0000    0.5000    0.6000
    0.6000    0.5000    1.0000    0.7000
    0.7000    0.6000    0.7000    1.0000
```

## More About

### Algorithms

The multivariate de-noising procedure is a generalization of the one-dimensional strategy. It combines univariate wavelet de-noising in the basis where the estimated noise covariance matrix is diagonal and non-centered Principal Component Analysis (PCA) on approximations in the wavelet domain or with final PCA.

The robust estimate of the noise covariance matrix given by the minimum covariance determinant estimator based on the matrix of finest details.

## References

Aminghafari, M.; Cheze, N.; Poggi, J-M. (2006), “Multivariate de-noising using wavelets and principal component analysis,” *Computational Statistics & Data Analysis*, 50, pp. 2381–2398.

Rousseeuw, P.; Van Driessen, K. (1999), “A fast algorithm for the minimum covariance determinant estimator,” *Technometrics,* 41, pp. 212–223.

## See Also

`wmspca`

## wnoise

Noisy wavelet test data

### Syntax

```
X = wnoise(FUN,N)
[X,XN] = wnoise(FUN,N,SQRT_SNR)
[X,XN] = wnoise(FUN,N,SQRT_SNR,INIT)
```

### Description

`X = wnoise(FUN,N)` returns values of the test signal given by `FUN`, on a $2^N$ grid of [0,1].

`[X,XN] = wnoise(FUN,N,SQRT_SNR)` returns a test vector `X` as above, rescaled such that `std(X) = SQRT_SNR`. The returned vector `XN` contains the same test vector corrupted by additive Gaussian white noise $N(0,1)$. Then, `XN` has a signal-to-noise ratio of `SNR = (SQRT_SNR)`$^2$.

`[X,XN] = wnoise(FUN,N,SQRT_SNR,INIT)` returns previous vectors `X` and `XN`, but the generator seed is set to `INIT` value.

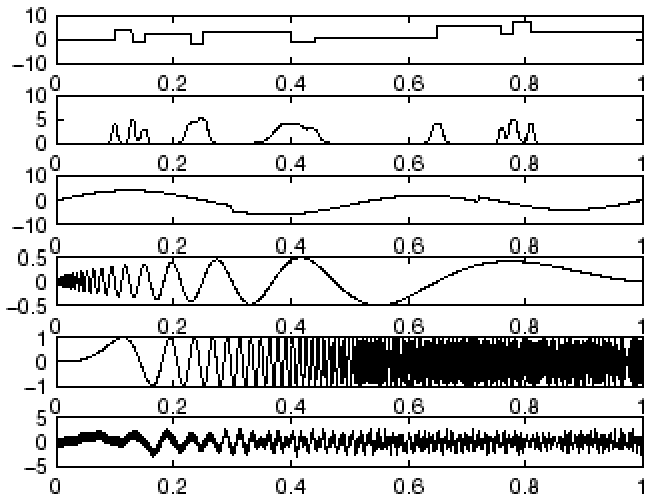
The six functions below are due to Donoho and Johnstone (See “References”).

| | |
|---|---|
| `FUN = 1` or | `'blocks'` |
| `FUN = 2` or | `'bumps'` |
| `FUN = 3` or | `'heavy sine'` |
| `FUN = 4` or | `'doppler'` |
| `FUN = 5` or | `'quadchirp'` |
| `FUN = 6` or | `'mishmash'` |

### Examples

```
% Generate 2^10 samples of 'Heavy sine' (item 3).
x = wnoise(3,10);
```

```
% Generate 2^10 samples of 'Doppler' (item 4) and of
% noisy 'Doppler' with a square root of signal-to-noise
% ratio equal to 7.
[x,noisyx] = wnoise(4,10,7);

% To introduce your own rand seed, a fourth
% argument is allowed:
init = 2055415866;
[x,noisyx] = wnoise(4,10,7,init);

% Plot all the test functions.
ind = linspace(0,1,2^10);
for i = 1:6
    x = wnoise(i,10);
    subplot(6,1,i), plot(ind,x)
end

% Editing some graphical properties,
% the following figure is generated.
```



## References

Donoho, D.L.; I.M. Johnstone (1994), “Ideal spatial adaptation by wavelet shrinkage,” *Biometrika*, vol. 81, pp. 425–455.

Donoho, D.L.; I.M. Johnstone (1995), "Adapting to unknown smoothness via wavelet shrinkage via wavelet shrinkage," *JASA*, vol. 90, 432, pp. 1200–1224.

## See Also

`wden`

# wnoisest

Estimate noise of 1-D wavelet coefficients

## Syntax

```
STDC = wnoisest(C,L,S)
STDC = wnoisest(C)
STDC = wnoisest(C)
```

## Description

`STDC = wnoisest(C,L,S)` returns estimates of the detail coefficients' standard deviation for levels contained in the input vector `S`. `[C,L]` is the input wavelet decomposition structure (see `wavedec` for more information).

If `C` is a one dimensional cell array, `STDC = wnoisest(C)` returns a vector such that `STDC(k)` is an estimate of the standard deviation of `C{k}`.

If `C` is a numeric array, `STDC = wnoisest(C)` returns a vector such that `STDC(k)` is an estimate of the standard deviation of `C(k,:)`.

The estimator used is Median Absolute Deviation / 0.6745, well suited for zero mean Gaussian white noise in the de-noising one-dimensional model (see `thselect` for more information).

## Examples

### Estimate Noise Standard Deviation in The Presence of Outliers

Estimate of the noise standard deviation in an N(0,1) white Gaussian noise vector with outliers.

Create an N(0,1) noise vector with 10 randomly-placed outliers.

```
rng default;
x = randn(1000,1);
```

```
P = randperm(length(x));
indices = P(1:10);
x(indices(1:5)) = 10;
x(indices(6:end)) = -10;
```

Obtain the discrete wavelet transform down to level 2 using the Daubechies' extremal phase wavelet with 3 vanishing moments.

```
[c,l] = wavedec(x,2,'db3');
stdc = wnoisest(c,l,1:2)
```

In spite of the outliers, `wnoisest` provides a robust estimate of the standard deviation.

## References

Donoho, D.L.; I.M. Johnstone (1994), "Ideal spatial adaptation by wavelet shrinkage," *Biometrika*, vol 81, pp. 425–455.

Donoho, D.L.; I.M. Johnstone (1995), "Adapting to unknown smoothness via wavelet shrinkage via wavelet shrinkage," *JASA*, vol 90, 432, pp. 1200–1224.

### See Also

`thselect` | `wden` | `wavedec`

# wp2wtree

Extract wavelet tree from wavelet packet tree

## Syntax

```
T = wp2wtree(T)
```

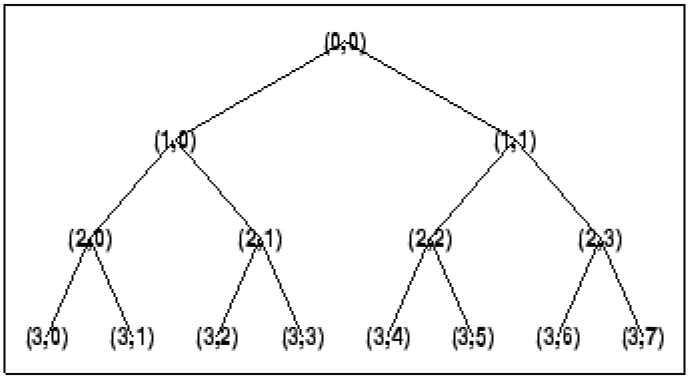
## Description

`wp2wtree` is a one- or two-dimensional wavelet packet analysis function.

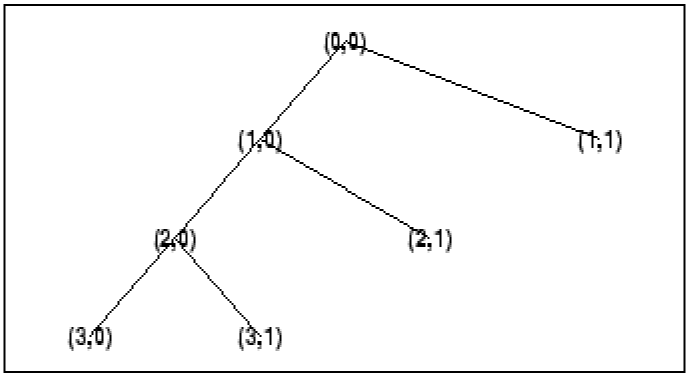
`T = wp2wtree(`*`T`*`)` computes the modified wavelet packet tree *T* corresponding to the wavelet decomposition tree.

## Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load signal.
    load noisdopp; x = noisdopp;

% Decompose x at depth 3 with db1 wavelet packets
% using shannon entropy.
wpt = wpdec(x,3,'db1');

% Plot wavelet packet tree wpt.
plot(wpt)
```



```
% Compute wavelet tree.
wt = wp2wtree(wpt);

% Plot wavelet tree wt.
plot(wt)
```



## See Also

wpdec | wpdec2

# wpbmpen

Penalized threshold for wavelet packet de-noising

## Syntax

```
THR = wpbmpen(T,SIGMA,ALPHA)
wpbmpen(T,SIGMA,ALPHA,ARG)
```

## Description

`THR = wpbmpen(T,SIGMA,ALPHA)` returns a global threshold `THR` for de-noising. `THR` is obtained by a wavelet packet coefficients selection rule using a penalization method provided by Birge-Massart.

`T` is a wavelet packet tree corresponding to the wavelet packet decomposition of the signal or image to be de-noised.

`SIGMA` is the standard deviation of the zero mean Gaussian white noise in the de-noising model (see `wnoisest` for more information).

`ALPHA` is a tuning parameter for the penalty term. It must be a real number greater than 1. The sparsity of the wavelet packet representation of the de-noised signal or image grows with `ALPHA`. Typically `ALPHA` = 2.

`THR` minimizes the penalized criterion given by

let $t^*$ be the minimizer of

```
crit(t) = -sum(c(k)^2,k≤t) + 2*SIGMA^2*t*(ALPHA + log(n/t))
```

where `c(k)` are the wavelet packet coefficients sorted in decreasing order of their absolute value and `n` is the number of coefficients, then THR=|c($t^*$)|.

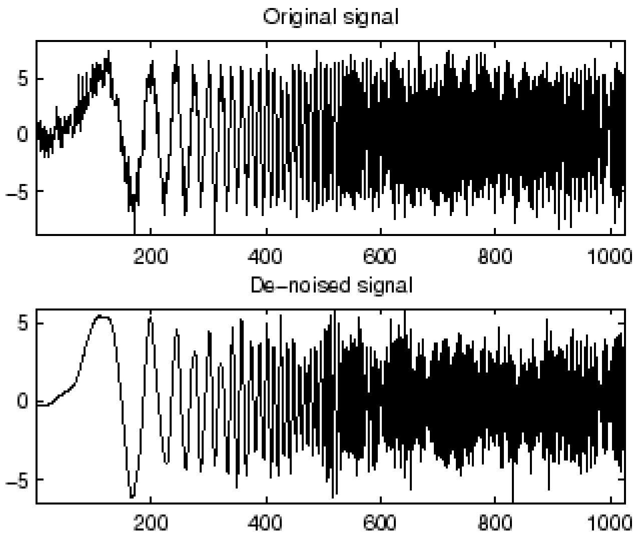
`wpbmpen(T,SIGMA,ALPHA,ARG)` computes the global threshold and, in addition, plots three curves:

- `2*SIGMA^2*t*(ALPHA + log(n/t))`

- `sum(c(k)^2,k£t)`
- `crit(t)`

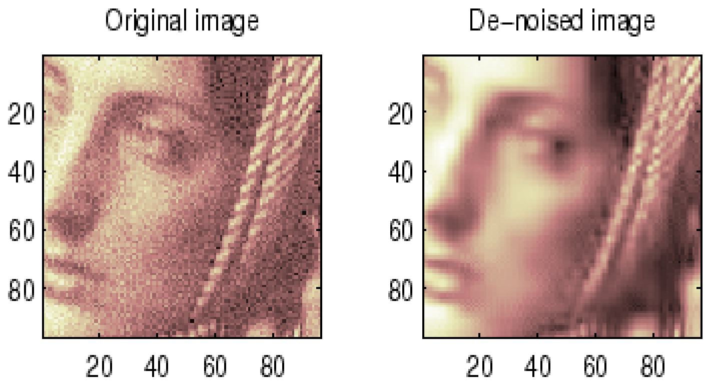
## Examples

```
% Example 1: Signal de-noising.
% Load noisy chirp signal.
load noischir; x = noischir;

% Perform a wavelet packet decomposition of the signal
% at level 5 using sym6.
wname = 'sym6'; lev = 5;
tree = wpdec(x,lev,wname);

% Estimate the noise standard deviation from the
% detail coefficients at level 1,
% corresponding to the node index 2.
det1 = wpcoef(tree,2);
sigma = median(abs(det1))/0.6745;

% Use wpbmpen for selecting global threshold
% for signal de-noising, using the recommended parameter.
alpha = 2;
thr = wpbmpen(tree,sigma,alpha)

thr =

    4.5740


% Use wpdencmp for de-noising the signal using the above
% threshold with soft thresholding and keeping the
% approximation.
keepapp = 1;
xd = wpdencmp(tree,'s','nobest',thr,keepapp);

% Plot original and de-noised signals.
figure(1)
subplot(211), plot(x),
title('Original signal')
subplot(212), plot(xd)
title('De-noised signal')
```



```
% Example 2: Image de-noising.
% Load original image.
load noiswom;
nbc = size(map,1);

% Perform a wavelet packet decomposition of the image
% at level 3 using coif2.
wname = 'coif2'; lev = 3;
tree = wpdec2(X,lev,wname);

% Estimate the noise standard deviation from the
% detail coefficients at level 1.
det1 = [wpcoef(tree,2) wpcoef(tree,3) wpcoef(tree,4)];
sigma = median(abs(det1(:)))/0.6745;

% Use wpbmpen for selecting global threshold
% for image de-noising.
alpha = 1.1;
thr = wpbmpen(tree,sigma,alpha)

thr =

   38.5125
```

```
% Use wpdencmp for de-noising the image using the above
% thresholds with soft thresholding and keeping the
% approximation.
keepapp = 1;
xd = wpdencmp(tree,'s','nobest',thr,keepapp);

% Plot original and de-noised images.
figure(2)
colormap(pink(nbc));
subplot(221), image(wcodemat(X,nbc))
title('Original image')
subplot(222), image(wcodemat(xd,nbc))
title('De-noised image')
```



## See Also

wbmpen | wden | wdencmp | wpdencmp

# wpcoef

Wavelet packet coefficients

## Syntax

```
X = wpcoef(T,N)
X = wpcoef(T)
```

## Description

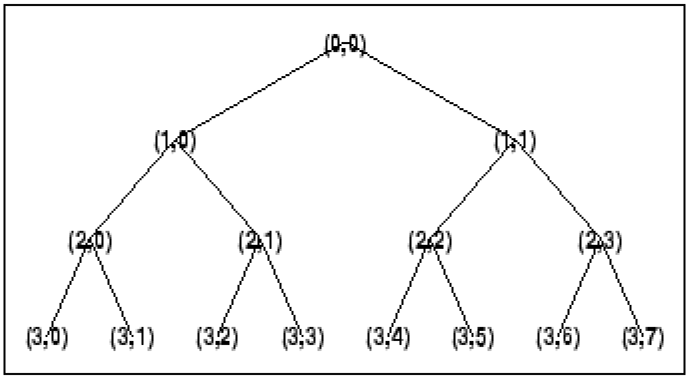
`wpcoef` is a one- or two-dimensional wavelet packet analysis function.

`X = wpcoef(T,N)` returns the coefficients associated with the node `N` of the wavelet packet tree *T*. If `N` doesn't exist, `X` = [ ];

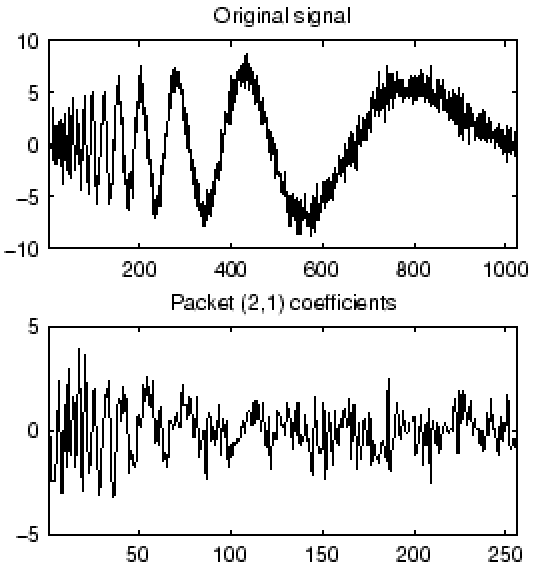
`X = wpcoef(T)` is equivalent to `X = wpcoef(T,0)`.

## Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load signal.
load noisdopp; x = noisdopp;

figure(1); subplot(211);
plot(x); title('Original signal');

% Decompose x at depth 3 with db1 wavelet packets
% using Shannon entropy.
wpt = wpdec(x,3,'db1');

% Plot wavelet packet tree wpt.
plot(wpt)
```



```
% Read packet (2,1) coefficients.
cfs = wpcoef(wpt,[2 1]);

figure(1); subplot(212);
plot(cfs); title('Packet (2,1) coefficients');
```



## More About

- "Reconstructing a Signal Approximation from a Node"

## See Also

`wpcoef` | `wpdec` | `wpdec2` | `wprcoef`

## wpcutree

Cut wavelet packet tree

### Syntax

```
T = wpcutree(T,L)
T
[T,RN] = wpcutree(T,L)
```

### Description

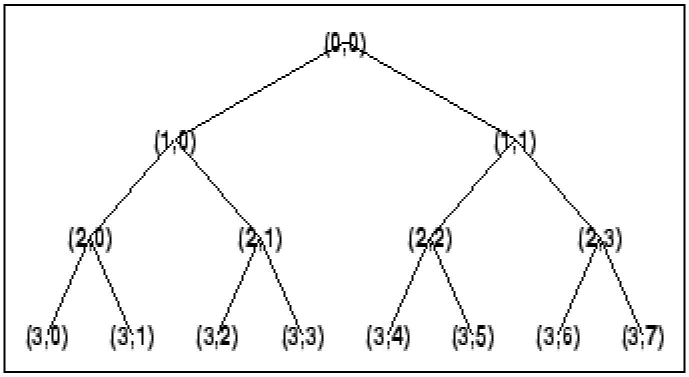
`wpcutree` is a one- or two-dimensional wavelet packet analysis function.

`T = wpcutree(T,L)` cuts the tree `T` at level `L`.

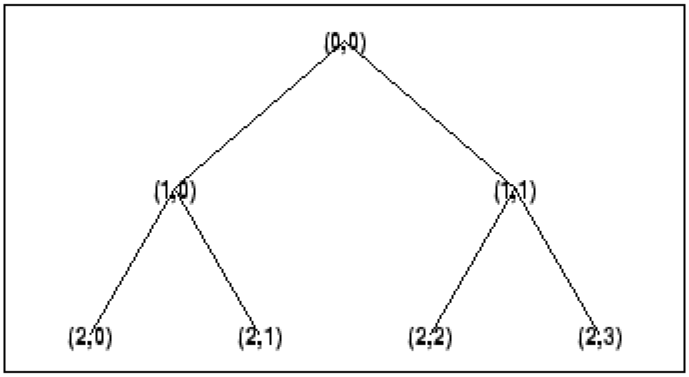
`[T,RN] = wpcutree(T,L)` returns the same arguments as above and, in addition, the vector `RN` contains the indices of the reconstructed nodes.

### Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load signal.
load noisdopp; x = noisdopp;

% Decompose x at depth 3 with db1 wavelet packets
% using Shannon entropy.
wpt = wpdec(x,3,'db1');

% Plot wavelet packet tree wpt.
plot(wpt)
```



```
% Cut wavelet packet tree at level 2.
nwpt = wpcutree(wpt,2);

% Plot new wavelet packet tree nwpt.
plot(nwpt)
```



## See Also

wpdec | wpdec2

# wpdec

Wavelet packet decomposition 1-D

## Syntax

```
T = wpdec(X,N,'wname',E,P)
T = wpdec(X,N,'wname')
T = wpdec(X,N,'wname','shannon')
T
```

## Description

`wpdec` is a one-dimensional wavelet packet analysis function.

`T = wpdec(X,N,'wname',E,P)` returns a wavelet packet tree *T* corresponding to the wavelet packet decomposition of the vector `X` at level `N`, with a particular wavelet (`'wname'`, see `wfilters` for more information).

`T = wpdec(X,N,'wname')` is equivalent to `T = wpdec(X,N,'wname','shannon')`.

`E` is a string containing the type of entropy and *P* is an optional parameter depending on the value of `T` (see `wentropy` for more information).

| Entropy Type Name (E) | Parameter (P) | Comments |
|---|---|---|
| `'shannon'` | | `P` is not used. |
| `'log energy'` | | `P` is not used. |
| `'threshold'` | $0 \le P$ | `P` is the threshold. |
| `'sure'` | $0 \le P$ | `P` is the threshold. |
| `'norm'` | $1 \le P$ | `P` is the power. |
| `'user'` | `string` | `P` is a string containing the file name of your own entropy function, with a single input `X`. |

| Entropy Type Name (E) | Parameter (P) | Comments |
|---|---|---|
| `FunName` | `No constraints on P` | `FunName` is any other string except those used for the previous Entropy Type Names listed above. `FunName` contains the file name of your own entropy function, with `X` as input and `P` as additional parameter to your entropy function. |

**Note** The `'user'` option is historical and still kept for compatibility, but it is obsoleted by the last option described in the table above. The `FunName` option do the same as the `'user'` option and in addition gives the possibility to pass a parameter to your own entropy function.

The wavelet packet method is a generalization of wavelet decomposition that offers a richer signal analysis. Wavelet packet atoms are waveforms indexed by three naturally interpreted parameters: position and scale as in wavelet decomposition, and frequency.

For a given orthogonal wavelet function, a library of wavelet packets bases is generated. Each of these bases offers a particular way of coding signals, preserving global energy and reconstructing exact features. The wavelet packets can then be used for numerous expansions of a given signal.

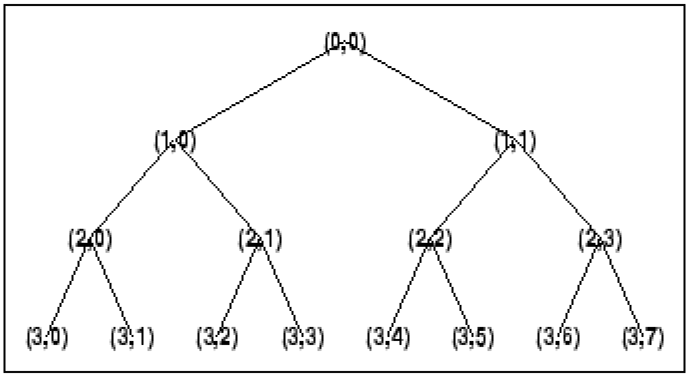
Simple and efficient algorithms exist for both wavelet packets decomposition and optimal decomposition selection. Adaptive filtering algorithms with direct applications in optimal signal coding and data compression can then be produced.

In the orthogonal wavelet decomposition procedure, the generic step splits the approximation coefficients into two parts. After splitting we obtain a vector of approximation coefficients and a vector of detail coefficients, both at a coarser scale. The information lost between two successive approximations is captured in the detail coefficients. The next step consists in splitting the new approximation coefficient vector; successive details are never re-analyzed.

In the corresponding wavelet packets situation, each detail coefficient vector is also decomposed into two parts using the same approach as in approximation vector splitting. This offers the richest analysis: the complete binary tree is produced in the one-dimensional case or a quaternary tree in the two-dimensional case.

## Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load signal.
load noisdopp; x = noisdopp;

% Decompose x at depth 3 with db1 wavelet packets
% using Shannon entropy.
wpt = wpdec(x,3,'db1','shannon');

% The result is the wavelet packet tree wpt.

% Plot wavelet packet tree (binary tree, or tree of order 2).
plot(wpt)
```



## More About

### Algorithms

The algorithm used for the wavelet packets decomposition follows the same line as the wavelet decomposition process (see `dwt` and `wavedec` for more information).

## References

Coifman, R.R.; M.V. Wickerhauser, (1992), “Entropy-based Algorithms for best basis selection,” *IEEE Trans. on Inf. Theory*, vol. 38, 2, pp. 713–718.

Meyer, Y. (1993), *Les ondelettes. Algorithmes et applications*, Colin Ed., Paris, 2nd edition. (English translation: *Wavelets: Algorithms and Applications*, SIAM).

Wickerhauser, M.V. (1991), “INRIA lectures on wavelet packet algorithms,” *Proceedings ondelettes et paquets d'ondes*, 17–21 June, Rocquencourt, France, pp. 31–99.

Wickerhauser, M.V. (1994), *Adapted wavelet analysis from theory to software algorithms*, A.K. Peters.

## See Also

`wavedec` | `waveinfo` | `wenergy` | `wpdec` | `wprec`

# wpdec2

Wavelet packet decomposition 2-D

## Syntax

```
T = wpdec2(X,N,'wname',E,P)
T = wpdec2(X,N,'wname')
T = wpdec2(X,N,wnam,'shannon')
```

## Description

`wpdec2` is a two-dimensional wavelet packet analysis function.

`T = wpdec2(X,N,'wname',E,P)` returns a wavelet packet tree `T` corresponding to the wavelet packet decomposition of the matrix `X`, at level `N`, with a particular wavelet (`'wname'`, see `wfilters` for more information).

`T = wpdec2(X,N,'wname')` is equivalent to `T = wpdec2(X,N,wnam,'shannon')`.

`E` is a string containing the type of entropy and `P` is an optional parameter depending on the value of `T` (see `wentropy` for more information).

| Entropy Type Name (E) | Parameter (P) | Comments |
|---|---|---|
| `'shannon'` | | `P` is not used. |
| `'log energy'` | | `P` is not used. |
| `'threshold'` | $0 \leq P$ | `P` is the threshold. |
| `'sure'` | $0 \leq P$ | `P` is the threshold. |
| `'norm'` | $1 \leq P$ | `P` is the power. |
| `'user'` | `string` | `P` is a string containing the file name of your own entropy function, with a single input `X`. |
| `STR` | `No constraints on P` | `STR` is any other string except those used for the previous Entropy Type Names listed above. |

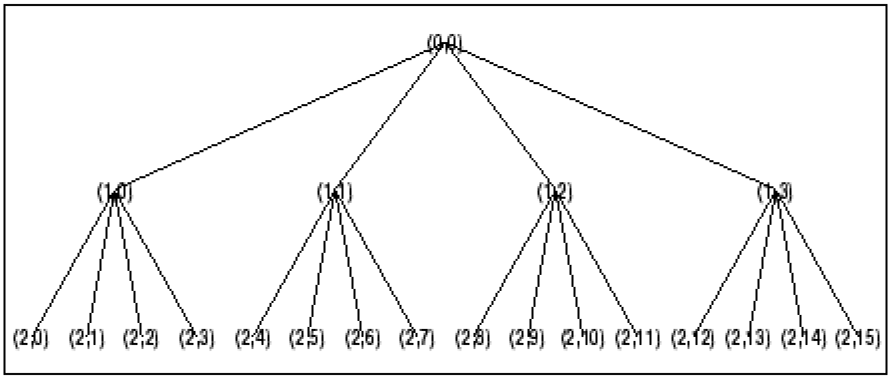
| Entropy Type Name (E) | Parameter (P) | Comments |
|---|---|---|
| | | STR contains the file name of your own entropy function, with X as input and P as additional parameter to your entropy function. |

**Note** The 'user' option is historical and still kept for compatibility, but it is obsoleted by the last option described in the preceding table. The FunName option does the same as the 'user' option and in addition, allows you to pass a parameter to your own entropy function.

See wpdec for a more complete description of the wavelet packet decomposition.

## Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load image.
load tire
% X contains the loaded image.

% For an image the decomposition is performed using:
t = wpdec2(X,2,'db1');
% The default entropy is shannon.

% Plot wavelet packet tree
% (quarternary tree, or tree of order 4).
plot(t)
```



## More About

### Tips

When X represents an indexed image, X is an `m`-by-`n` matrix. When X represents a truecolor image, it is an `m`-by-`n`-by-3 array, where each `m`-by-`n` matrix represents a red, green, or blue color plane concatenated along the third dimension.

For more information on image formats, see the `image` and `imfinfo` reference pages.

### Algorithms

The algorithm used for the wavelet packets decomposition follows the same line as the wavelet decomposition process (see `dwt2` and `wavedec2` for more information).

## References

Coifman, R.R.; M.V. Wickerhauser (1992), “Entropy-based algorithms for best basis selection,” *IEEE Trans. on Inf. Theory*, vol. 38, 2, pp. 713–718.

Meyer, Y. (1993), *Les ondelettes. Algorithmes et applications*, Colin Ed., Paris, 2nd edition. (English translation: *Wavelets: Algorithms and Applications*, SIAM).

Wickerhauser, M.V. (1991), “INRIA lectures on wavelet packet algorithms,” *Proceedings ondelettes et paquets d'ondes*, 17–21 June, Rocquencourt, France, pp. 31–99.

Wickerhauser, M.V. (1994), *Adapted wavelet analysis from theory to software Algorithms*, A.K. Peters.

## See Also

`wavedec2` | `waveinfo` | `wenergy` | `wpdec` | `wprec2`

# wpdencmp

De-noising or compression using wavelet packets

## Syntax

```
[XD,TREED,PERFO,PERFL2] =
wpdencmp(X,SORH,N,'wname',CRIT,PAR,KEEPAPP)
[XD,TREED,PERFO,PERFL2] = wpdencmp(TREE,SORH,CRIT,PAR,KEEPAPP)
```

## Description

`wpdencmp` is a one- or two-dimensional de-noising and compression oriented function.

`wpdencmp` performs a de-noising or compression process of a signal or an image, using wavelet packet. The ideas and the procedures for de-noising and compression using wavelet packet decomposition are the same as those used in the wavelets framework (see `wden` and `wdencmp` for more information).

`[XD,TREED,PERFO,PERFL2] =`
`wpdencmp(X,SORH,N,'wname',CRIT,PAR,KEEPAPP)` returns a de-noised or compressed version `XD` of input signal `X` (one- or two-dimensional) obtained by wavelet packets coefficients thresholding.

The additional output argument `TREED` is the wavelet packet best tree decomposition (see `besttree` for more information) of `XD`. `PERFL2` and `PERFO` are $L^2$ energy recovery and compression scores in percentages.

`PERFL2 = 100 *` (vector-norm of WP-cfs of `XD` / vector-norm of WP-cfs of `X`$^2$.

If `X` is a one-dimensional signal and `'wname'` an orthogonal wavelet, `PERFL2` is reduced to

$$\frac{100\|XD\|^2}{\|X\|^2}$$

`SORH` equal to `'s'` or `'h'` is for soft or hard thresholding (see `wthresh` for more information).

Wavelet packet decomposition is performed at level `N` and `'wname'` is a string containing the wavelet name. Best decomposition is performed using entropy criterion defined by string `CRIT` and parameter `PAR` (see `wentropy` for more information). Threshold parameter is also `PAR`. If `KEEPAPP = 1`, approximation coefficients cannot be thresholded; otherwise, they can be.

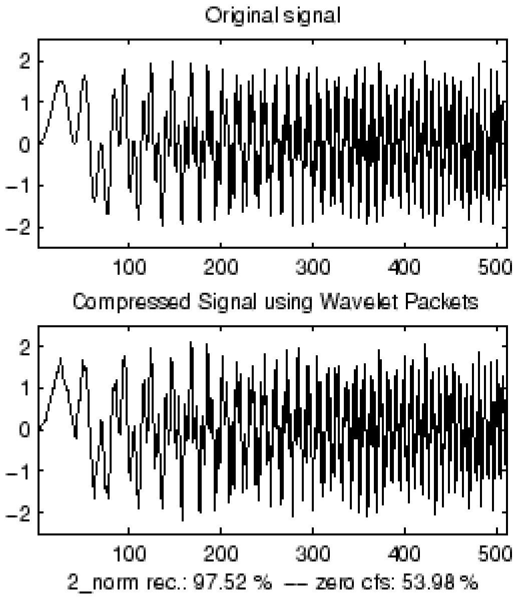
`[XD,TREED,PERF0,PERFL2] = wpdencmp(TREE,SORH,CRIT,PAR,KEEPAPP)` has the same output arguments, using the same options as above, but obtained directly from the input wavelet packet tree decomposition `TREE` (see `wpdec` for more information) of the signal to be de-noised or compressed.

In addition if `CRIT = 'nobest'` no optimization is done and the current decomposition is thresholded.

## Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load original signal.
load sumlichr; x = sumlichr;

% Use wpdencmp for signal compression.
% Find default values (see ddencmp).
[thr,sorh,keepapp,crit] = ddencmp('cmp','wp',x)

thr =
    0.5193

sorh =
h

keepapp =
    1

crit =
    threshold

% De-noise signal using global thresholding with
% threshold best basis.
```

```
[xc,wpt,perf0,perfl2] = ...
wpdencmp(x,sorh,3,'db2',crit,thr,keepapp);

% Using some plotting commands,
% the following figure is generated.
```



```
% Load original image.
load sinsin

% Generate noisy image.
x = X/18 + randn(size(X));

% Use wpdencmp for image de-noising.
% Find default values (see ddencmp).
[thr,sorh,keepapp,crit] = ddencmp('den','wp',x)

thr =
    4.9685

sorh =
h
keepapp =
    1
```

```
crit =
sure
% De-noise image using global thresholding with
% SURE best basis.
xd = wpdencmp(x,sorh,3,'sym4',crit,thr,keepapp);

% Using some plotting commands,
% the following figure is generated.

% Generate heavy sine and a noisy version of it.
init = 1000;
[xref,x] = wnoise(5,11,7,init);

% Use wpdencmp for signal de-noising.
n = length(x);
thr = sqrt(2*log(n*log(n)/log(2)));
xwpd = wpdencmp(x,'s',4,'sym4','sure',thr,1);

% Compare with wavelet-based de-noising result.
xwd = wden(x,'rigrsure','s','one',4,'sym4');
```

## References

Antoniadis, A.; G. Oppenheim, Eds. (1995), *Wavelets and statistics*, Lecture Notes in Statistics, 103, Springer Verlag.

Coifman, R.R.; M.V. Wickerhauser (1992), “Entropy-based algorithms for best basis selection,” *IEEE Trans. on Inf. Theory*, vol. 38, 2, pp. 713–718.

DeVore, R.A.; B. Jawerth, B.J. Lucier (1992), “Image compression through wavelet transform coding,” *IEEE Trans. on Inf. Theory*, vol. 38, No 2, pp. 719–746.

Donoho, D.L. (1993), “Progress in wavelet analysis and WVD: a ten minute tour,” in Progress in wavelet analysis and applications, Y. Meyer, S. Roques, pp. 109–128. Frontières Ed.

Donoho, D.L.; I.M. Johnstone (1994), “Ideal spatial adaptation by wavelet shrinkage,” *Biometrika*, vol. 81, pp. 425–455.

Donoho, D.L.; I.M. Johnstone, G. Kerkyacharian, D. Picard (1995), “Wavelet shrinkage: asymptopia,” *Jour. Roy. Stat. Soc.*, series B, vol. 57 no. 2, pp. 301–369.

## See Also

`besttree` | `ddencmp` | `wdencmp` | `wenergy` | `wpbmpen` | `wpdec` | `wpdec2` | `wthresh`

# wpfun

Wavelet packet functions

## Syntax

```
[WPWS,X] = wpfun('wname',NUM,PREC)
[WPWS,X] = wpfun('wname',NUM)
[WPWS,X] = wpfun('wname',NUM,7)
```

## Description

`wpfun` is a wavelet packet analysis function.

`[WPWS,X] = wpfun('wname',NUM,PREC)` computes the wavelet packets for a wavelet `'wname'` (see `wfilters` for more information), on dyadic intervals of length $2^{-PREC}$.

`PREC` must be a positive integer. Output matrix `WPWS` contains the $W$ functions of index from 0 to `NUM`, stored row-wise as `[` $W_0$; $W_1$; ... ; $W_{NUM}$ `]`. Output vector `X` is the corresponding common `X`-grid vector.

`[WPWS,X] = wpfun('wname',NUM)` is equivalent to
`[WPWS,X] = wpfun('wname',NUM,7)`.

The computation scheme for wavelet packets generation is easy when using an orthogonal wavelet. We start with the two filters of length $2N$, denoted $h(n)$ and $g(n)$, corresponding to the wavelet.

Now by induction let us define the following sequence of functions ($W_n(x)$ , $n$ = 0,1,2,...) by

$$W_{2n}(x) = \sqrt{2} \sum_{k=0,\dots,2N-1} h(k) W_n(2x-k)$$
$$W_{2n+1}(x) = \sqrt{2} \sum_{k=0,\dots,2N-1} g(k) W_n(2x-k)$$

where $W_0(x) = \phi\ (x)$ is the scaling function and $W_1(x) = \psi(x)$ is the wavelet function.

For example for the Haar wavelet we have

$$N = 1, h(0) = h(1) = \frac{1}{\sqrt{2}}$$

and

$$g(0) = -g(1) = \frac{1}{\sqrt{2}}$$

The equations become

$$W_{2n}(x) = W_n(2x) + W_n(2x-1)$$

and

$$(W_{2n+1}(x) = W_n(2x) - W_n(2x-1))$$

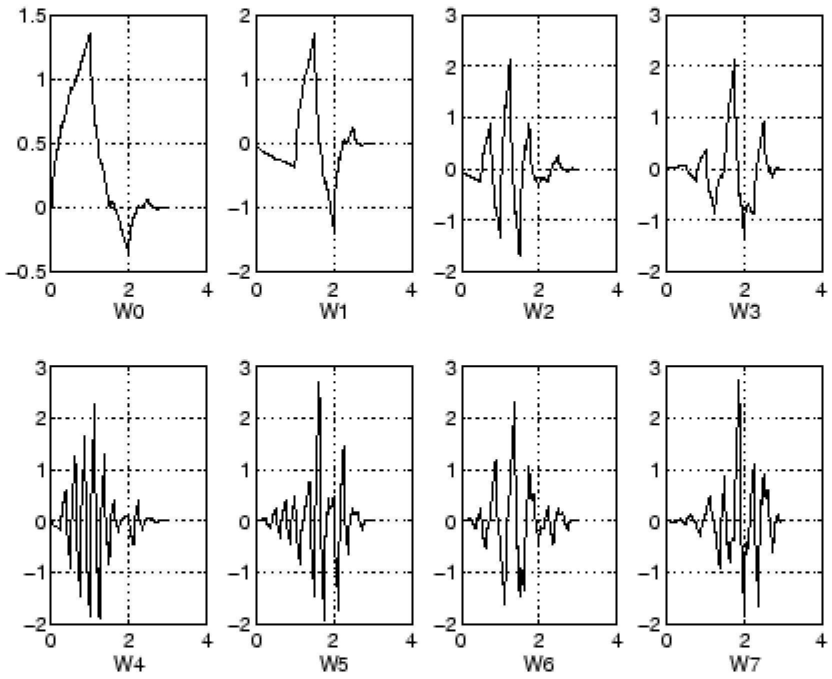
$W_0(x) = \phi(x)$ is the `haar` scaling function and $W_1(x) = \psi(x)$ is the `haar` wavelet, both supported in [0,1].

Then we can obtain $W_{2\,n}$ by adding two 1/2-scaled versions of $W_n$ with distinct supports [0,1/2] and [1/2,1], and obtain $W_{2n+1}$ by subtracting the same versions of $W_n$.

Starting from more regular original wavelets, using a similar construction, we obtain smoothed versions of this system of $W$-functions, all with support in the interval [0, 2$N$-1].

## Examples

```
% Compute the db2 Wn functions for n = 0 to 7, generating
% the db2 wavelet packets.
[wp,x] = wpfun('db2',7);

% Using some plotting commands,
% the following figure is generated.
```



## References

Coifman, R.R.; M.V. Wickerhauser (1992), "Entropy-based Algorithms for best basis selection," *IEEE Trans. on Inf. Theory*, vol. 38, 2, pp. 713–718.

Meyer, Y. (1993), *Les ondelettes. Algorithmes et applications*, Colin Ed., Paris, 2nd edition. (English translation: *Wavelets: Algorithms and applications*, SIAM).

Wickerhauser, M.V. (1991), "INRIA lectures on wavelet packet algorithms," *Proceedings ondelettes et paquets d'ondes*, 17–21 June, Rocquencourt, France, pp. 31–99.

Wickerhauser, M.V. (1994), *Adapted wavelet analysis from theory to software algorithms*, A.K. Peters.

## See Also

`wavefun` | `waveinfo`

# wpjoin

Recompose wavelet packet

## Syntax

```
T = wpjoin(T,N)
[T,X] = wpjoin(T,N)
T = wpjoin(T)
T = wpjoin(T,O)
[T,X] = wpjoin(T)
[T,X] = wpjoin(T,O)
```

## Description

`wpjoin` is a one- or two-dimensional wavelet packet analysis function.

`wpjoin` updates the wavelet packet tree after the recomposition of a node.

The nodes are numbered from left to right and from top to bottom. The root index is 0.

`T = wpjoin(T,N)` returns the modified wavelet packet tree `T` corresponding to a recomposition of the node `N`.

`[T,X] = wpjoin(T,N)` also returns the coefficients of the node.

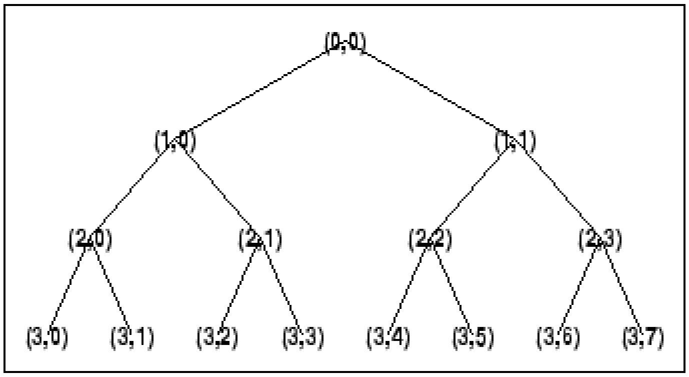
`T = wpjoin(T)` is equivalent to `T = wpjoin(T,O)`.

`[T,X] = wpjoin(T)` is equivalent to `[T,X] = wpjoin(T,O)`.

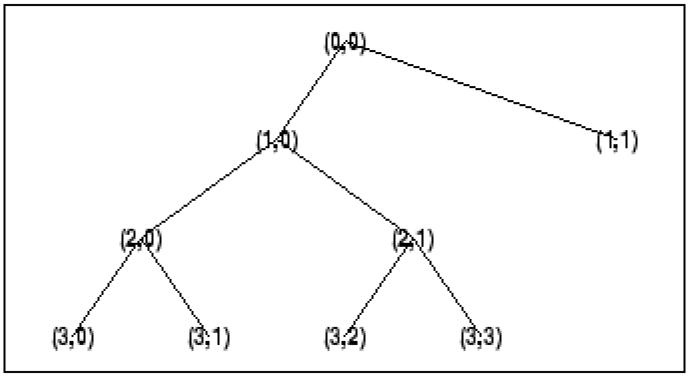
## Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load signal.
load noisdopp; x = noisdopp;
```

```
% Decompose x at depth 3 with db1 wavelet packets.
wpt = wpdec(x,3,'db1');

% Plot wavelet packet tree wpt.
plot(wpt)
```



```
% Recompose packet (1,1) or 2
wpt = wpjoin(wpt,[1 1]);

% Plot wavelet packet tree wpt.
plot(wpt)
```



## See Also

wpdec | wpdec2 | wpsplt

## wprcoef

Reconstruct wavelet packet coefficients

### Syntax

```
X = wprcoef(T,N)
X = wprcoef(T)
X = wprcoef(T,0)
```

### Description

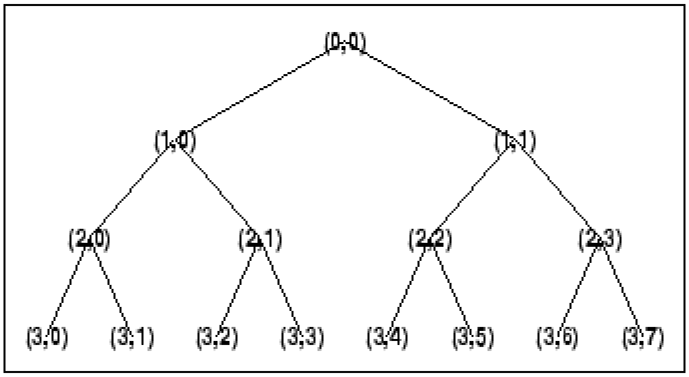
`wprcoef` is a one- or two-dimensional wavelet packet analysis function.

`X = wprcoef(T,N)` computes reconstructed coefficients of the node `N` of the wavelet packet tree *T*.

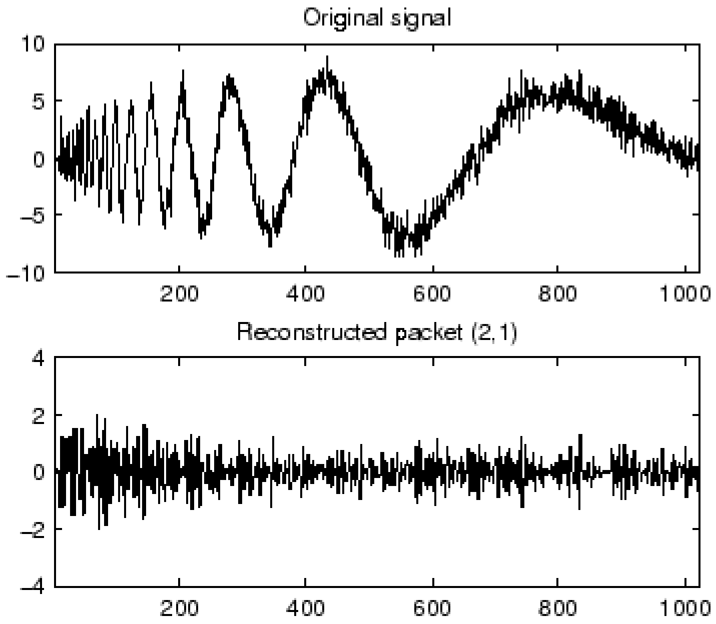
`X = wprcoef(T)` is equivalent to `X = wprcoef(T,0)`.

### Examples

```
% The current extension mode is zero-padding (see dwtmode)

% Load signal.
load noisdopp; x = noisdopp;

figure(1); subplot(211);
plot(x); title('Original signal');

% Decompose x at depth 3 with db1 wavelet packets
% using Shannon entropy.
t = wpdec(x,3,'db1','shannon');

% Plot wavelet packet tree.
plot(t)
```



```
% Reconstruct packet (2,1).
rcfs = wprcoef(t,[2 1]);

figure(1); subplot(212);
plot(rcfs); title('Reconstructed packet (2,1)');
```



## More About

- "Reconstructing a Signal Approximation from a Node"

## See Also

`wpdec` | `wpdec2` | `wprec` | `wprec2`

# wprec

Wavelet packet reconstruction 1-D

## Syntax

```
X = wprec(T)
wprec(wpdec(X,'wname'))
```

## Description

`wprec` is a one-dimensional wavelet packet analysis function.

`X = wprec(T)` returns the reconstructed vector `X` corresponding to a wavelet packet tree *`T`*.

`wprec` is the inverse function of `wpdec` in the sense that the abstract statement `wprec(wpdec(X,'wname'))` would give back `X`.

### See Also

`wpdec` | `wpdec2` | `wpjoin` | `wprec2` | `wpsplt`

## wprec2

Wavelet packet reconstruction 2-D

### Syntax

```
X = wprec2(T)
wprec2(wpdec2(X,'wname'))
```

### Description

`wprec2` is a two-dimensional wavelet packet analysis function.

`X = wprec2(T)` returns the reconstructed matrix `X` corresponding to a wavelet packet tree *T*.

`wprec2` is the inverse function of `wpdec2` in the sense that the abstract statement `wprec2(wpdec2(X,'wname'))` would give back `X`.

### More About

#### Tips

If *T* is obtained from an indexed image analysis or a truecolor image analysis, `X` is an `m`-by-`n` matrix or an `m`-by-`n`-by-3 array, respectively.

For more information on image formats, see the `image` and `imfinfo` reference pages.

### See Also

`wpdec` | `wpdec2` | `wprec` | `wpjoin` | `wpsplt`

# wpspectrum

Wavelet packet spectrum

## Syntax

```
[SPEC,TIMES,FREQ] = wpspectrum(WPT,Fs)
[...] = wpspectrum(WPT,Fs,'plot')
[...,TNFO] = wpspectrum(...)
```

## Description

`[SPEC,TIMES,FREQ] = wpspectrum(WPT,Fs)` returns a matrix of wavelet packet spectrum estimates, SPEC, for the binary wavelet packet tree object, WPT. Fs is the sampling frequency in Hertz. SPEC is a $2^J$-by-$N$ matrix where $J$ is the level of the wavelet packet transform and $N$ is the length of the time series. TIMES is a 1-by-$N$ vector of times and FREQ is a 1-by-$2^J$ vector of frequencies.

`[...] = wpspectrum(WPT,Fs,'plot')` displays the wavelet packet spectrum.

`[...,TNFO] = wpspectrum(...)` returns the terminal nodes of the wavelet packet tree in frequency order.

## Input Arguments

### `WPT`

WPT is a binary wavelet packet tree of class `wptree`.

### `Fs`

Sampling frequency in Hertz as a scalar of class double.

**Default:** 1

### `plot`

The string `'plot'` displays the wavelet packet spectrum. Enter `'plot'` after Fs to produce a plot of the wavelet packet spectrum.

## Output Arguments

**SPEC**

Wavelet packet spectrum. SPEC is a $2^J$-by-$N$ matrix where $J$ is the level of the wavelet packet transform and $N$ is the length of node 0 in the wavelet packet tree object.

The frequency spacing between the rows of SPEC is $Fs/2^{J+1}$.

**TIMES**

Time vector. TIMES is a vector of times in seconds equal in length to node 0 of the wavelet packet tree object. The time spacing between elements is $1/Fs$.

**FREQ**

Frequency vector. FREQ is a vector of frequencies of length $2^J$ where $J$ is the level of the wavelet packet tree object. The frequency spacing in FREQ is $Fs/2^{J+1}$.

**TNFO**

Terminal nodes. TNFO is a vector of the terminal nodes of the wavelet packet tree object in frequency order.

## Examples

Wavelet packet spectrum for signal consisting of two sinusoids with disjoint support:

```
fs = 500;
t = 0:1/fs:4;
y = sin(32*pi*t).*(t<2) + sin(128*pi*t).*(t>=2);
subplot(2,1,1);
plot(t,y);
axis tight
title('Analyzed Signal');

% Wavelet packet spectrum
level = 6;
wpt = wpdec(y,level,'sym6');
subplot(2,1,2);
```

```
[S,T,F] = wpspectrum(wpt,fs,'plot');
```

Wavelet packet spectrum of chirp:

```
fs = 1000;
t = 0:1/fs:2;
% create chirp signal
y = sin(256*pi*t.^2);

% Plot the analyzed signal
subplot(2,1,1);
plot(t,y);
axis tight
title('Analyzed Signal');

% Wavelet packet spectrum
level = 6;
wpt = wpdec(y,level,'sym8');
subplot(2,1,2);
[S,T,F] = wpspectrum(wpt,fs,'plot');
```

## More About

### Wavelet Packet Spectrum

The wavelet packet spectrum contains the absolute values of the coefficients from the frequency-ordered terminal nodes of the input binary wavelet packet tree. The terminal nodes provide the finest level of frequency resolution in the wavelet packet transform. If $J$ denotes the level of the wavelet packet transform and $Fs$ is the sampling frequency, the terminal nodes approximate bandpass filters of the form:

$$[\frac{nFs}{2^{J+1}},\frac{(n+1)Fs}{2^{J+1}}) \quad n = 0,1,2,3,\ldots 2^J - 1$$

At the terminal level of the wavelet packet tree, the transform divides the interval from 0 to the Nyquist frequency into bands of approximate width $Fs / 2^{J+1}$.

### Algorithms

`wpspectrum` computes the wavelet packet spectrum as follows:

- Extract the wavelet packet coefficients corresponding to the terminal nodes. Take the absolute value of the coefficients.
- Order the wavelet packet coefficients by frequency ordering.
- Determine the time extent on the original time axis corresponding to each wavelet packet coefficient. Repeat each wavelet packet coefficient to fill in the time gaps between neighboring wavelet packet coefficients and create a vector equal in length to node 0 of the wavelet packet tree object.

- “Wavelet Packet Spectrum”

## References

Wickerhauser, M.V. *Lectures on Wavelet Packet Algorithms*, Technical Report, Washington University, Department of Mathematics, 1992.

### See Also

`otnodes` | `wpdec`

# wpsplt

Split (decompose) wavelet packet

## Syntax

```
T = wpsplt(T,N)
[T,cA,cD] = wpsplt(T,N)
[T,cA,cH,cV,cD] = wpsplt(T,N)
```

## Description

`wpsplt` is a one- or two-dimensional wavelet packet analysis function.

`wpsplt` updates the wavelet packet tree after the decomposition of a node.

`T = wpsplt(T,N)` returns the modified wavelet packet tree *T* corresponding to the decomposition of the node `N`.

For a one-dimensional decomposition,

`[T,cA,cD] = wpsplt(T,N)` with `cA` = approximation and `cD` = detail of node `N`.

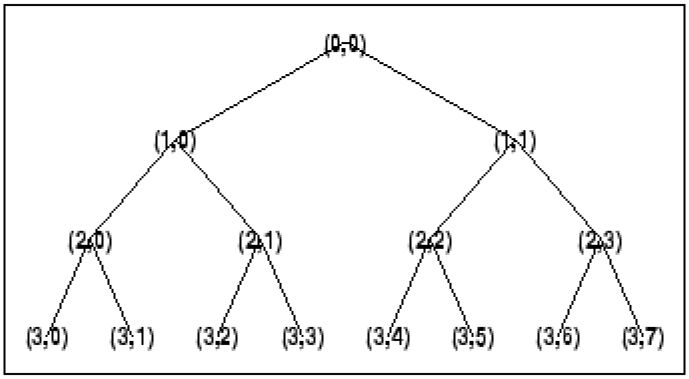
For a two-dimensional decomposition,

`[T,cA,cH,cV,cD] = wpsplt(T,N)` with `cA` = approximation and `cH,cV,c` = horizontal, vertical, and diagonal details of node `N`.

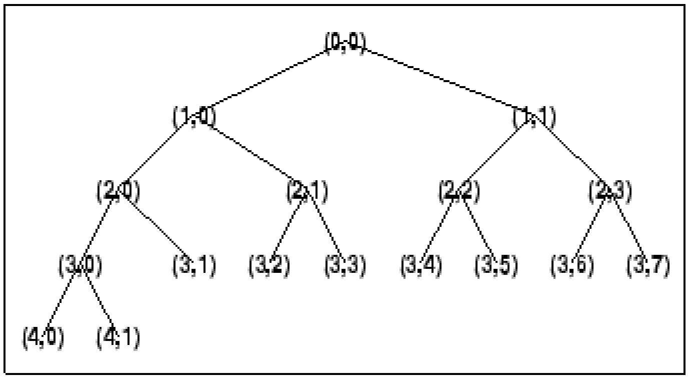
## Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load signal.
load noisdopp;
x = noisdopp;

% Decompose x at depth 3 with db1 wavelet packets.
```

```
wpt = wpdec(x,3,'db1');

% Plot wavelet packet tree wpt.
plot(wpt)
```



```
% Decompose packet (3,0).
wpt = wpsplt(wpt,[3 0]);
% or equivalently wpsplt(wpt,7).

% Plot wavelet packet tree wpt.
plot(wpt)
```



## See Also

wavedec | wpdec | wavedec2 | wpdec2 | wpjoin

# wpthcoef

Wavelet packet coefficients thresholding

## Syntax

```
NT = wpthcoef(T,KEEPAPP,SORH,THR)
```

## Description

`wpthcoef` is a one- or two-dimensional de-noising and compression utility.

`NT = wpthcoef(T,KEEPAPP,SORH,THR)` returns a new wavelet packet tree `NT` obtained from the wavelet packet tree `T` by coefficients thresholding.

If `KEEPAPP` = 1, approximation coefficients are not thresholded; otherwise, they can be thresholded.

If `SORH` = `'s'`, soft thresholding is applied; if `SORH = 'h'`, hard thresholding is applied (see `wthresh` for more information).

`THR` is the threshold value.

### See Also

`wpdec` | `wpdec2` | `wpdencmp` | `wthresh`

# wptree

WPTREE constructor

## Syntax

```
T = wptree(ORDER,DEPTH,X,WNAME,ENT_TYPE,PARAMETER)
T = wptree(ORDER,DEPTH,X,WNAME)
T = wptree(ORDER,DEPTH,X,WNAME,'shannon')
T = wptree(ORDER,DEPTH,X,WNAME,ENT_TYPE,ENT_PAR,USERDATA)
```

## Description

`T = wptree(ORDER,DEPTH,X,WNAME,ENT_TYPE,PARAMETER)` returns a complete wavelet packet tree `T`.

`ORDER` is an integer representing the order of the tree (the number of “children” of each non terminal node). `ORDER` must be equal to 2 or 4.

If `ORDER = 2`, `T` is a WPTREE object corresponding to a wavelet packet decomposition of the vector (signal) `X`, at level `DEPTH` with a particular wavelet `WNAME`.

If `ORDER = 4`, `T` is a WPTREE object corresponding to a wavelet packet decomposition of the matrix (image) `X`, at level `DEPTH` with a particular wavelet `WNAME`.

`ENT_TYPE` is a string containing the entropy type and `ENT_PAR` is an optional parameter used for entropy computation ( see `wentropy`, `wpdec`, or `wpdec2` for more information).

`T = wptree(ORDER,DEPTH,X,WNAME)` is equivalent to `T = wptree(ORDER,DEPTH,X,WNAME,'shannon')`

With `T = wptree(ORDER,DEPTH,X,WNAME,ENT_TYPE,ENT_PAR,USERDATA)` you may set a userdata field.

The function `wptree` returns a WPTREE object.

For more information on object fields, see the `get` function or type

```
help wptree/get
```

Class WPTREE (Parent class: DTREE)

## Fields

| | |
|---|---|
| `'dtree'` | DTREE parent object |
| `'wavInfo'` | Structure (wavelet information) |
| `'entInfo'` | Structure (entropy information) |

The wavelet information structure, `'wavInfo'`, contains

| | |
|---|---|
| `'wavName'` | Wavelet name |
| `'Lo_D'` | Low Decomposition filter |
| `'Hi_D'` | High Decomposition filter |
| `'Lo_R'` | Low Reconstruction filter |
| `'Hi_R'` | High Reconstruction filter |

The entropy information structure, `'entInfo'`, contains

| | |
|---|---|
| `'entName'` | Entropy name |
| `'entPar'` | Entropy parameter |

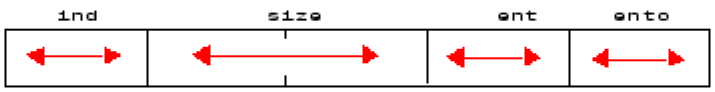
Fields from the DTREE parent object:

| | |
|---|---|
| 'allNI' | All nodes information |

`'allNI'` is an array of size `nbnode` by `5`, which contains

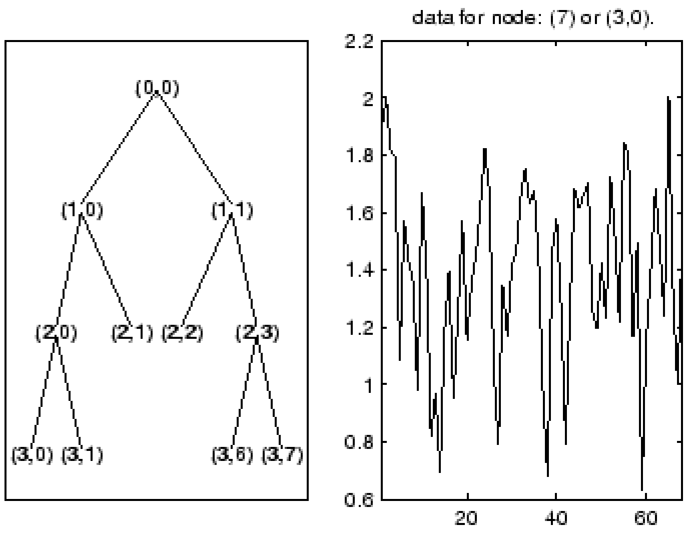
| | |
|---|---|
| `ind` | Index |
| `size` | Size of data |
| `ent` | Entropy |
| `ento` | Optimal entropy |

Each line is built based on the following scheme:



## Examples

```
% Create a wavelet packet tree.
x = rand(1,512);
t = wptree(2,3,x,'db3');
t = wpjoin(t,[4;5]);

% Plot tree t4.
plot(t);

% Click the node (3,0), (see the plot function).
```



## See Also

dtree | ntree

# wpviewcf

Plot wavelet packets colored coefficients

## Syntax

```
wpviewcf(T,CMODE)
wpviewcf(T,CMODE,NBCOL)
```

## Description

`wpviewcf(T,CMODE)` plots the colored coefficients for the terminal nodes of the tree *T*.

*T* is a wavelet packet tree and `CMODE` is an integer, which represents the color mode. The color modes are listed in the table below.

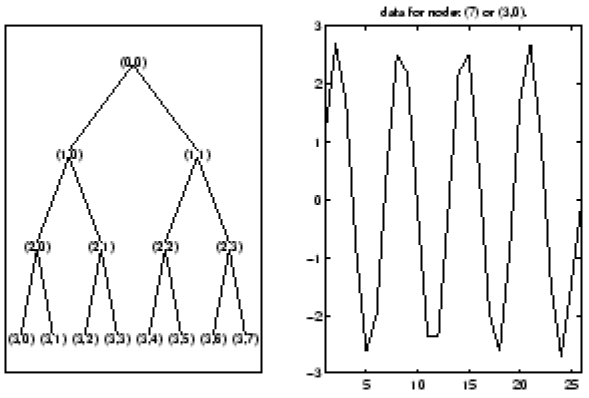
| Color Mode | Description |
|---|---|
| 1 | Frequency order – Global coloration – Absolute values |
| 2 | Frequency order – By level – Absolute values |
| 3 | Frequency order – Global coloration – Values |
| 4 | Frequency order – By level coloration – Values |
| 5 | Natural order – Global coloration – Absolute values |
| 6 | Natural order – By level – Absolute values |
| 7 | Natural order – Global coloration – Values |
| 8 | Natural order – By level coloration – Values |

`wpviewcf(T,CMODE,NBCOL)` uses `NBCOL` colors.

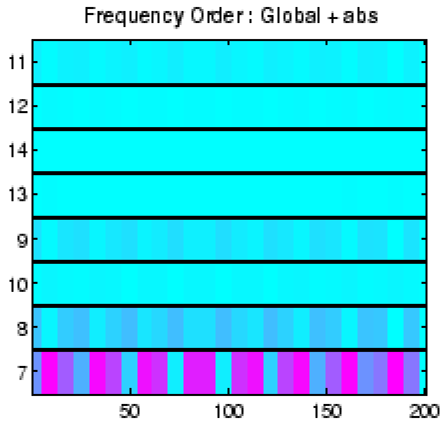
## Examples

```
% Create a wavelet packet tree.
x = sin(8*pi*[0:0.005:1]);
t = wpdec(x,3,'db1');
```

```
% Plot tree t.
% Click the node (3,0), (see the plot function)
plot(t);
```



```
% Plot the colored wavelet packet coefficients.
wpviewcf(t,1);
```



## See Also

`wpdec`

# wrcoef

Reconstruct single branch from 1-D wavelet coefficients

## Syntax

```
X = wrcoef('type',C,L,'wname',N)
X = wrcoef('type',C,L,Lo_R,Hi_R,N)
X = wrcoef('type',C,L,'wname')
X = wrcoef('type',C,L,Lo_R,Hi_R)
```

## Description

`wrcoef` reconstructs the coefficients of a one-dimensional signal, given a wavelet decomposition structure (`C` and `L`) and either a specified wavelet (`'wname'`, see `wfilters` for more information) or specified reconstruction filters (`Lo_R` and `Hi_R`).

`X = wrcoef('type',C,L,'wname',N)` computes the vector of reconstructed coefficients, based on the wavelet decomposition structure `[C,L]` (see `wavedec` for more information), at level `N`. `'wname'` is a string containing the wavelet name.

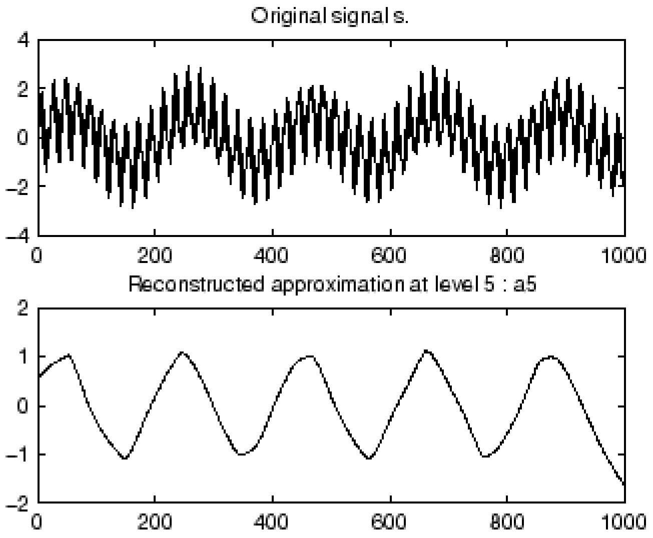
Argument `'type'` determines whether approximation (`'type' = 'a'`) or detail (`'type' = 'd'`) coefficients are reconstructed. When `'type' = 'a'`, `N` is allowed to be 0; otherwise, a strictly positive number `N` is required. Level `N` must be an integer such that `N` ≤ `length(L)-2`.

`X = wrcoef('type',C,L,Lo_R,Hi_R,N)` computes coefficients as above, given the reconstruction filters you specify.

`X = wrcoef('type',C,L,'wname')` and `X = wrcoef('type',C,L,Lo_R,Hi_R)` reconstruct coefficients of maximum level `N = length(L)-2`.

## Examples

```
% The current extension mode is zero-padding (see dwtmode).
% Load a one-dimensional signal.
load sumsin; s = sumsin;

% Perform decomposition at level 5 of s using sym4.
[c,l] = wavedec(s,5,'sym4');

% Reconstruct approximation at level 5,
% from the wavelet decomposition structure [c,l].
a5 = wrcoef('a',c,l,'sym4',5);

% Using some plotting commands,
% the following figure is generated.
```



## See Also

appcoef | detcoef | wavedec

# wrcoef2

Reconstruct single branch from 2-D wavelet coefficients

## Syntax

```
X = wrcoef2('type',C,S,'wname',N)
X = wrcoef2('type',C,S,Lo_R,Hi_R,N)
X = wrcoef2('type',C,S,'wname')
X = wrcoef2('type',C,S,Lo_R,Hi_R)
```

## Description

`wrcoef2` is a two-dimensional wavelet analysis function. `wrcoef2` reconstructs the coefficients of an image.

`X = wrcoef2('type',C,S,'wname',N)` computes the matrix of reconstructed coefficients of level `N`, based on the wavelet decomposition structure `[C,S]` (see `wavedec2` for more information).

`'wname'` is a string containing the name of the wavelet (see `wfilters` for more information). If `'type' = 'a'`, approximation coefficients are reconstructed; otherwise if `'type' = 'h'` (`'v'` or `'d'`, respectively), horizontal (vertical or diagonal, respectively) detail coefficients are reconstructed.

Level `N` must be an integer such that $0 \le N \le$ `size(S,1)-2` if `'type' = 'a'` and such that $1 \le N \le$ `size(S,1)-2` if `'type' = 'h'`, `'v'`, or `'d'`.

Instead of giving the wavelet name, you can give the filters.

For `X = wrcoef2('type',C,S,Lo_R,Hi_R,N)`, `Lo_R` is the reconstruction low-pass filter and `Hi_R` is the reconstruction high-pass filter.

`X = wrcoef2('type',C,S,'wname')` or `X = wrcoef2('type',C,S,Lo_R,Hi_R)` reconstruct coefficients of maximum level `N = size(S,1)-2`.

## Examples

```
% The current extension mode is zero-padding (see dwtmode).
% Load an image.
load woman;
% X contains the loaded image.

% Perform decomposition at level 2
% of X using sym5.
[c,s] = wavedec2(X,2,'sym5');

% Reconstruct approximations at
% levels 1 and 2, from the wavelet
% decomposition structure [c,s].
a1 = wrcoef2('a',c,s,'sym5',1);
a2 = wrcoef2('a',c,s,'sym5',2);

% Reconstruct details at level 2,
% from the wavelet decomposition
% structure [c,s].
% 'h' is for horizontal,
% 'v' is for vertical,
% 'd' is for diagonal.
hd2 = wrcoef2('h',c,s,'sym5',2);
vd2 = wrcoef2('v',c,s,'sym5',2);
dd2 = wrcoef2('d',c,s,'sym5',2);

% All these images are of same size sX.
sX = size(X)

sX =
 256 256

sa1 = size(a1)

sa1 =
 256 256

shd2 = size(hd2)

shd2 =
 256 256
```

## More About

### Tips

If `C` and `S` are obtained from an indexed image analysis (respectively a truecolor image analysis) then `X` is an `m`-by-`n` matrix (respectively an `m`-by-`n`-by-3 array).

For more information on image formats, see the reference pages of `image` and `imfinfo` functions.

## See Also

`appcoef2` | `detcoef2` | `wavedec2`

## wrev

Flip vector

### Syntax

```
Y = wrev(X)
```

### Description

`wrev` is a general utility.

`Y = wrev(X)` reverses the vector `X`.

### Examples

```
v = [1 2 3];
wrev(v)
wrev(v')
```

### See Also

`fliplr` | `flipud`

# write

Write values in WPTREE fields

## Syntax

```
T = write(T,'cfs',NODE,COEFS)
T = write(T,'cfs',N1,CFS1,'cfs',N2,CFS2, ...)
```

## Description

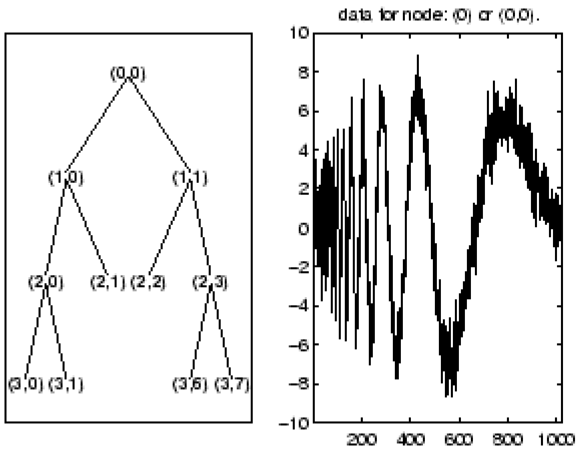
`T = write(T,'cfs',NODE,COEFS)` writes coefficients for the terminal node `NODE`.

`T = write(T,'cfs',N1,CFS1,'cfs',N2,CFS2, ...)` writes coefficients `CFS1, CFS2, ...` for the terminal nodes `N1, N2, ...`.

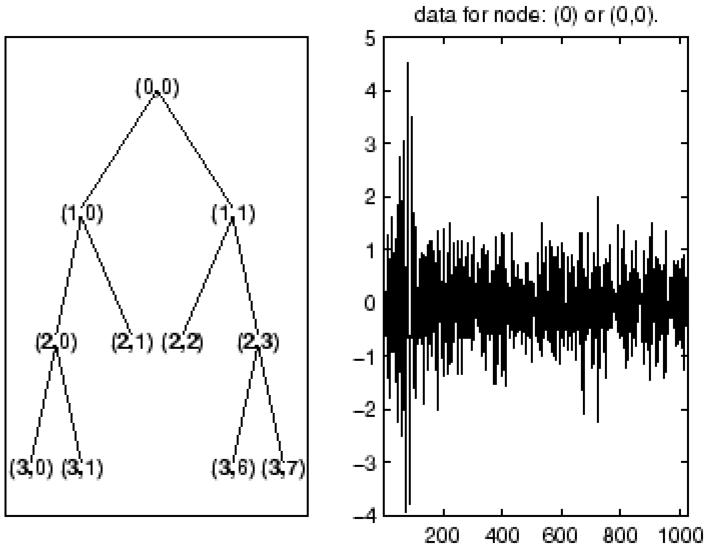
**Caution** The coefficients values must have the suitable size. You can use `S = read(T,'sizes',NODE)` or `S = read(T,'sizes',[N1;N2; ...])` in order to get those sizes.

## Examples

```
% Create a wavelet packet tree.
load noisdopp; x = noisdopp;
t = wpdec(x,3,'db3');
t = wpjoin(t,[4;5]);

% Plot tree t and click the node (0,0) (see the plot function).
plot(t);
```



```
% Write values.
sNod = read(t,'sizes',[4,5,7]);
cfs4 = zeros(sNod(1,:));
cfs5 = zeros(sNod(2,:));
cfs7 = zeros(sNod(3,:));
t = write(t,'cfs',4,cfs4,'cfs',5,cfs5,'cfs',7,cfs7);

% Plot tree t and click the node (0,0) (see the plot function).
plot(t)
```



## See Also

`disp` | `get` | `read` | `set`

# wscalogram

Scalogram for continuous wavelet transform

## Syntax

```
SC = wscalogram(TYPEPLOT,COEFS)
SC = wscalogram(TYPEPLOT,COEFS,'PropName1',PropVal1,...)
```

## Description

`SC = wscalogram(TYPEPLOT,COEFS)` computes the scalogram `SC` which represents the percentage of energy for each coefficient. `COEFS` is the matrix of the continuous wavelet coefficients (see `cwt`).

The scalogram is obtained by computing:

```
S = abs(coefs.*coefs); SC = 100*S./sum(S(:))
```

When `TYPEPLOT` is equal to `'image'`, a scaled image of scalogram is displayed. When `TYPEPLOT` is equal to `'contour'`, a contour representation of scalogram is displayed. Otherwise, the scalogram is returned without plot representation.

`SC = wscalogram(TYPEPLOT,COEFS,'PropName1',PropVal1,...)` allows you to modify some properties. The valid choices for `PropName` are:

| | |
|---|---|
| `'scales'` | Scales used for the CWT. |
| `'ydata'` | Signal used for the CWT. |
| `'xdata'` | *x* values corresponding to the signal values. |
| `'power'` | Positive real value. Default value is zero. |

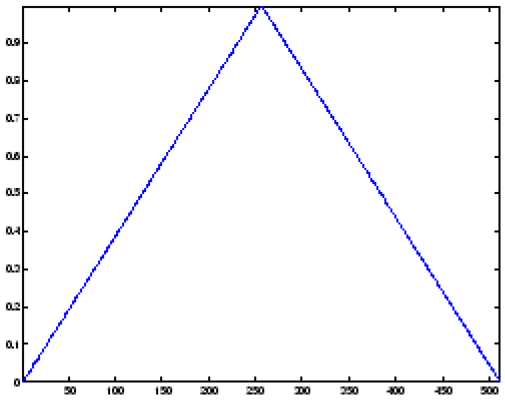
If `power > 0`, coefficients are first normalized

```
coefs(k,:) = coefs(k,:)/(scales(k)^power)
```

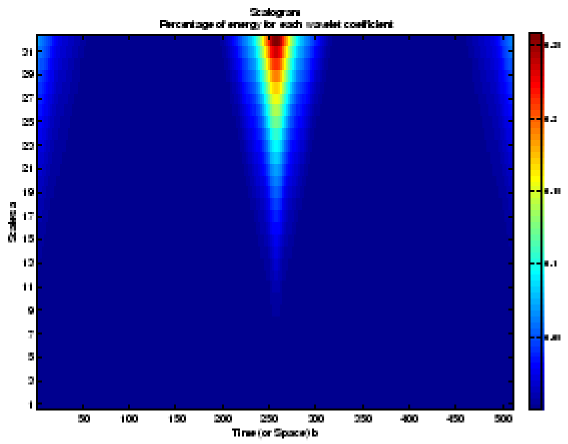
and then the scalogram is computed as explained above.

## Examples

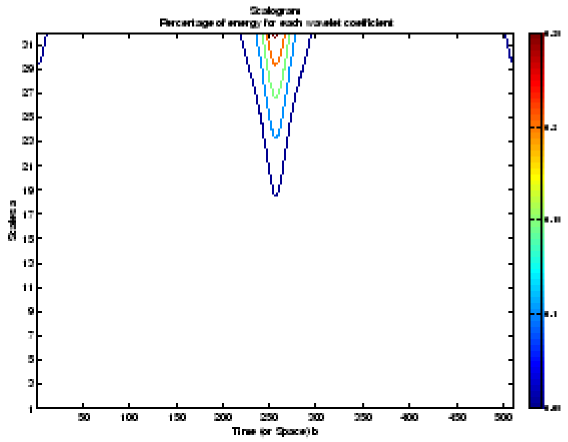
```
% Compute signal s
t = linspace(-1,1,512);
s = 1-abs(t);

% Plot signal s
figure;
plot(s), axis tight
```



```
% Compute coefficients COEFS using cwt
COEFS = cwt(s,1:32,'cgau4');

% Compute and plot the scalogram (image option)
figure;
SC = wscalogram('image',COEFS);
```



```
% Compute and plot the scalogram (contour option)
figure;
SC = wscalogram('contour',COEFS);
```



## See Also

`cwt`

# wtbo

WTBO constructor

## Syntax

```
OBJ = wtbo
OBJ = wtbo(USERDATA)
```

## Description

`OBJ = wtbo` returns a WTBO object. Any object in the Wavelet Toolbox software is parented by a WTBO object.

With `OBJ = wtbo(USERDATA)` you can set a userdata field.

Class WTBO (Parent class: none)

## Fields

| | |
|---|---|
| `wtboInfo` | Object information (not used in the current version of the toolbox) |
| `ud` | Userdata field |

## wtbxmngr

Wavelet Toolbox manager

### Syntax

```
wtbxmngr(OPTION)
V = wtbxmngr('version')
```

### Description

`wtbxmngr` or `wtbxmngr('version')` displays the current version of Wavelet Toolbox software.

`wtbxmngr(OPTION)` sets a toolbox option. Available options are

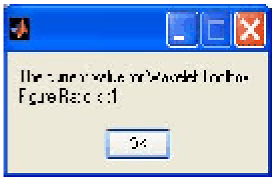
| Option | Description |
|---|---|
| `'LargeFonts'` | Sets the size of future-created figures to use large fonts. |
| `'DefaultSize'` | Restores the default figure size for future- created figures. |
| `'FigRatio'` | Returns the current figure ratio value. |
| `'FigRatio',ratio` | Changes the size of future-created figures by multiplying the default size by the specified ratio, where ratio must be between 0.75 and 1.25. |

`V = wtbxmngr('version')` saves the current version of the toolbox to variable `V`.

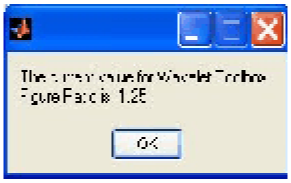
### Examples

```
wtbxmngr('version')

*************************************
**  Wavelet Toolbox Version: V3.1  **
*************************************
```

```
wtbxmngr('FigRatio')       % Display the current figure ratio
wtbxmngr('FigRatio',1.25) % Set the figure ratio to 1.25
wtbxmngr('FigRatio')       % Display the current figure ratio
wtbxmngr('DefaultSize')    % Return to the default figure ratio
```



The current value for Wavelet Toolbox
Figure Ratio is: 1



The current value for Wavelet Toolbox
Figure Ratio is: 1.25

# wthcoef

1-D wavelet coefficient thresholding

## Syntax

```
NC = wthcoef('d',C,L,N,P)
NC = wthcoef('d',C,L,N)
NC = wthcoef('a',C,L)
NC = wthcoef('t',C,L,N,T,SORH)
```

## Description

`wthcoef` thresholds wavelet coefficients for the denoising or compression of a 1-D signal.

`NC = wthcoef('d',C,L,N,P)` returns coefficients obtained from the wavelet decomposition structure `[C,L]` (see `wavedec` for more information), by rate compression defined in vectors `N` and `P`. `N` contains the detail levels to be compressed and `P` the corresponding percentages of lower coefficients to be set to zero. `N` and `P` must be of same length. Vector `N` must be such that `1` ≤ `N(i)` ≤ `length(L)-2`.

`NC = wthcoef('d',C,L,N)` returns coefficients obtained from `[C,L]` by setting all the coefficients of detail levels defined in `N` to zero.

`NC = wthcoef('a',C,L)` returns coefficients obtained by setting approximation coefficients to zero.

`NC = wthcoef('t',C,L,N,T,SORH)` returns coefficients obtained from the wavelet decomposition structure `[C,L]` by soft (if `SORH ='s'`) or hard (if `SORH ='h'`) thresholding (see `wthresh` for more information) defined in vectors `N` and `T`. `N` contains the detail levels to be thresholded and `T` the corresponding thresholds. `N` and `T` must be of the same length.

`[NC,L]` is the modified wavelet decomposition structure.

### See Also

`wavedec` | `wthresh`

# wthcoef2

Wavelet coefficient thresholding 2-D

## Syntax

```
NC = wthcoef2('type',C,S,N,T,SORH)
NC = wthcoef2('type',C,S,N)
NC = wthcoef2('a',C,S)
NC = wthcoef2('t',C,S,N,T,SORH)
```

## Description

`wthcoef2` is a two-dimensional de-noising and compression oriented function.

For `'type' = 'h'` ( `'v'` or `'d'`), `NC = wthcoef2('type',C,S,N,T,SORH)` returns the horizontal (vertical or diagonal, respectively) coefficients obtained from the wavelet decomposition structure `[C,S]` (see `wavedec2` for more information), by soft (if `SORH ='s'`) or hard (if `SORH ='h'`) thresholding defined in vectors `N` and `T`. `N` contains the detail levels to be thresholded and `T` the corresponding thresholds. `N` and `T` must be of the same length. The vector `N` must be such that `1 ≤ N(i) ≤ size(S,1)-2`.

For `'type' = 'h'` (`'v'` or `'d'`), `NC = wthcoef2('type',C,S,N)` returns the horizontal (vertical or diagonal, respectively) coefficients obtained from `[C,S]` by setting all the coefficients of detail levels defined in `N` to zero.

`NC = wthcoef2('a',C,S)` returns the coefficients obtained by setting approximation coefficients to zero.

`NC = wthcoef2('t',C,S,N,T,SORH)` returns the detail coefficients obtained from the wavelet decomposition structure `[C,S]` by soft (if `SORH ='s'`) or hard (if `SORH ='h'`) thresholding (see `wthresh` for more information) defined in vectors `N` and `T`. `N` contains the detail levels to be thresholded and `T` the corresponding thresholds which are applied in the three detail orientations. `N` and `T` must be of the same length.

`[NC,S]` is the modified wavelet decomposition structure.

## See Also

`wavedec2` | `wthresh`

# wthresh

Soft or hard thresholding

## Syntax

```
Y = wthresh(X,SORH,T)
Y = wthresh(X,'s',T)
Y = wthresh(X,'h',T)
```

## Description

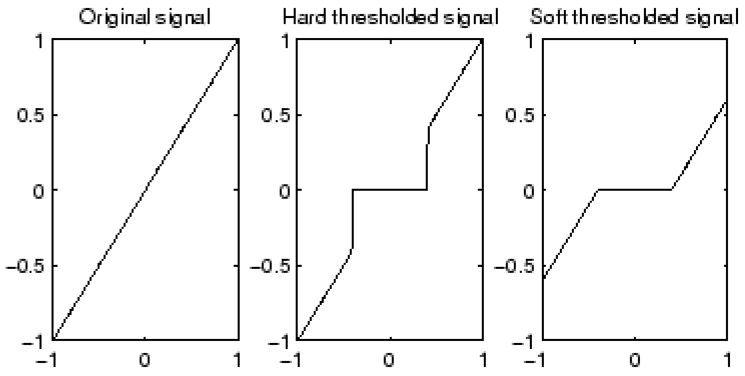
`Y = wthresh(X,SORH,T)` returns the soft (if `SORH = 's'`) or hard (if `SORH = 'h'`) thresholding of the input vector or matrix `X`. *T* is the threshold value.

`Y = wthresh(X,'s',T)` returns $Y = sign(X) \cdot (|X| - T)_+$, soft thresholding is wavelet shrinkage ( $(x)_+ = 0$ if $x < 0$; $(x)_+ = x$, if $x \geq 0$ ).

`Y = wthresh(X,'h',T)` returns $Y = X.1_{(|X| > T)}$, hard thresholding is cruder.

## Examples

```
% Generate signal and set threshold.
y = linspace(-1,1,100);
thr = 0.4;

% Perform hard thresholding.
ythard = wthresh(y,'h',thr);

% Perform soft thresholding.
ytsoft = wthresh(y,'s',thr);

% Using some plotting commands,
% the following figure is generated.
```



## See Also

`wden` | `wdencmp` | `wpdencmp`

# wthrmngr

Threshold settings manager

## Syntax

```
THR = wthrmngr(OPTION,METHOD,VARARGIN)
```

## Description

`THR = wthrmngr(OPTION,METHOD,VARARGIN)` returns a global threshold or level dependent thresholds depending on `OPTION`. The inputs, `VARARGIN`, depend on the `OPTION` and `METHOD` values.

This file returns the thresholds used throughout the Wavelet Toolbox software for de-noising and compression tools (command line files or GUI tools).

Valid options for the `METHOD` parameter are listed in the table below.

| `METHOD` | Description |
|---|---|
| `'scarcehi'` | See `wdcbm` or `wdcbm2` when used with `'high'` predefined value of parameter `M`. |
| `'scarceme'` | See `wdcbm` or `wdcbm2` when used with `'medium'` predefined value of parameter `M`. |
| `'scarcelo'` | See `wdcbm` or `wdcbm2` when used with `'low'` predefined value of parameter `M`. |
| `'sqtwolog'` | See `'sqtwolog'` option in `thselect`, and see also `wden`. |
| `'sqtwologuwn'` | See `'sqtwolog'` option in `thselect`, and see also `wden` when used with `'sln'` option. |
| `'sqtwologswn'` | See `'sqtwolog'` option in `thselect`, and see also `wden` when used with `'mln'` option. |
| `'rigsure'` | See `'rigsure'` option in `thselect`, and see also `wden`. |
| `'heursure'` | See `'heursure'` option in `thselect`, and see also `wden`. |
| `'minimaxi'` | See `'minimaxi'` option in `thselect`, and see also `wden`. |

| METHOD | Description |
|---|---|
| 'penalhi' | See wbmpen or wpbmpen when used with 'high' value of parameter ALPHA. |
| 'penalme' | See wbmpen or wpbmpen when used with 'medium' value of parameter ALPHA. |
| 'penallo' | See wbmpen or wpbmpen when used with 'low' value of parameter ALPHA. |
| 'rem_n0' | This option returns a threshold close to 0. A typical THR value is median(abs(coefficients)). |
| 'bal_sn' | This option returns a threshold such that the percentages of retained energy and number of zeros are the same. |
| 'sqrtbal_sn' | This option returns a threshold equal to the square root of the value such that the percentages of retained energy and number of zeros are the same. |

## Discrete Wavelet 1-D Options

For 1–D wavelet transforms, the expansion coefficients are in the vector C and the lengths of the expansion coefficient vectors are stored in L.

### Compression using a global threshold.

X is the signal to be compressed and [C,L] is the wavelet decomposition structure of the signal to be compressed.

```
THR = wthrmngr('dw1dcompGBL','rem_n0',X)
THR = wthrmngr('dw1dcompGBL','bal_sn',X)
```

### Compression using level dependent thresholds.

X is the signal to be compressed and [C,L] is the wavelet decomposition structure of the signal to be compressed.

ALFA is a sparsity parameter (see wdcbm for more information).

```
THR = wthrmngr('dw1dcompLVL','scarcehi',C,L,ALFA)
      ALFA must be such that 2.5 < ALFA < 10
THR = wthrmngr('dw1dcompLVL','scarceme',C,L,ALFA)
      ALFA must be such that 1.5 < ALFA < 2.5
```

```
THR = wthrmngr('dw1dcompLVL','scarcelo',C,L,ALFA)
      ALFA must be such that 1 < ALFA < 2
```

#### De-noising using level dependent thresholds.

[C,L] is the wavelet decomposition structure of the signal to be de-noised, SCAL defines the multiplicative threshold rescaling (see wden for more information) and ALFA is a sparsity parameter (see wbmpen for more information).

```
THR = wthrmngr('dw1ddenoLVL','sqtwolog',C,L,SCAL)
THR = wthrmngr('dw1ddenoLVL','rigrsure',C,L,SCAL)
THR = wthrmngr('dw1ddenoLVL','heursure',C,L,SCAL)
THR = wthrmngr('dw1ddenoLVL','minimaxi',C,L,SCAL)
THR = wthrmngr('dw1ddenoLVL','penalhi',C,L,ALFA)
      ALFA must be such that 2.5 < ALFA < 10
THR = wthrmngr('dw1ddenoLVL','penalme',C,L,ALFA)
      ALFA must be such that 1.5 < ALFA < 2.5
THR = wthrmngr('dw1ddenoLVL','penallo',C,L,ALFA)
      ALFA must be such that 1 < ALFA < 2
```

## Discrete Stationary Wavelet 1-D Options

#### De-noising using level dependent thresholds.

SWTDEC is the stationary wavelet decomposition structure of the signal to be de-noised, SCAL defines the multiplicative threshold rescaling (see wden for more information) and ALFA is a sparsity parameter (see wbmpen for more information).

```
THR = wthrmngr('sw1ddenoLVL',METHOD,SWTDEC,SCAL)
THR = wthrmngr('sw1ddenoLVL',METHOD,SWTDEC,ALFA)
```

The options for METHOD are the same as in the 'dw1ddenoLVL'case.

## Discrete Wavelet 2-D Options

For 2–D wavelet transforms, the expansion coefficients are in the vector C and the size of the coefficient matrices at each level is stored in S.

#### Compression using a global threshold.

X is the image to be compressed and [C,S] is the wavelet decomposition structure of the image to be compressed.

```
THR = wthrmngr('dw2dcompGBL','rem_n0',X)
THR = wthrmngr('dw2dcompGBL','bal_sn',C,S)
THR = wthrmngr('dw2dcompGBL','sqrtbal_sn',C,S)
```

#### Compression using level dependent thresholds.

X is the image to be compressed and [C,S] is the wavelet decomposition structure of the image to be compressed. ALFA is a sparsity parameter (see wdcbm2 for more information).

```
THR = wthrmngr('dw2dcompLVL','scarcehi',C,S,ALFA)
      ALFA must be such that 2.5 < ALFA < 10
THR = wthrmngr('dw2dcompLVL','scarceme',C,S,ALFA)
      ALFA must be such that 1.5 < ALFA < 2.5
THR = wthrmngr('dw2dcompLVL','scarcelo',C,S,ALFA)
      ALFA must be such that 1 < ALFA < 2
```

#### De-noising using level dependent thresholds.

[C,S] is the wavelet decomposition structure of the image to be de-noised, SCAL defines the multiplicative threshold rescaling (see wden for more information) and ALFA is a sparsity parameter (see wbmpen for more information).

```
THR = wthrmngr('dw2ddenoLVL','penalhi',C,S,ALFA)
      ALFA must be such that 2.5 < ALFA < 10
THR = wthrmngr('dw2ddenoLVL','penalme',C,S,ALFA)
      ALFA must be such that 1.5 < ALFA < 2.5
THR = wthrmngr('dw2ddenoLVL','penallo',C,S,ALFA)
      ALFA must be such that 1 < ALFA < 2
THR = wthrmngr('dw2ddenoLVL','sqtwolog',C,S,SCAL)
THR = wthrmngr('dw2ddenoLVL','sqrtbal_sn',C,S)
```

### Discrete Stationary Wavelet 2-D Options

#### De-noising using level dependent thresholds.

SWTDEC is the stationary wavelet decomposition structure of the image to be de-noised, SCAL defines the multiplicative threshold rescaling (see wden for more information) and ALFA is a sparsity parameter (see wbmpen for more information).

```
THR = wthrmngr('sw2ddenoLVL',METHOD,SWTDEC,SCAL)
THR = wthrmngr('sw2ddenoLVL',METHOD,SWTDEC,ALFA)
```

The options for METHOD are the same as in the 'dw2ddenoLVL' case.

## Discrete Wavelet Packet 1-D Options

### Compression using a global threshold.

X is the signal to be compressed and WPT is the wavelet packet decomposition structure of the signal to be compressed.

```
THR = wthrmngr('wp1dcompGBL','bal_sn',WPT)
THR = wthrmngr('wp1dcompGBL','rem_n0',X)
```

### De-noising using a global threshold.

WPT is the wavelet packet decomposition structure of the signal to be de-noised.

```
THR = wthrmngr('wp1ddenoGBL','sqtwologuwn',WPT)
THR = wthrmngr('wp1ddenoGBL','sqtwologswn',WPT)
THR = wthrmngr('wp1ddenoGBL','bal_sn',WPT)
THR = wthrmngr('wp1ddenoGBL','penalhi',WPT)
      see wbmpen with ALFA = 6.25
THR = wthrmngr('wp1ddenoGBL','penalme',WPT)
      see wbmpen with ALFA = 2
THR = wthrmngr('wp1ddenoGBL','penallo',WPT)
      see wbmpen with ALFA = 1.5
```

## Discrete Wavelet Packet 2-D Options

### Compression using a global threshold.

X is the image to be compressed and WPT is the wavelet packet decomposition structure of the image to be compressed.

```
THR = wthrmngr('wp2dcompGBL','bal_sn',WPT)
THR = wthrmngr('wp2dcompGBL','rem_n0',X)
THR = wthrmngr('wp2dcompGBL','sqrtbal_sn',WPT)
```

### De-noising using a global threshold.

WPT is the wavelet packet decomposition structure of the image to be de-noised.

```
THR = wthrmngr('wp2ddenoGBL','sqtwologuwn',WPT)
THR = wthrmngr('wp2ddenoGBL','sqtwologswn',WPT)
THR = wthrmngr('wp2ddenoGBL','sqrtbal_sn',WPT)
THR = wthrmngr('wp2ddenoGBL','penalhi',WPT)
      see wbmpen with ALFA = 6.25
THR = wthrmngr('wp2ddenoGBL','penalme',WPT)
      see wbmpen with ALFA = 2
THR = wthrmngr('wp2ddenoGBL','penallo',WPT)
      see wbmpen with ALFA = 1.5
```

## Examples

### Level-Independent Threshold — Stationary Wavelet Transform

This example uses a level-independent threshold based on the finest-scale wavelet coefficients to implement hard thresholding with the stationary wavelet transform.

Load the noisy blocks signal. Obtain the stationary wavelet transform down to level 5 using the Haar wavelet.

```
load noisbloc;
L = 5;
swc = swt(noisbloc,L,'db1');
```

Make a copy of the wavelet transform coefficients. Determine the Donoho-Johnstone universal threshold based on the first-level detail coefficients. Using the `'sln'` option, `wthrmngr` returns a 1-by-L vector with every element equal to the same value. Take the mean of the vector to obtain a scalar threshold.

```
swcnew = swc;
ThreshSL = mean(wthrmngr('sw1ddenoLVL','sqtwolog',swc,'sln'));
```

Use the universal threshold to implement hard thresholding. The same threshold is applied to the wavelet coefficients at every level.

```
for jj = 1:L
swcnew(jj,:) = wthresh(swc(jj,:),'h',ThreshSL);
end
```

Invert the stationary wavelet transform on the thresholded coefficients, `swcnew`. Plot the original signal and the denoised signal for comparison.

```
noisbloc_denoised = iswt(swcnew,'db1');
plot(noisbloc); hold on;
plot(noisbloc_denoised,'r','linewidth',2);
```

## Level-Dependent Threshold — Stationary Wavelet Transform

This example uses a level-dependent threshold derived from the wavelet coefficients at each scale to implement hard thresholding with the stationary wavelet transform.

Load the noisy blocks signal. Obtain the stationary wavelet transform down to level 5 using the Haar wavelet.

```
load noisbloc;
L = 5;
swc = swt(noisbloc,L,'db1');
```

Make a copy of the wavelet transform coefficients. Determine the Donoho-Johnstone universal threshold based on the detail coefficients for each scale. Using the `'mln'` option, `wthrmngr` returns a 1-by-L vector with each element of the vector equal to the universal threshold for the corresponding scale.

```
swcnew = swc;
ThreshML = wthrmngr('sw1ddenoLVL','sqtwolog',swc,'mln');
```

Use the universal thresholds to implement hard thresholding. The thresholds are applied in a scale-dependent manner.

```
for jj = 1:L
swcnew(jj,:) = wthresh(swc(jj,:),'h',ThreshML(jj));
end
```

Invert the stationary wavelet transform on the thresholded coefficients, `swcnew`. Plot the original signal and the denoised signal for comparison.

```
noisbloc_denoised = iswt(swcnew,'db1');
plot(noisbloc); hold on;
plot(noisbloc_denoised,'r','linewidth',2);
```

## Image Compression— Birgé-Massart Thresholds

This example compresses an image using the Birgé-Massart strategy.

Load the image and add white Gaussian noise.

```
load sinsin
```

```
x = X+18*randn(size(X));
```

Obtain the 2-D discrete wavelet transform down to level 2 using the Daubechies' least-asymmetric wavelet with 8 vanishing moments. Obtain the compression thresholds using the Birgé-Massart strategy with `alpha` equal to 2.

```
[C,L] = wavedec2(x,2,'sym8');
alpha = 2;
THR = wthrmngr('dw2dcompLVL','scarcehi',C,L,alpha);
```

Compress the image and display the result.

```
 Xd = wdencmp('lvd',X,'sym8',2,THR,'s');
image(X); title('Original image');
figure;
image(x); title('Noisy image');
figure;
image(Xd); title('Denoised image');
```

# wtreemgr

NTREE manager

## Syntax

## Description

`wtreemgr` is a tree management utility.

This function returns information on the tree *T* depending on the value of the `OPT` parameter.

Allowed values for `OPT` are listed in the table below.

| | |
|---|---|
| `'allnodes'` | Tree nodes |
| `'isnode'` | True for existing node |
| `'istnode'` | True for terminal nodes |
| `'nodeasc'` | Node ascendants |
| `'nodedesc'` | Node descendants |
| `'nodepar'` | Node parent |
| `'ntnode'` | Number of terminal nodes |
| `'tnodes'` | Terminal nodes |
| `'leaves'` | Terminal nodes |
| `'noleaves'` | Not terminal nodes |
| `'order'` | Tree order |
| `'depth'` | Tree depth |

## See Also

`allnodes` | | `istnode` | `leaves` | `nodeasc` | `nodedesc` | `nodepar` | `noleaves` | `ntnode` | `tnodes` | `treedpth` | `treeord`

# wvarchg

Find variance change points

## Syntax

```
[PTS_OPT,KOPT,T_EST] = wvarchg(Y,K,D)
```

## Description

`[PTS_OPT,KOPT,T_EST] = wvarchg(Y,K,D)` computes estimated variance change points for the signal `Y` for j change points, with j = 0, 1, 2, ... , `K`.

Integer `D` is the minimum delay between two change points.

Integer `KOPT` is the proposed number of change points (0 ≤ `KOPT` ≤ `K`). The vector `PTS_OPT` contains the corresponding change points.

For `1` ≤ `k` ≤ `K`, `T_EST(k+1,1:k)` contains the `k` instants of the variance change points and then, if `KOPT > 0`, `PTS_OPT = T_EST(KOPT+1,1:KOPT)` else `PTS_OPT = []`.

`K` and `D` must be integers such that 1 < `K` << length(`Y`) and 1 ≤ `D` << length(`Y`).

The signal `Y` should be zero mean.

`wvarchg(Y,K)` is equivalent to `wvarchg(Y,K,10)`.

`wvarchg(Y)` is equivalent to `wvarchg(Y,6,10)`.

## Examples

### Detect Variance Change Points

Add two variance change points to the blocks signal. Detect the variance change points using `wvarchg`.

Load the blocks signal. Add white noise with two variance change points located at index 180 and 600.

```
x = wnoise(1,10);
rng default;
bb = 1.5*randn(1,length(x));
cp1 = 180; cp2 = 600;
x = x + [bb(1:cp1),bb(cp1+1:cp2)/4,bb(cp2+1:end)];
```

Obtain the level-1 wavelet coefficients. Replace the top 2% of values with the mean value of the wavelet coefficients to remove all signal.

```
wname = 'db3'; lev = 1;
[c,l] = wavedec(x,lev,wname);
det = wrcoef('d',c,l,wname,1);
y = sort(abs(det));
v2p100 = y(fix(length(y)*0.98));
ind = find(abs(det)>v2p100);
det(ind) = mean(det);
```

Estimate the variance change points using the wavelet coefficients.

```
[pts_Opt,kopt,t_est] = wvarchg(det,5);
sprintf('The estimated change points are %d and %d\n',pts_Opt)
```

## References

Lavielle, M. (1999), “Detection of multiple changes in a sequence of dependent variables,” *Stoch. Proc. and their Applications*, 83, 2, pp. 79–102.